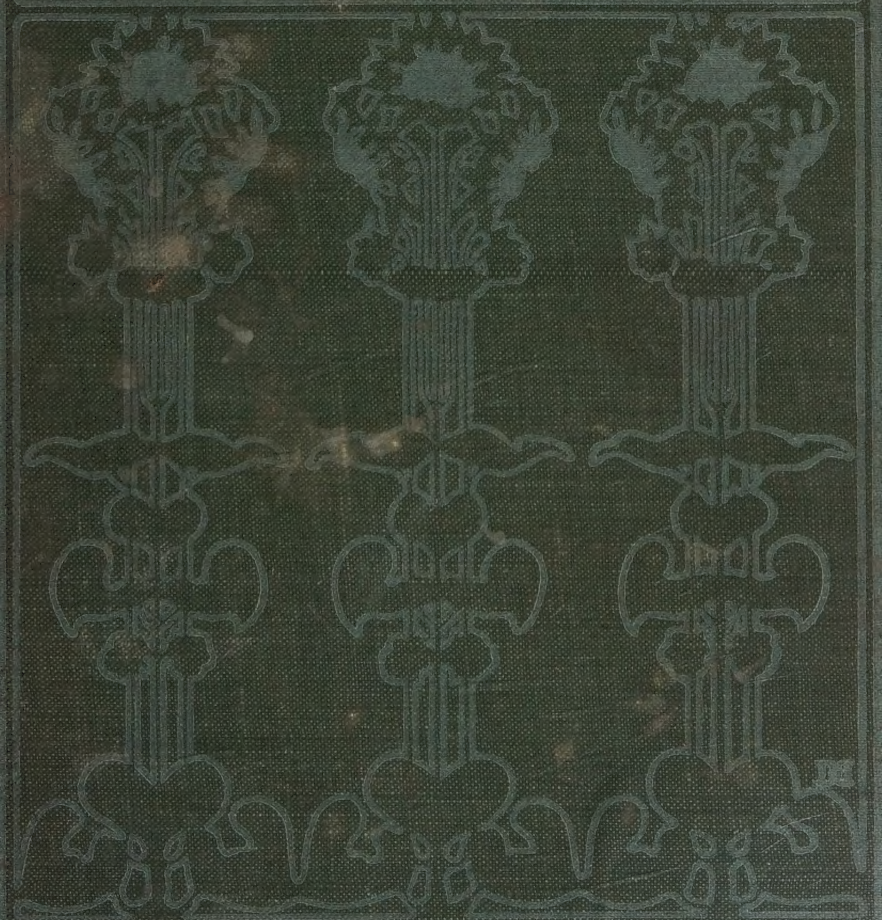
PRACTICAL ARITHMETIC



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PRACTICAL ARITHMETIC

BY

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PREFACE

The following have been controlling ideas in the preparation of this book:

1. In sequence of topics, to follow the plan adopted in the author's Primary Arithmetic, that of recognizing the value of the various courses of study in use in different parts of the country. Whatever originality may be demanded and legitimately shown in the preparation of a text-book, an author is bound to recognize the consensus of opinion as to topics and sequence. For example, modern courses invariably suggest the repetition of the most important portions of arithmetic from time to time, but they favor a somewhat exhaustive treatment of each subject whenever it is under discussion. The extreme spiral system, in which no topic is ever thoroughly treated at one time, but each is repeated until the pupil wearies of it, is psychologically too unwarranted to be considered seriously. On the other hand, the old-time plan of presenting important chapters but once is equally unscientific. Between these extremes lies the mean of the modern courses of study.

2. In arrangement by grades, to recognize the prevailing courses of study in the country, and to outline the work usually given in the fifth, sixth, seventh, and eighth school years, thus covering the work ordinarily taken beyond the primary grades. For schools that have no time for such an extended course, or that wish to introduce a considerable amount of work in algebra and elementary geometry, the author's Grammar School Arithmetic will be found to supply sufficient material.

3. In the selection of problems, to touch the actual life of this country at this time; to give correct ideas of the business customs of to-day; to embody the mathematical principles in interesting and instructive groups of problems; to touch the genuine interests of pupils in the story of our national resources and industries rather than to dwell upon the technicalities of minor trades in which they have no immediate or prospective concern; and to come in contact with human life rather than with those phases of science which are quite as foreign to the interests of boys and girls as are the mere abstract problems of numbers.

4. In the matter of abstract drill work, to recognize the fact that a large number of "problems without content" are necessary to concentrate the attention on the operations and to acquire the computing habit. The numbers selected have been those demanded by the conditions of the present day, the fractions and compound numbers being those in common use rather than those never met in business, and the integers being the ordinary ones of daily life. Very large numbers have generally been used only in such applied problems as represent the real conditions that the children meet in their geography, their elementary science, and their newspaper reading.

In fine, the book is written for the use of those teachers who wish to preserve the best that was in the old-style arithmetic, with its topical system and its abundant drill, while giving to it a modern arrangement and securing "mental discipline" through problems of to-day rather than through the tiresome, meaningless, unreal inheritances of the past.

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CONTENTS

CHAPTER I

I. REVIEW OF PRIMARY ARITHMETIC. COMMON FRACTIONS

	PAGE
Introduction	1
Notation and Numeration	2
Addition	6
Subtraction	10
Multiplication	14
Division	20
Factors and Multiples	30
Common Fractions	36
Decimal Fractions	76
Denominate Numbers	90

II. DECIMAL FRACTIONS AND PERCENTAGE

Addition and Subtraction of Decimals	105
Multiplication of Decimals	106
Division of Decimals	110
How to solve Problems	115
Pictures of Magnitudes	120
Bills and Receipts	125
Percentage	129
Discount	147
Review of Denominate Numbers	149
Measurements and Comparison	151

CHAPTER II

I. PERCENTAGE AND ITS APPLICATIONS. SIMPLE INTEREST. MEASUREMENTS

The Formal Solution of Problems	159
Percentage	181
Profit and Loss on Purchases	194
Commission	199

PAGE

INTEREST 202
CHECKS AND RECEIPTS 204
DENOMINATE NUMBERS REVIEWED 206

II. SIMPLE INTEREST. RATIO AND PROPORTION

SIMPLE INTEREST 211
RATIO 217
PROPORTION 223
MEASURES 228
GENERAL BUSINESS APPLICATIONS 242
MISCELLANEOUS PROBLEMS 254

CHAPTER III

I. A GENERAL REVIEW OF ARITHMETIC

WRITING NUMBERS 265
ADDITION REVIEWED 272
SUBTRACTION REVIEWED 276
MULTIPLICATION REVIEWED 279
DIVISION REVIEWED 284
GENERAL PRINCIPLES OF THE OPERATIONS 291
PRACTICAL SHORT METHODS 292
MEASURES 311

II. MEASURES. PERCENTAGES. PROPORTION

VOLUMES 329
LONGITUDE AND TIME 337
PERCENTAGE REVIEWED 346
SIMPLE INTEREST REVIEWED 361
RATIO AND PROPORTION REVIEWED 379
REVIEW PROBLEMS 390

CHAPTER IV

I. BUSINESS APPLICATIONS

GOING INTO BUSINESS 400
BANK ACCOUNTS 403
PARTIAL PAYMENTS 416
TRADE DISCOUNT 420
SIMPLE ACCOUNTS 424
PARTNERSHIP 426

PAGE

EXCHANGE 429
METRIC SYSTEM 440
TAXES 452
FIRE INSURANCE 463
MARINE INSURANCE 467
LIFE INSURANCE 468

II. BUSINESS ARITHMETIC CONTINUED. MENSURATION

CORPORATIONS 472
BUYING PRODUCE 481
INDUSTRIAL PROBLEMS 483
POWERS AND ROOTS 492
MENSURATION 510
MISCELLANEOUS PROBLEMS 528
TABLES FOR REFERENCE 543
DEFINITIONS 548

INDEX 557

PRACTICAL ARITHMETIC

CHAPTER I

I. REVIEW OF PRIMARY ARITHMETIC: COMMON FRACTIONS

INTRODUCTION

1. To the pupil. You have already learned much about Arithmetic. You have learned to count and to write numbers, at least to a million, to add, subtract, multiply, and divide integers, and to do some work with fractions and mixed numbers.

You have also learned how to write United States money and decimal fractions, how to measure things, and the ordinary tables of denominate numbers. Besides all this, you have seen that you have need for Arithmetic at the store, in your games, in reading, and in many other ways.

There is, however, much more to learn about the uses of Arithmetic, and some more to learn about how to work with numbers. Every one of us is interested in what the world is doing, and in our country and its products. We all like to know about our farms, our mines, our forests, our great industries, and the accounts that we must sometime keep, and we find much of interest about these matters in Arithmetic.

NOTATION AND NUMERATION

ORAL EXERCISE

1. What name do you give to 3 tens? to 10 tens?

2. What name do you give to 1000 thousands?

3. Looking at your fingers, do you see why people came to count as they do, by tens? Why was it?

4. Read aloud these numbers:

1,427	2,341	4,004	99,999	10,010
234,567	381,426	500,500	203,203	990,990
3,505,246	8,421,205	6,006,006	17,243,321	82,603,627

2. Notation. The writing of numbers by means of symbols is called *notation.*

3. Numeration. The naming of numbers is called *numeration.*

Our common notation started in India about 2000 years ago. The Arabs took it to Europe. Therefore these symbols are called *Arabic numerals.* You know that there are nine of these numerals besides the *zero,* which is also called *naught* or *cipher.*

4. Periods. When more than three figures are used in an integer they are separated into groups of three so as to be read more easily. The groups are called *periods.*

For example, 1,397,352, sometimes printed as 1 397 352.

5. Separatrix. The comma used in separating periods is often called a *separatrix.*

6. Place value. The value of a period and of a figure depends upon the place it occupies. Therefore each has a *place value.*

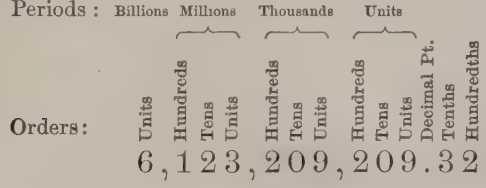
Such definitions as the above need not be memorized. The teacher should, however, be sure that the terms and the ideas which they suggest are understood.

ORAL EXERCISE

1. In the number 234, what place is occupied by the 4? by the 3? by the 2?

2. In the number 123,456,789, what name is given to the period 789? to the period 456? to the period 123? to the whole number?

7. **Reading numbers.** In a number like 6,123,209,209.32 we speak of the three places in each period as orders.



Teachers should require pupils to read numbers accurately.

100.023 is read one hundred and twenty-three thousandths,

.123 " " one hundred twenty-three thousandths,

123 " " one hundred twenty-three.

Number names beyond billions are too rarely used to be taught.

Omit the following exercise with classes not ready for the multiplications.

WRITTEN EXERCISE

Let us see what a large number like a million means:

1. It is 1000 mi. from Chicago to New York. How many feet does this equal?

2. How many seconds in 1 min.? in 1 hr.? in 1 da.? in 11 da.? Have you a million seconds yet?

3. How many hours in 1 yr.? in 100 yr.? Have you a million hours yet?

4. If there are 25 pupils in your room, how many rooms would be needed for 100 pupils? for 1000? for 1,000,000?

Some Uses for Large Numbers

ORAL EXERCISE

Read the following aloud:

1. The number of different books published in a certain year in the United States was 8141, in England 6044, in France 13,053, and in Germany 25,331.

2. In the same year the value of the wheat produced in the United States was $467,350,156; of corn, $921,555,768; of oats, $293,658,777.

3. In a certain year the United States coined $99,272,943 worth of gold, Great Britain $63,769,609, and Russia $83,221,525.

4. In the common schools of our country there were in a certain year 15,341,220 pupils, and the average number present each day was 10,513,518. To teach them required 421,288 teachers.

5. From 1800 to 1900 the population of New York City increased from 60,489 to 3,437,202; that of Philadelphia, from 41,220 to 1,293,697; that of Boston, from 24,937 to 560,892; and that of Chicago, from nothing to 1,698,575.

6. Suppose Germany keeps an army of 576,666 men; France, 545,962; Great Britain and India, 347,042; Russia, 1,049,003, while the United States has only about 50,000. Can you explain this great difference?

7. Our country mined in one year 228,717,579 tons of coal. Great Britain mined 223,616,279 tons; Germany, 135,844,419 tons; France, 32,862,712 tons; Russia, 12,800,000 tons; Canada, 4,142,242 tons; Mexico, 409,125 tons; and the rest of the world mined enough to bring the total to 723,617,836 tons.

WRITTEN EXERCISE

Copy the following, writing the numbers in Arabic figures:

1. In Alabama there were at one time five thousand, six hundred two factories, employing fifty-two thousand, nine hundred two persons.

2. In California there were two hundred five million, three hundred ninety-five thousand, twenty-five dollars invested in manufacturing, and the value of the product was three hundred two million, eight hundred seventy-four thousand, seven hundred sixty-one dollars a year.

3. New York has the largest amount of manufacturing of any of our states. It employed recently eight hundred forty-nine thousand wage earners and produced two billion, one hundred seventy-five million, seven hundred sixty-six thousand, nine hundred dollars' worth of goods a year.

4. The expenses of our government are very great. In eighteen ninety-seven they were three hundred two million, seven hundred eighty-six thousand, three hundred eighty-six dollars. In eighteen ninety-nine they were six hundred seventy-three million, fifty thousand, two hundred ninety-three dollars. They are now probably between four and five hundred million dollars a year.

5. One of the great industries in the United States is supplying the people with meat. The value of the products was recently seven hundred eighty-six million, six hundred three thousand, six hundred seventy dollars in one year. Next to that comes the iron industry. It recently produced five hundred ninety-six million, five hundred eighty-eight thousand, thirty-four dollars' worth of goods in one year.

ADDITION

ORAL EXERCISE

1. State rapidly two one-figure numbers whose sum is 11; also two whose sum is 12; 13; 14; 15; 16; 17; 18.

2. Write upon the blackboard the series 7, 0, 8, 6, 9, 5, 4, 2, 1, 3, and read the numbers, each increased by 2; by 7.

Much drill work of this kind should be given. A short time daily devoted to such oral work is wisely spent.

8. Addition. The process of finding a number that equals two or more other numbers is called *addition*.

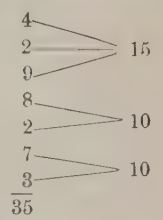
9. Addends. Numbers to be added are called *addends*.

10. Sum. The result in addition is called the *sum*.

11. *Only numbers considered as like numbers can be added.*

For example, \$3 + \$5 = \$8. But 3 turnips + 5 potatoes are neither turnips nor potatoes, although if we consider them as vegetables we may say, "3 vegetables + 5 vegetables are 8 vegetables."

12. Remember to read a column like a word. Try to see the 10's, or the 5's and 15's. In this column you should see the two 10's and the 15, and write the answer, 35. It is, however, not wise to skip numbers in order to group by 10's.



13. Checks. To *check* or *prove* your work, read the column from the top down, if you first read it upwards.

WRITTEN EXERCISE

1.	2.	3.	4.	5.
3,288	2,287	33,042	317,675	768,265
13,590	9,954	128,600	1,240,443	1,838,921
4,057	5,320	41,584	292,248	1,159,356
5,841	1,386	65,936	533,502	662,973

Our Country's Extent and Resources

WRITTEN EXERCISE

1. The census at the beginning of this century showed 65,843,302 native and 10,460,085 foreign-born persons in the United States. What was the total population?

2. At that time the population of our three largest cities was as follows: New York, 3,437,202; Chicago, 1,698,575; Philadelphia, 1,293,697. What was their total population?

3. We sold $1,487,764,991 worth of goods and produce to foreign countries in 1901; $1,381,719,401 in 1902; and $1,420,141,679 in 1903. What was the total?

4. The amount of gold in the United States in a certain year was $1,174,600,000; of silver, $660,000,000; and of paper money, $437,800,000. What is the total amount of money in the country?

5. The area of the United States is 3,602,990 sq. mi.; of the Philippine Islands, 143,000 sq. mi.; of Porto Rico, 3600 sq. mi.; of Hawaii, 6740 sq. mi.; and of other islands, 554 sq. mi. What is the total area?

6. The four countries producing the most wheat at the opening of this century were as follows: United States, 756,269,573 bushels a year; Russia, 427,780,477 bushels; France, 307,388,463 bushels; and India, 248,593,946 bushels. How much did all four produce?

7. The number of children in our elementary public schools in a certain year was 14,662,488, and in private schools 1,193,882; the number of pupils in public high schools was 488,549, and in private ones 166,679. There were also 103,251 in colleges and universities, and 123,514 in professional schools. Required the total.

8. Our country produced 302,502,328 lb. of wool in 1901, 316,341,032 lb. in 1902, 287,450,000 lb. in 1903. How many pounds did it produce in these three years?

9. The amount of coal mined in this country in 1900 was 225,759,980 tons of soft coal and 67,538,536 tons of hard coal. How many tons were there of both?

10. Of the world's cotton supply in 1903, America produced 10,877,000 bales, the East Indies 1,050,000 bales, and other countries 1,230,000 bales. What was the total production for that year?

11. The amount of money deposited in our savings banks in a certain year was as follows: New England states, $1,000,175,255; other eastern states, $1,308,940,078; southern states, $16,530,046; central states, $257,491,072; other states, $180,438,675. Required the total amount.

12. Of our 200,000 miles of railroad, the greatest mileage for any state is in Illinois, which had not long ago 11,119 mi. Next in amount of mileage was Pennsylvania with 10,538 mi., and then came Texas with 10,189 mi. What was the total mileage of these three states?

13. The number of bushels of grain grown in the United States in a recent year was as follows: corn, 2,105,102,516 bu.; wheat, 756,269,573 bu.; oats, 800,125,989 bu.; barley, 58,925,833 bu.; rye, 23,995,927 bu.; and buckwheat, 9,566,966 bu., each year. Required the total.

14. The annual income of the colleges and universities in various parts of the country was recently as follows: North Atlantic division, $9,187,016; South Atlantic division, $2,015,910; South Central division, $2,061,615; North Central division, $7,865,045; Western division, $1,659,468. What was the total for all these divisions?

14. If the column is very long, business men often write on a separate slip of paper the partial sums as here shown, so as to check mistakes more easily. Whether you do this or not, always check your work by adding again in the opposite direction.

$2.35
3.46
9.87
7.03
21
15
21
$22.71

The following exercise is intended for rapid drill work. To save the time of copying, teachers may prefer to have the pupils place a slip of paper below the line and write the sum on that.

WRITTEN EXERCISE

Add the following:

1.	2.	3.	4.	5.
124,483	69,869	316,499	15,755	241,400
438,432	22,410	202,991	22,328	162,393
179,136	61,830	412,279	44,151	332,823
158,912	50,675	773,703	85,564	620,923
173,188	76,953	3,524,102	335,653	3,188,448
98,251	18,263	2,452,542	267,405	1,952,034
27,341	62,483	73,192	134,207	412,375
41,273	21,276	147,628	29,632	208,607

6.	7.	8.	9.	10.
257	1,094	40,936	12,665	221,295
1,102	10,048	419,216	114,760	226,821
184	1,831	70,206	24,079	418,496
334	2,546	87,261	27,388	507,254
355	1,431	65,563	17,380	493,263
604	1,433	52,647	15,037	257,770
253	2,508	71,038	28,446	201,619
429	2,479	76,987	36,461	578,298
415	1,195	64,883	11,270	424,853
315	1,503	65,109	20,483	434,017
459	3,488	111,778	29,918	573,200

SUBTRACTION

ORAL EXERCISE

1. What number added to the lower of these numbers makes the upper one?

7	8	10	20	25	47	92
4	3	4	16	16	28	76

2. If you hand a merchant \$1, how much change should you get if you owe him 75 ct.? 60 ct.? 25 ct.? 88 ct.?

Throughout the course there should be much oral drill on such problems as the above. It is expeditious, in the case of 75−48, to think: "48 and 2 are 50, and 25 more are 75; 2 + 25 = 27."

75
48
27

15. Difference. The number which added to one number makes another is called the *difference* between them.

That is, the difference between 75 and 48 is 27, because 48 + 27 = 75.

16. Subtraction. The operation of finding the difference between two numbers is called *subtraction.*

Subtraction may also be defined as the operation of taking one number from another, and the result may be defined as the remainder or difference. But since a great deal of subtraction is "making change," where the work is based on addition, the definitions first given are better.

17. Subtrahend. The number to which the difference is added, or the number which is subtracted, is called the *subtrahend.*

18. Minuend. The sum of the difference and subtrahend, or the number from which we subtract, is called the *minuend.*

563 minuend
127 subtrahend
436 difference or remainder

19. The process. In subtracting 756 from 1632,

We write only this:	*But this assists in explaining:*
1632	$1632 = 1500 + 120 + 12$
756	$756 = \ 700 + \ 50 + \ 6$
876	$800 + \ 70 + \ 6 = 876$

We cannot subtract 6 from 2, but $12 - 6 = 6$. We have now used 10 of the 30, leaving 20. We cannot subtract 50 from 20, but $120 - 50 = 70$. We have now used 100 of the 1600, and $1500 - 700 = 800$.

The above is one of the most common methods of subtracting, but the class should follow the method learned in the earlier grades. It is confusing to learn new methods that are little better than the old ones. The teacher is advised to give but little time to asking for such definitions as those on page 10, for the terms are rarely used in business. The business man says, "*Deduct* \$10 and pay me the *balance*," instead of "*Subtract* \$10 and pay me the *remainder*."

20. Making change. If you owe 70 cents and give the merchant \$1, he says: "70 and 5 are 75, and 25 more makes a dollar," at the same time laying down 5 cents and 25 cents. He thus sees that $\$1 - \$0.70 = \$0.30$.

ORAL EXERCISE

1. Make change for \$1, when you owe 55 ct.; 37 ct.
2. Also for \$2, when you owe \$1.49; \$1.62; \$1.16.
3. Also for \$5, when you owe \$2.75; \$3.10; \$1.48.

WRITTEN EXERCISE

1. \$143.92 − \$127.65.
2. \$237.50 − \$148.75.
3. \$926.50 − \$735.75.
4. \$432.75 − \$246.50.
5. \$329.50 − \$137.75.
6. \$823.45 − \$696.75.
7. 437,625 − 129,735.
8. 1,237,635 − 982,987.

Interesting Facts in Our History

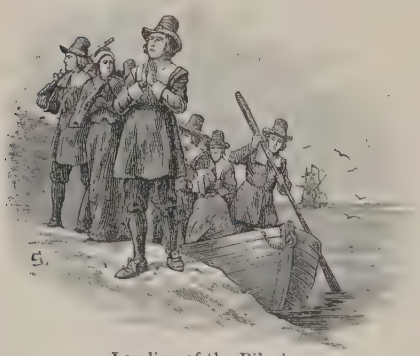
A few important dates

1492. Columbus discovered America.
1609. The Hudson River first explored.
1620. Landing of the Pilgrims at Plymouth.
1664. New York City captured from the Dutch.
1776. Signing of the Declaration of Independence.
1789. George Washington inaugurated President.

WRITTEN EXERCISE

1. How many years since Columbus discovered America?

2. How long after that was the Hudson explored?



Landing of the Pilgrims

3. How many years since Washington was inaugurated?

4. How long ago did the *Mayflower* reach Plymouth?

5. How many years after that did the English capture New York from the Dutch?

6. How many years from the Declaration of Independence to the next Fourth of July after to-day?

7. How long after Columbus discovered America was the landing of the Pilgrims?

8. It lacked only 2 years of being a century and a quarter from the battle of Lexington to that of Manila, 1898. When was the battle of Lexington?

ORAL EXERCISE

1. The subtrahend is 7, the minuend 21; what is the difference? How much is 21 minus 7?

2. The difference is 11, the minuend 16; what is the subtrahend? What number minus 16 equals 11?

3. The subtrahend is 27, the remainder 14; what is the minuend? 27 minus what number equals 14?

4. The minuend is 25 more than the subtrahend; what is the difference?

5. The subtrahend is 47 less than the minuend; what is the difference?

6. The subtrahend and difference together equal 39; what is the minuend?

7. Mr. Roberts owed Mr. Lambert $75 yesterday. To-day he has paid him $25. What balance is due?

WRITTEN EXERCISE

Perform these subtractions, and check every answer by adding the remainder and subtrahend. See how many you can perform in five minutes.

1. 20,265 – 5927.	2. 32,502 – 27,635.
3. 28,765 – 21,976.	4. 41,232 – 36,525.
5. 81,463 – 29,658.	6. 75,000 – 29,127.
7. 234,684 – 102,793.	8. 648,329 – 409,681.
9. 642,306 – 609,008.	10. 920,102 – 735,008.
11. $435.27 – $107.62.	12. $308.08 – $127.09.
13. $2087.50 – $1296.49.	14. $3042.50 – $1209.87.
15. $62,341.75 – $12,986.50.	16. $42,350.50 – $19,625.08.
17. $48,002.75 – $31,609.50.	18. $29,875.25 – $20,975.26.

MULTIPLICATION

ORAL EXERCISE

1. Read the products by 2: 4, 7, 1, 9, 6, 2, 8, 3, 11, 5, 10.

This or a similar set of numbers should be written on the board.

2. In the same way, read rapidly the products by the various numbers from 3 to 10. Also by 20, 30, 100.

3. Count by 2's from 2 to 20, and give the multiplication table of 2's. Give the multiplication tables from 3 to 10.

4. How much is $4 + 4 + 4 + 4 + 4$? 5 times 4? Multiplication is a short form of what other process?

21. Abstract numbers. A number that does not refer to any particular kind of object or measure is called an *abstract number.*

For example, 10, 42, $3\frac{1}{2}$, are abstract numbers.

22. Concrete numbers. A number that refers to some particular kind of object or measure is called a *concrete number.*

For example, \$10, 32 ft., 6 mo., are concrete numbers.

23. Multiplication. The process of taking one number as many times as there are units in another is called *multiplication.*

This means multiplication by an abstract integer. Thus, to multiply \$3 by 2 means that \$3 is taken 2 times.

24. Multiplicand. The number multiplied is called the *multiplicand.*

25. Multiplier. The number by which we multiply is called the *multiplier.*

\$325 multiplicand
3 multiplier
\$975 product

26. Product. The result of multiplying is called the *product.*

27. Nature of the numbers. It is therefore seen that

1. *The multiplier must be thought of as abstract.*
2. *The product is like the multiplicand.*

That is, if we have 3 times 17 ft., the 3 is abstract, and the product is feet, like the multiplicand.

28. Multiple. The product of two abstract integers is called a *multiple* of either.

For example, 35 is a multiple of 7 and of 5.

29. Factors. The numbers which multiplied together make another number are called its *factors.*

For example, the factors of 30 are 2, 3, and 5.

Teachers are advised not to require much memorizing of definitions. The important thing is that the words shall be used correctly. Those just defined are rarely heard in business. In reading $2 \times \$6$ and $\$6 \times 2$, teachers should follow the custom of the school. The former is preferably read "two times \$6," and the latter "\$6 multiplied by 2." In order not to confuse children who have learned other ways of reading, this book uses the word *times* where there might be any misunderstanding.

30. Power. The result of taking a number any number of times as a factor is called a *power* of the number.

For example, $2 \times 2 = 4$, and 4 is called the second power, or square, of 2. 2×2 is written 2^2. So $2 \times 2 \times 2$, or 2^3, is the third power of 2. It is also called the *cube* of 2.

ORAL EXERCISE

1. State four multiples of 7; of 3; of 9; of 11; of 15.
2. State the factors of 35; of 77; of 21; of 49; of 121.
3. State the squares of 7, 6, 9, 8, 10, 11, 20, 100, 1000.
4. What is the square of 5? the cube of 3? the fourth power of 2? the cube of 5? of 2? of 10?

31. The process. You have already learned that in multiplying, for example, \$635.50 by 215,

This is the complete operation:				*But we write only this:*
\$635.50				\$635.50
215				215
\$3177.50	5	times	\$635.50	3177 50
6355.00	10	"	"	6355 0
127100.00	200	"	"	127100
\$136632.50	215	"	"	\$136632.50

32. Zero in the multiplier. If a zero appears in the multiplier,

This long process might be taken:		*But we need only this:*	
247	247	247	247
305	350	305	350
1235	000	1235	12350
000	1235	741	741
741	741	75335	86450
75335	86450		

WRITTEN EXERCISE

Multiply:

1. 923×936.
2. 237×243.
3. 406×572.
4. 560×437.
5. 206×302.
6. 725×725.
7. \$41.27 by 237.
8. \$68.95 by 419.
9. \$309.25 by 406.
10. \$723.30 by 507.
11. \$234.56 by 129.
12. \$507.75 by 625.
13. \$802.75 by 308.
14. \$7235 by 5200.
15. \$1245.25 by 209.
16. \$3005.75 by 2002.
17. \$8126.05 by 3030.
18. \$4040.40 by 4040.

Multiplying by Powers of 10

ORAL EXERCISE

1. Multiply by 10: 10 ct., 50 ct., \$1.20, \$3.50.

2. Multiply by 100: 3, 42, 565, \$1.50, \$21.75.

3. Multiply by 1000: 4, 7, 10, 40, 52, 100, 250.

4. State a short way of multiplying an integer by 10; by 100; by 1000.

5. State a short way of multiplying a decimal by 10; by 100; by 1000.

33. Multiplying by powers of 10. *To multiply an integer by 10, annex a 0; by 100, two 0's; by 1000, three 0's.*

34. *To multiply a decimal by 10, move the decimal point 1 place to the right; by 100, 2 places; by 1000, 3 places.*

Hence to multiply 27 by 40, multiply by 4 and annex a 0.

$$\begin{array}{r} 27 \\ 40 \\ \hline 1080 \end{array}$$

Also to multiply \$25.16 by 300, multiply by 3 and move the decimal point 2 places to the right.

$$\begin{array}{r} \$25.16 \\ 300 \\ \hline \$7548 \end{array}$$

WRITTEN EXERCISE

1. 210 times \$346.
2. 320 times \$421.
3. 800 times 932 ft.
4. 700 times 864 rd.
5. 9600 times \$4300.
6. 4300 times \$8250.
7. 7000 times \$15.42.
8. 9000 times \$21.75.
9. 6520 times \$12.50.
10. 2750 times \$31.02.

11. If a train goes 48.2 mi. an hour, how far will it go in 20 hr., at the same rate?

12. If it costs \$14.75 a year to educate you, how much does it cost to educate 300 children, at the same rate?

ORAL EXERCISE

1. On the blackboard, multiply 68 by 5. Divide 680 by 2.

2. On the blackboard, multiply \$12.25 by 5. Also divide \$122.50 by 2. How do the results compare? Tell a short way of multiplying by 5.

3. Because $5 = 10 \div 2$, to multiply an integer by 5 is the same as to annex 0 and divide by what number?

4. On the blackboard, multiply \$42.74 by 50. Also divide \$4274 by 2. Compare the results.

5. Because $50 = 100 \div 2$, to multiply by 50 we may first multiply by what number, and then divide by what one?

6. On the blackboard, multiply \$32.56 by 25. Also divide \$3256 by 4. Compare the results. Because $25 = 100 \div 4$, to multiply by 25 we may first multiply by what number, and then divide by what other number?

35. Aliquot part. A number that will exactly divide another is called an *aliquot part* of that number.

For example, 5 is an aliquot part of 10, and both 25 and 50 are aliquot parts of 100.

36. Multiplying by aliquot parts. *To multiply by 5, multiply by 10 and divide by 2.*

To multiply by 50, multiply by 100 and divide by 2.

To multiply by 25, multiply by 100 and divide by 4.

WRITTEN EXERCISE

1. Multiply by 5: 723, \$2500, \$47.40, \$666.60.
2. Multiply by 50: 237, 425 ft., \$68.50, \$987.40.
3. Multiply by 25: 134, 802 ft., \$49.60, \$384.40.
4. Multiply by 500: 16, 64, 726, \$42.80, \$124.40.

The American City

WRITTEN EXERCISE

1. There were in a certain year 19,075,847 of our people living in cities having a population of 25,000 or more. If this was $\frac{1}{4}$ of our total population, what was the total?

2. In the United States there were recently 161 places of more than 25,000 inhabitants, 356 between 8000 and 25,000, and 532 between 4000 and 8000. How many were there having more than 4000?

3. If the valuation of the land and buildings in Buffalo is $221,405,290; in Indianapolis, $94,935,180; in New Orleans, $103,753,915; and in St. Louis, $316,041,190, by how much does the St. Louis valuation exceed that of each of these other cities?

4. Including the suburbs, the population of Chicago at the beginning of the century was 1,939,327; of Philadelphia, 1,738,192; of Baltimore, 656,047; and of Louisville, 292,502. If the population of New York City and its suburbs was as much as that of all these four cities, how much was it?

DIVISION

ORAL EXERCISE

1. How much is 10 times \$17? How many times is \$17 contained in \$170? How much is $\frac{1}{10}$ of \$170?

2. How many times is \$3 contained in \$12? in \$14, with what remainder? in \$20? in \$42? in \$61?

3. If the product of two numbers is 35 ft., and one of the numbers is 7 ft., what is the other? If one of the numbers is 5, what is the other?

37. Division. The operation by which, given the product of two numbers and one of them, the other is found, is called *division.*

38. Dividend and divisor. The given product is called the *dividend* and the other given number is called the *divisor.*

39. Quotient and remainder. The number found by division is called the *quotient.* The part of the dividend remaining when the division is not exact is called the *remainder.*

The teacher should recognize that it is possible to give too much attention to such terms. In business the word *dividend* is commonly used in another way, and the word *quotient* is rarely employed.

40. Two kinds of division. Because 10 times \$17 = \$170, therefore \$170 ÷ \$17 = 10, and \$170 ÷ 10 = \$17.

41. *If the dividend and divisor are concrete, they must be alike, and the quotient is abstract.*

For example, \$170 ÷ \$17 = 10. This is sometimes called *measuring*, because the \$170 is measured by \$17.

42. *If the dividend is concrete and the divisor is abstract, the quotient is like the dividend.*

For example, \$170 ÷ 10 = \$17. This is sometimes called *partition*, because the \$170 is separated into 10 equal parts.

ORAL EXERCISE

1. Divide by 2: 16, 24, 480, 248, 408, 418.
2. Divide by 3: 27, 36, 630, 963, 906, 615.
3. Divide by 4: 40, 84, 480, 844, 408, 816.
4. Divide by 5: 55, 60, 550, 555, 600, 655.
5. Divide by 6: 30, 66, 360, 606, 612, 726, 636.
6. Divide by 7: 63, 77, 350, 560, 497, 714, 775.
7. Divide by 8: 72, 56, 640, 648, 816, 968, 641.
8. Divide by 9: 63, 99, 720, 279, 819, 558, 633.
9. If 6 sheep cost $33, how much will 1 cost? 2?
10. If 4 dictionaries cost $36, how much will 1 cost?
11. If 1 dictionary costs $9, how many can be bought for $36? for $63? for $72? for $99? for 180?
12. A school paid $96 for school desks at $3 each. How many were bought? How many at $4 each?
13. If 4 school desks cost $13, how much will 1 cost? How much, if 5 cost $16? if 8 cost $25? if 6 cost $21?

WRITTEN EXERCISE

1. If 11 acres of land are worth $1485, what is the average price per acre? How much are 4 acres worth?
2. If a dealer pays $336 a dozen for suits of ready-made clothes, how much does he pay per suit? for 7 suits?
3. A man pays $1276 for the rent of a city house for 11 months. At this rate, how much does he pay for 5 months?
4. A wagon dealer pays $1215 for 9 wagons. At this rate, how much do 4 wagons cost? 7 wagons?
5. At the rate of $788 for 8 acres of land, how much must a man pay for 3 acres? for 5 acres? for 100 acres?

43. The process. You have already learned how to divide. For example, to divide \$1375.44 by 521,

This is the complete operation:		*But we write only this:*
\$2.64		\$2.64
521)\$1375.44		521)\$1375.44
1042	= 521 times \$2	1042
333.44	still to be divided	3334
312.60	= 521 times \$0.60	3126
20.84	still to be divided	2084
20.84	= 521 times \$0.04	2084
	Total quotient = \$2.64	

We see that \$1000 ÷ 521 equals no thousands of dollars; \$1300 ÷ 521 equals no hundreds; \$1370 ÷ 521 equals no tens; but \$1375 ÷ 521 equals about \$2. (We really notice that 13 ÷ 5 = 2 + something.) Subtracting 521 times \$2, there is \$333.44 still to be divided. \$333.40 ÷ 521 equals about \$0.60 (for 33 ÷ 5 = 6 + something). Subtracting 521 times \$0.60, there remains \$20.84, and \$20.84 ÷ 521 = \$0.04. Therefore the quotient is \$2.64.

44. To check the result. We have seen that the dividend is the product of the quotient and divisor, plus the remainder, if any.

In the above example, \$1375.44 = 521 times \$2.64.

45. Remainders. In 109 ÷ 33 = 3, with a remainder 10 still to be divided, the quotient is written $3\frac{10}{33}$.

$$\begin{array}{r} 3\frac{10}{33} \\ 33\overline{)109} \\ \underline{99} \\ 10 \end{array}$$

WRITTEN EXERCISE

1. \$863.24 ÷ 321.
2. \$326.34 ÷ 333.
3. \$626 ÷ 25.
4. \$260.02 ÷ 302.
5. \$1000.01 ÷ 201.
6. \$975 ÷ 18.
7. \$923.58 ÷ 233.
8. \$9000.90 ÷ 107.
9. \$456 ÷ 33.
10. \$425.30 ÷ 120.
11. \$1235.40 ÷ 225.
12. \$221 ÷ 16.

ORAL EXERCISE

1. $16 \div 8$; $160 \div 80$; $16{,}000 \div 8000$.

2. $96 \div 12$; $9600 \div 1200$; $96{,}000 \div 12{,}000$.

3. $132 \div 11$; $13{,}200 \div 1100$; $132{,}000 \div 11{,}000$.

4. $200\overline{)8600}$; $200\overline{)8600 + 100}$; $200\overline{)8700}$; $200\overline{)8750}$.

5. Instead of dividing 17,280 by 90, we will get the same result by dividing what number by 9?

6. Instead of dividing 172,800 by 1200, we will get the same result by dividing what number by 12?

46. Dividing by multiples of 10. We have seen that just as 86 ft. $\div$ 2 ft. $= 86 \div 2$, so $8600 \div 200 = 86 \div 2$. In the same way, in dividing 8637 by 200 we have

$$200\overline{)8600 + 37} \qquad 43 \quad + \tfrac{37}{200}$$

or

$$200\overline{)86|37} \qquad 43\ \tfrac{37}{200}$$

WRITTEN EXERCISE

1. $46{,}400 \div 800$.
2. $46{,}800 \div 900$.
3. $42{,}000 \div 3000$.
4. $160{,}000 \div 5000$.
5. $102{,}000 \div 3400$.
6. $294{,}000 \div 4200$.
7. $276{,}000 \div 7000$.
8. $814{,}200 \div 3000$.
9. $354{,}200 \div 75{,}000$.
10. $489{,}100 \div 63{,}000$.

11. At 260 mi. a day, how long would it take a steamer to go from San Francisco to Guam, 5200 miles?

12. At a certain temperature sound travels 1132 ft. a second. How long will it take it to travel 23,772 ft.?

13. In one year the receipts of the Chicago post office were \$7,713,800. How much did they average per day?

14. When the annual expenses of our government were \$479,250,000, how much were the average daily expenses?

ORAL EXERCISE

47. Rapid oral work. In adding 26 and 45 begin with the tens, saying: 26, 66, 71. In subtracting follow the first plan on page 10, or in the case of $75 - 48$ think: $75 - 40 - 8 = 35 - 8 = 27$. In multiplying by a two-figure multiplier it is better to begin with the tens, as in 12 times 24, where we should think: $240 + 48 = 288$.

1. $29 + 67$.	2. $48 + 96$.	3. $63 + 78$.
4. $78 + 96$.	5. $89 + 48$.	6. $67 + 93$.
7. $69 + 92$.	8. $83 + 49$.	9. $78 + 87$.
10. $160 + 75$.	11. $230 + 93$.	12. $420 + 88$.
13. $125 + 86$.	14. $137 + 68$.	15. $142 + 81$.
16. $562 + 120$.	17. $287 + 130$.	18. $329 + 120$.
19. $63 - 21$.	20. $52 - 22$.	21. $67 - 38$.
22. $81 - 57$.	23. $92 - 39$.	24. $83 - 68$.
25. $121 - 30$.	26. $146 - 60$.	27. $153 - 73$.
28. $260 - 140$.	29. $320 - 110$.	30. $430 - 160$.
31. $275 - 185$.	32. $263 - 143$.	33. $328 - 108$.

Multiply in Exs. 34–45:

34. 241 by 2.	35. 323 by 3.	36. 420 by 2.
37. 210 by 3.	38. 410 by 5.	39. 320 by 4.
40. 430 by 3.	41. 530 by 3.	42. 610 by 6.
43. 810 by 9.	44. 800 by 11.	45. 500 by 12.
46. $819 \div 9$.	47. $784 \div 8$.	48. $455 \div 7$.
49. $715 \div 5$.	50. $312 \div 4$.	51. $273 \div 3$.
52. $612 \div 12$.	53. $781 \div 11$.	54. $340 \div 20$.
55. $720 \div 12$.	56. $891 \div 11$.	57. $620 \div 20$.
58. $840 \div 40$.	59. $990 \div 30$.	60. $840 \div 60$.

ORAL EXERCISE

Find the cost of all the items in Exs. 1–5:

1. Tea @ 98¢, and coffee @ 49¢.
2. 4 yd. silk @ $1.20 a yard.
3. Flour @ 39¢, and cheese @ 28¢.
4. Biscuits @ 24¢, and cake @ 37¢.
5. A book @ 65¢, and a hat @ $1.75.

Find the price per pound, ounce, or piece, in Exs. 6–11:

6. 6 lb. of figs cost $1.08.
7. 9 cans of soup cost $1.17.
8. 20 oz. of olive oil cost 64¢.
9. 60 cakes of soap cost $2.70.
10. 50 lb. of dried apples cost $5.25.
11. 8 lb. of English walnuts cost $1.28.

WRITTEN EXERCISE

1. $273 + 421 + 68 + 9026 + 34{,}379 + 75 + 286$.
2. $3972 + 8298 + 4000 + 17{,}625 + 31 + 9 + 407$.
3. $4269 + 193{,}481 + 17{,}286 + 81{,}298 + 4876 + 923$.
4. $876{,}481 - 298{,}327$.
5. $298{,}008 - 43{,}661$.
6. $173{,}209 - 106{,}492$.
7. $381{,}200 - 179{,}699$.
8. $400{,}100 - 199{,}298$.
9. $673{,}100 - 194{,}287$.
10. $65{,}664 \div 456$.
11. $77{,}112 \div 189$.
12. $221{,}336 \div 102$.
13. $328{,}320 \div 360$.
14. $7{,}605{,}920 \div 112$.
15. $7{,}913{,}576 \div 209$.
16. 628×325.
17. 872×896.
18. 469×832.
19. 1268×3472.
20. 2081×6235.
21. 4287×3009.

Our Country's Food Supply

48. Wheat. This great food product brings to our farmers three or four hundred million dollars a year.

WRITTEN EXERCISE

1. If 43,525,000 acres of wheat in this country produce on an average 13 bu. to the acre, what is the total yield?

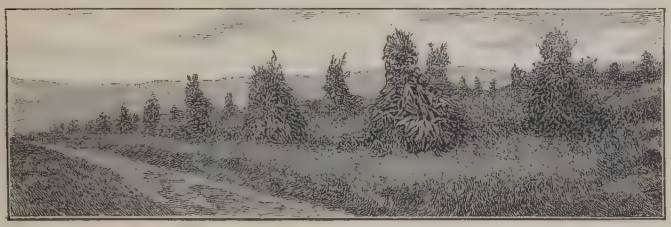
2. If Kansas produces 82,368,000 bu. a year, which is 12,000 times as much as Connecticut, how much does Connecticut produce?

3. If the wheat crop of the world is 2,586,000,000 bu. a year, and if the United States produces $\frac{1}{5}$ of it, how many bushels does it produce?

4. If the United States produces 547,303,000 bu. of wheat in one year, worth 70¢ a bushel, what is the total value of the crop?

5. If the value of the Minnesota yield of wheat averages $32,560,000 a year, and this is 4400 times that of New Hampshire, what is the value of the yield of the latter?

49. Corn. Another of the great food products of the country is corn. The greatest corn-producing states are Iowa, Illinois, Nebraska, Missouri, Kansas, and Indiana.



WRITTEN EXERCISE

1. When this country produced 2,105,102,400 bu. of corn a year, averaging 25 bu. to the acre, how many acres had we in corn?

2. If 3 bu. of corn could then be bought for $1, what was the total value of this yield of 2,105,102,400 bu.?

3. When Iowa's annual product amounted to 305,800,000 bu., this was how many times the 440,000 bu. produced by Maine?

4. To transport 1000 lb. of corn from St. Louis to New Orleans by river costs $1. How much will it cost to transport 1750 tons?

5. If the average value of corn for each of the 46,610 acres given to it in Connecticut in a certain year was $21, and for each of the 4,031,600 acres in Indiana $13, what was the entire value of the corn crop of each state?

6. If the average annual corn crop per acre is 40 bu. in Wisconsin, 36 bu. in Maine, 37 bu. in New Hampshire, 38 bu. in Massachusetts, 38 bu. in Indiana, and 38 bu. in Iowa, find the average by adding and dividing by 6.

50. Cattle. The greatest beef-producing states are Texas, Iowa, and Kansas. The greatest stock yards and packing houses in the world are in Chicago.

A Texas Cattle Ranch

WRITTEN EXERCISE

1. If Texas has 4,533,897 head of cattle, valued at $17 a head, how much are they all worth?

2. If Kansas has 2,076,489 head of cattle, valued at $27 a head, how much are they all worth?

3. If Iowa has 2,163,584 head of cattle, valued at $31 a head, how much are they all worth?

4. If this country sold 55,553,750 lb. of canned beef in a certain year, at 9 ct. a pound, what was the total value?

5. In a year when the returns showed 69,335 cattle in a certain county, their value was estimated at $1,525,370. What was the average value per head?

6. In a year when the returns showed 18,108 cows kept for milk in a certain county, their value was estimated at $543,240. What was the average value per head?

7. The number of cattle being raised for food in the United States in 1880 was 21,231,000, and our population was 50,155,783. In 1900 the number of cattle was 27,610,054, and our population was 76,303,387. What was the increase in cattle and in population in the 20 years?

51. The stock yards. The Chicago stock yards are the center of the great beef industry. From the packing houses the meat is shipped in refrigerator cars, and goes to various parts of the world.

WRITTEN EXERCISE

1. In one year the receipts from the beef industry of the country were \$800,000,000, of which $\frac{1}{16}$ was profit. How much was profit?

2. If, out of every 100 lb. of an animal's weight, 56 lb. is salable as meat, how much salable meat is there in a 1150-lb. animal? What is it worth at \$8.00 a hundred, adding \$1.50 for preparing the meat for shipment?

3. If 4,489,500 cattle were shipped to some stock yards in one year, what was the average per day?

4. An eastern dealer buys 10,000 lb. of beef a month, at 14 ct. a pound, and sells it on an average of 19 ct. a pound. How much are his profits a month?

5. In 50 years the value of the beef produced annually in this country rose from \$11,981,642 to \$785,562,442. How much was the average annual increase in value?

6. If 700 lb. of meat are sold by a packing house for \$63, and it costs \$1 a hundred to ship it to New York, how much does it cost a pound delivered in New York?

FACTORS AND MULTIPLES

ORAL EXERCISE

1. What do you mean by the factors of a number?

2. Name two factors of 35; of 77; of 26; of 34.

3. Name three factors of 30; of 70; of 66; of 110.

4. Name a factor that is common to both 35 and 45.

5. Name two factors common to both 30 and 48.

6. In reducing $\frac{2}{4}$ to $\frac{1}{2}$ we found in primary arithmetic that we could divide both terms by 2, or *cancel* the 2. What factor do we cancel in reducing $\frac{6}{8}$ to its *lowest terms*, $\frac{3}{4}$?

7. Name the factors to be canceled in reducing to lowest terms $\frac{4}{8}$, $\frac{2}{8}$, $\frac{4}{10}$, $\frac{6}{9}$, $\frac{8}{12}$, $\frac{10}{15}$, $\frac{15}{20}$. What need have we found for factors in arithmetic?

52. Common divisor. A factor that is common to two or more numbers is called a *common divisor.*

For example, 5 is a common divisor of 35 and 45, as we found in Ex. 4. It might be called a common factor, but common divisor is the name usually given it.

53. Greatest common divisor. The greatest factor common to two or more numbers is called their *greatest common divisor* (g.c.d.).

For example, although 3 is a common divisor of 12, 18, and 48, the number 6 is the *greatest* common divisor.

54. Finding the g.c.d. The greatest common divisor of such numbers as we shall meet in fractions is easily found.

For example, consider 12 and 30. We see that

$$12 = 2 \times 3 \times 2, \qquad 30 = 2 \times 3 \times 5,$$

and that 2 and 3 are the only common factors. Therefore 2×3, or 6, is the greatest common divisor.

ORAL EXERCISE

1. Name a common divisor of 20 and 22; 16 and 36.

2. Name the greatest common divisor of 75 and 100.

3. Which of these numbers have no factors excepting one and themselves: 4, 5, 6, 7, 10, 11, 12, 13?

4. In reducing $\frac{12}{16}$ to lowest terms is it sufficient to cancel the common divisor 2? What kind of common divisor must be canceled in reducing a fraction to lowest terms?

55. Prime number. A number that has no factors except itself and 1 is called a *prime number.*

For example, 7, 11, 17 are prime numbers.

56. Composite number. A number not prime is called *composite.*

57. Even numbers. A number which contains the factor 2 is called an *even number.*

58. Odd numbers. Numbers that are not even are called *odd numbers.*

59. Prime factors. Factors that are prime numbers are called *prime factors.*

For example, the prime factors of 68 are 2, 2, and 17.

60. Mutually prime numbers. Numbers that have no common factors are said to be *prime to each other.*

For example, 12 and 35 are prime to each other.

WRITTEN EXERCISE

1. Make a list of prime numbers to 100; of odd numbers.

2. Find the prime factors of 54; of 48; of 64; of 77.

3. Find the greatest common divisor of 96, 60, and 36.

ORAL EXERCISE

1. Is 2 a factor of 14? of 27? of 50? of 45? of 600? How do you tell whether 2 is a factor of a number?

2. Is 2 a factor of 10? of any number of 10's? Then is it a factor of any number if it is a factor of the units?

3. Is 5 a factor of 20? of 78? of 45? of 72? of 800? How do you tell whether 5 is a factor of a number?

4. Is 5 a factor of 10? of any number of 10's? Then is it a factor of any number if it is a factor of the units?

61. Divisible numbers. When we speak of one number being divisible by another we mean exactly divisible.

62. Divisibility by 2. *A number is divisible by 2 if the units are so divisible.*

For example, 64, or 60 + 4, must be divisible by 2 if 4 is so divisible, because 60 is divisible by 2.

63. Divisibility by 5. *A number is divisible by 5 if it ends in 0 or 5.*

64. Divisibility by 3. *A number is divisible by 3 if the sum of its digits is so divisible.*

For example, 411 is divisible by 3 because 4 + 1 + 1 is.

WRITTEN EXERCISE

1. Which of these are divisible by 2: 660, 4907, 6255, 3027, 1356, 5790, 2371, 4196, 2005, 37,268, 125,474?

2. Which of these are divisible by 5: 660, 7620, 4867, 7075, 3200, 4035, 9636, 8124, 3672, 12,475, 374,465?

3. Which of these are divisible by 3: 660, 1236, 5778, 9102, 8328, 3444, 1239, 9876, 4004, 7117, 31,476, 307,983, 206,340, 475,633, 2,124,816, 4,216,872, 15,681,123?

ORAL EXERCISE

1. State the prime factors of 15, 18, 27, 35, 42, 75.
2. State one factor of 395, 123, 777, 692, 1275, 1263.
3. Which of these are divisible by 3: 77, 609, 1203?
4. Which of these are divisible by 2: 68, 4973, 2870?

65. Other tests. There are various tests of divisibility besides those on page 32. They are easily illustrated or explained, and may be given or not as the teacher prefers. The more important are the following.

66. Divisibility by 4. A number is divisible by 4 if the number represented by the two right-hand figures is so divisible.

67. Divisibility by 6. A number is divisible by 6 if it is even and if the sum of its digits is divisible by 3.

68. Divisibility by 8. A number is divisible by 8 if the number represented by the three right-hand figures is so divisible.

69. Divisibility by 9. A number is divisible by 9 if the sum of its digits is so divisible.

70. Divisibility by 11. A number is divisible by 11 if the difference between the sums of the digits in its even and odd places is so divisible.

For example, 430,507 is divisible by 11, for $7 + 5 + 3 = 15$, and $0 + 0 + 4 = 4$, and $15 - 4 = 11$.

71. Finding prime factors. Find the prime factors of 2310.

$$\begin{array}{r} 2\,)2310 \\ 5\,)1155 \\ 3\,)231 \\ 7\,)77 \\ \hline 11 \end{array}$$

By § 62, 2 is a factor. By § 63, 5 is a factor of the other factor, 1155. By § 64, 3 is a factor of the other factor, 231. It is easy to see that 7 and 11 are factors of 77. Hence the prime factors are 2, 5, 3, 7, 11.

72. Finding the g.c.d. In the same way it is easy to find the greatest common divisor of numbers.

For example, to find the greatest common divisor of 231 and 660. Factoring 660 and 231 as here shown, we have

$$231 = 3 \times 7 \times 11,$$

and

$$660 = 2 \times 2 \times 3 \times 5 \times 11,$$

in which 3 and 11 are the only common factors.

Therefore 3×11, or 33, is the greatest common divisor.

$$\begin{array}{r|l} 3 & 231 \\ 7 & 77 \\ & 11 \end{array} \qquad \begin{array}{r|l} 2 & 660 \\ 2 & 330 \\ 3 & 165 \\ 5 & 55 \\ & 11 \end{array}$$

WRITTEN EXERCISE

Find the prime factors of the numbers in Exs. 1–30:

1. 775.	2. 420.	3. 385.	4. 352.	5. 320.
6. 891.	7. 147.	8. 297.	9. 176.	10. 336.
11. 130.	12. 462.	13. 390.	14. 285.	15. 375.
16. 8910.	17. 1250.	18. 2430.	19. 1280.	20. 1024.
21. 2970.	22. 1375.	23. 5390.	24. 1568.	25. 1331.
26. 6300.	27. 1875.	28. 1215.	29. 3645.	30. 10,935.

Find the greatest common divisor in Exs. 31–48:

31. 147 and 350.	32. 128, 352, and 160.
33. 130 and 195.	34. 125, 175, and 275.
35. 444 and 888.	36. 165, 385, and 220.
37. 720 and 1728.	38. 1024, 1280, and 1792.
39. 1536 and 3584.	40. 1001, 5390, and 7700.
41. 3125 and 1000.	42. 1650, 1155, and 1485.
43. 1440 and 1728.	44. 1250, 1875, and 2500.
45. 1875 and 2500.	46. 1010, 2121, and 2727.
47. 1275 and 1775.	48. 2575, 2060, and 9270.

ORAL EXERCISE

1. Name two multiples of 5; of 7; of 9; of 11.

2. Name a multiple that is common to 3 and 5.

3. The multiples 35 and 70 are common to 5 and 7. Which is the least multiple that is common to 5 and 7?

4. In the same way name the least common multiple of 2 and 3, and then name two other multiples.

73. Common multiple. A multiple of each of two or more numbers is called a *common multiple* of the numbers.

74. Least common multiple. Of all the common multiples of two or more numbers the least is called the *least common multiple* (l.c.m.).

For example, 24 is a common multiple of 4 and 6, but 12 is the least common multiple.

75. Finding the l.c.m. The least common multiple of two numbers is easily found if their factors are known.

Thus, to find the least common multiple of 16 and 40. We see that $16 = 2 \times 2 \times 2 \times 2$, and $40 = 2 \times 2 \times 2 \times 5$.

To be a multiple of 16, the least common multiple must contain $2 \times 2 \times 2 \times 2$, and to contain 40 it must also contain 5. Therefore $2 \times 2 \times 2 \times 2 \times 5$, or 80, is the least common multiple.

WRITTEN EXERCISE

Find the least common multiple of:

1. 30 and 70.	2. 77 and 110.	3. 28 and 32.
4. 64 and 96.	5. 9, 12, and 50.	6. 3, 8, and 96.
7. 5, 15, and 75.	8. 6, 33, and 96.	9. 10, 25, and 50.
10. 8, 16, and 64.	11. 4, 63, and 135.	12. 16, 72, and 96.
13. 7, 15, and 51.	14. 27, 54, and 84.	15. 72, 66, and 111.

COMMON FRACTIONS

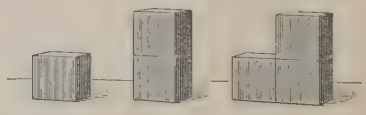
ORAL EXERCISE

1. When anything is divided into 3 equal parts, what is each part called? into 5 equal parts? into 12 equal parts?

2. What part of A is shaded? of B? of C? From A we see that $\frac{1}{2}$ equals how many sixths? from B, that $\frac{1}{3}$ equals how many sixths?

3. How many sixths of the rectangle make the whole rectangle? How many thirds? How many halves?

4. Cube A is what part as large as B? as C? B is what part as large as C?



5. If C is called 1, what is A? B? If B is called 1, what is A? C? If A is called 1, what is B? C?

6. Each small rectangle is what part of the whole rectangle? Each black triangle is what part of the whole rectangle? What does this tell you about $\frac{1}{2}$ of $\frac{1}{3}$?

7. From the picture show that $\frac{3}{18}$ of the rectangle is $\frac{1}{2}$ of $\frac{1}{3}$ of it; also that $\frac{3}{18} = \frac{1}{6}$; also that $\frac{6}{18} = \frac{1}{3}$; also that $\frac{9}{18} = \frac{1}{2}$; also that $\frac{10}{18} = \frac{5}{9}$; also that $\frac{12}{18} = \frac{2}{3}$.

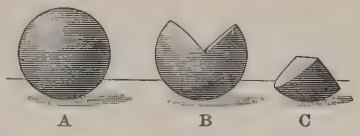
8. Name other fractions to which the following are equal: $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{9}$, $\frac{7}{21}$, $\frac{5}{25}$, $\frac{8}{40}$, $\frac{16}{24}$, $\frac{14}{21}$, $\frac{15}{25}$, $\frac{18}{27}$, $\frac{12}{28}$.

76. Unit. Any one thing is called a *unit*.

77. Fraction. One or more of the equal parts of a unit is called a *fraction*.

78. Denominator. The number which shows into how many equal parts a unit has been divided is called the *denominator.*

For example, a sphere has been divided into 4 equal parts, and B is $\frac{3}{4}$ of it. Then 4 is the denominator.



We read $\frac{1}{3}$ ft. "one third *of a* foot."

79. Numerator. The number which shows how many parts have been taken to make a fraction is called the *numerator.*

In the fraction $\frac{3}{4}$, 3 shows the number of fourths that have been taken, and is, therefore, the numerator.

80. Terms. The numerator and denominator together are called the *terms of the fraction.*

81. Common fraction. A fraction which has both terms expressed is called a *common fraction.*

For example, $\frac{2}{3}$, $\frac{5}{5}$, $\frac{7}{6}$, $\frac{6}{10}$, but not a decimal fraction like 0.6.

82. Proper fraction. A fraction whose numerator is less than the denominator is called a *proper fraction.*

For example, $\frac{2}{3}$, $\frac{1}{27}$, $\frac{3}{100}$, $\frac{1}{275}$.

83. Unit fraction. If the numerator is 1, the fraction is called a *unit fraction.*

For example, C is $\frac{1}{4}$ of a sphere.

84. Improper fraction. A fraction whose numerator equals or exceeds the denominator is called an *improper fraction.*

For example, A is $\frac{4}{4}$ of a sphere, and A + C is $\frac{5}{4}$.

85. Mixed number. The sum of a whole number and a fraction is called a *mixed number.*

For example, if sphere A is 1, A + B is $1\frac{3}{4}$, A + C is $1\frac{1}{4}$.

ORAL EXERCISE

Read aloud the following:

1. $\frac{1}{1}\frac{2}{3}$.
2. $\frac{1}{1}\frac{5}{6}$.
3. $\frac{1}{3}\frac{7}{2}$ in.
4. $\frac{9}{2}\frac{}{0}$.
5. $\frac{7}{1}\frac{}{6}\frac{}{0}$ A.
6. $\frac{7}{3}\frac{}{2}$.
7. $\frac{7}{1}\frac{}{2}$.
8. $\frac{1}{3}\frac{9}{2}\frac{}{0}$ mi.
9. $\frac{4}{3}\frac{}{3}$ rd.
10. $\frac{7}{1}\frac{}{4}\frac{}{4}$ in.
11. $2\frac{4}{5}$.
12. $3\frac{7}{8}$.
13. $9\frac{2}{3}$.
14. $8\frac{3}{1}\frac{}{6}$.
15. $25\frac{2}{3}$ in.
16. $\$12\frac{1}{2}$.
17. $33\frac{1}{3}$.
18. $15\frac{4}{2}\frac{}{1}$.
19. $9\frac{3}{3}\frac{}{2}$.
20. $157\frac{7}{8}$.
21. A cubic inch is $\frac{1}{1}\frac{}{7}\frac{1}{2}\frac{}{8}$ cu. ft., or $\frac{1}{4}\frac{}{6}\frac{1}{6}\frac{}{5}\frac{}{6}$ cu. yd.
22. An inch is $\frac{1}{1}\frac{}{2}$ ft., $\frac{1}{3}\frac{}{6}$ yd., $\frac{1}{1}\frac{}{9}\frac{}{8}$ rd., or $\frac{}{6}\frac{}{3}\frac{1}{3}\frac{}{6}\frac{}{0}$ mi.
23. An ounce is $\frac{1}{1}\frac{}{6}$ lb., or $\frac{}{3}\frac{}{2}\frac{1}{0}\frac{}{0}\frac{}{0}$ T., and 1 lb. is $\frac{1}{2}\frac{}{0}\frac{}{0}\frac{}{0}$ T.

WRITTEN EXERCISE

Write the fractions or mixed numbers in Exs. 1–10:

1. Sixty eighty-firsts.
2. Twenty-seven fiftieths.
3. Thirty-five fortieths.
4. Seventeen eighteenths.
5. Nineteen thousandths.
6. Seventy-seven sixtieths.
7. Eight two-hundredths.
8. Five one-hundred-tenths.
9. Two hundred twenty-seven one-hundred-sixteenths.
10. Five hundred one two-thousand-eight-hundred-sevenths.

Write in words the following:

11. $\frac{5}{7}$, $\frac{7}{8}$, $\frac{6}{1}\frac{}{1}$, $\frac{5}{1}\frac{}{9}$.
12. $\frac{7}{1}\frac{}{0}$, $\frac{3}{1}\frac{}{0}\frac{}{0}$, $\frac{5}{1}\frac{}{0}\frac{}{0}\frac{}{0}$, $\frac{2}{1}\frac{}{0}\frac{0}{1}$.
13. $\frac{1}{1}\frac{5}{7}$, $\frac{2}{3}\frac{7}{9}$, $\frac{4}{6}\frac{1}{1}$, $\frac{7}{9}\frac{5}{1}$.
14. $\frac{8}{1}\frac{}{5}$, $\frac{1}{2}\frac{2}{3}\frac{1}{0}$, $\frac{6}{1}\frac{2}{4}\frac{}{4}\frac{5}{}$, $\frac{7}{2}\frac{}{0}\frac{2}{0}\frac{}{0}\frac{9}{}$.
15. $1\frac{2}{3}$, $25\frac{4}{5}$, $62\frac{5}{8}$, $93\frac{1}{3}$.
16. $1\frac{9}{1}\frac{}{1}$, $66\frac{2}{3}$, $125\frac{5}{8}$, $137\frac{7}{8}$.
17. $2\frac{3}{7}$, $13\frac{5}{2}\frac{}{7}$, $68\frac{1}{2}\frac{9}{1}$, $71\frac{1}{1}\frac{3}{7}$.
18. $4\frac{6}{1}\frac{}{9}$, $73\frac{2}{1}\frac{}{1}$, $234\frac{5}{1}\frac{}{8}$, $341\frac{9}{1}\frac{}{6}$.
19. $\$125.12\frac{1}{2}$, $\$6.66\frac{2}{3}$.
20. $\$333.33\frac{1}{3}$, $\$275.16\frac{2}{3}$.
21. $\$135.87\frac{1}{2}$, $\$14.37\frac{1}{2}$.
22. $\$112.16\frac{7}{8}$, $\$235.45\frac{1}{3}$.

In practical business, $\frac{1}{2}$ ct. and over, in the final result, is called 1 ct.; less than $\frac{1}{2}$ ct. is neglected in most cases.

Reduction of Fractions

ORAL EXERCISE

1. If A is 1, B shows there are how many halves in 1?

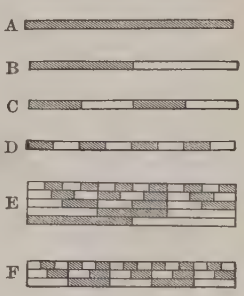
2. C shows how many fourths in 1? in $\frac{1}{2}$?

3. D shows that $\frac{1}{4}$ equals how many eighths? $\frac{1}{2} = \frac{?}{8}$.

4. E shows that $\frac{1}{3} = \frac{?}{6}$. $\frac{2}{3} = \frac{?}{6}$. $\frac{1}{3} = \frac{?}{9}$. $\frac{2}{3} = \frac{?}{9}$. $\frac{3}{6} = \frac{?}{2}$.

5. In E point to the twelfths. Tell how many twelfths equal a certain number of ninths, sixths, thirds, and halves.

6. In F tell as you did in E the relations between the fifths and tenths; the fifths and fifteenths.



86. Multiplying or dividing terms. We have found, as in primary arithmetic, that

Multiplying or dividing both terms of a fraction by the same number does not change its value.

For example, $\frac{2}{3} = \frac{2 \times 2}{2 \times 3} = \frac{4}{6}$, and $\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$.

87. Canceling. Dividing both terms of a fraction by the same factor is called *canceling* the factor.

88. Reduction. Changing the form of a fraction without changing the value is called *reducing the fraction.*

WRITTEN EXERCISE

Draw pictures like those above to show the following:

1. $\frac{1}{7} = \frac{2}{14}$.

2. $\frac{8}{16} = \frac{4}{8} = \frac{2}{4} = \frac{1}{2}$.

3. $\frac{3}{4} = \frac{6}{8} = 1\frac{2}{16} = 2\frac{4}{32}$.

ORAL EXERCISE

1. From the picture $\frac{1}{2}$ equals how many eighths? how many fourths?

89. Reduction to higher terms. A fraction is said to be *reduced to higher terms* when the terms are made larger.

2. From the picture $\frac{6}{8}$ equals how many fourths?

90. Reduction to lower terms. A fraction is said to be *reduced to lower terms* when the terms are made smaller.

91. Reduce $\frac{30}{45}$ to lower terms.

By § 86 we may divide both terms by any number without changing the value of the fraction. By § 63, 5 is a factor, and, by § 64, 3 is a factor. Dividing by 5 and 3, $\frac{30}{45} = \frac{2}{3}$.

92. Reduce $\frac{3}{7}$ to twenty-firsts.

By § 86 we may multiply both terms by any number without changing the value of the fraction. Since $21 \div 7 = 3$, we multiply both terms by 3, and $\frac{3}{7} = \frac{9}{21}$.

WRITTEN EXERCISE

1. Reduce to lower terms: $\frac{6}{16}$, $\frac{10}{22}$, $\frac{12}{14}$, $\frac{16}{32}$, $\frac{18}{27}$, $\frac{48}{60}$.
2. Reduce to fifteenths: $\frac{1}{5}$, $\frac{2}{3}$, $\frac{4}{30}$, $\frac{27}{45}$, $\frac{16}{60}$, $\frac{30}{75}$, $\frac{42}{90}$, $\frac{210}{75}$.
3. Reduce to sixteenths: $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{2}$, $\frac{10}{32}$, $\frac{28}{64}$, $\frac{48}{96}$, $\frac{30}{160}$, $\frac{25}{80}$.
4. Reduce to fiftieths: $\frac{1}{2}$, $\frac{3}{5}$, $\frac{9}{10}$, $\frac{7}{25}$, $\frac{4}{100}$, $\frac{27}{150}$, $\frac{36}{200}$, $\frac{35}{250}$.
5. Reduce to thirty-seconds: $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{16}$, $\frac{66}{64}$, $\frac{42}{96}$, $\frac{45}{160}$, $\frac{54}{192}$.
6. Reduce to hundredths: $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{7}{20}$, $\frac{15}{20}$, $\frac{4}{25}$, $\frac{30}{200}$, $\frac{402}{600}$.
7. Reduce to one-hundred-forty-fourths: $\frac{7}{12}$, $\frac{2}{3}$, $\frac{16}{288}$, $\frac{11}{12}$.
8. Reduce $\frac{7}{8}$ and $\frac{7}{12}$ to fractions having the denominator 24; the denominator 72; the denominator 216.

Integers and Mixed Numbers to Improper Fractions

ORAL EXERCISE

1. How many half hours in an hour? in 3 hours?
2. How many quarter hours in an hour? in 2 hours?
3. How many halves in 1? in 3? How many fourths?
4. How many quarters in \$1? in \$$1\frac{1}{2}$? in \$$2\frac{1}{4}$? in \$$3\frac{3}{4}$?
5. Express as fourths: 1, $1\frac{1}{2}$, $2\frac{1}{4}$, $3\frac{3}{4}$, 5, 7, $8\frac{1}{4}$, 10, 50, 80.
6. Express as halves: 1, $2\frac{1}{2}$, 3, 4, $4\frac{1}{2}$, 6, 15, $37\frac{1}{2}$, 70, 100.

93. Reduction to an improper fraction. Reduce $5\frac{7}{8}$ to an improper fraction.

Since $1 = \frac{8}{8}$, $5 = \frac{5 \times 8}{8} = \frac{40}{8}$; $\frac{40}{8} + \frac{7}{8} = \frac{47}{8}$.

7. How do you reduce a mixed number to an improper fraction?

WRITTEN EXERCISE

Reduce as directed in Exs. 1–9:

1. 7 to sixths.
2. 9 to sevenths.
3. 6 to tenths.
4. 8 to elevenths.
5. 11 to sixths.
6. 13 to fourths.
7. 9 to halves.
8. 9 to twentieths.
9. 15 to thirds.

Reduce to improper fractions:

10. $5\frac{2}{3}$.
11. $6\frac{7}{8}$.
12. $7\frac{2}{3}$.
13. $8\frac{9}{10}$.
14. $6\frac{2}{3}$.
15. $3\frac{7}{8}$.
16. $4\frac{8}{9}$.
17. $6\frac{4}{9}$.
18. $9\frac{3}{16}$.
19. $7\frac{4}{11}$.
20. $8\frac{5}{8}$.
21. $6\frac{3}{7}$.
22. $8\frac{4}{15}$.
23. $7\frac{3}{14}$.
24. $8\frac{5}{12}$.
25. $16\frac{2}{3}$.
26. $12\frac{1}{2}$.
27. $33\frac{1}{3}$.
28. $66\frac{2}{3}$.
29. $2\frac{13}{15}$.
30. $333\frac{1}{3}$.
31. $166\frac{2}{3}$.
32. $666\frac{2}{3}$.
33. $125\frac{1}{2}$.
34. $43\frac{27}{32}$.
35. $146\frac{3}{5}$.
36. $235\frac{5}{9}$.
37. $825\frac{6}{7}$.
38. $999\frac{1}{9}$.
39. $675\frac{2}{3}$.

Improper Fractions to Integers or Mixed Numbers

ORAL EXERCISE

1. Express $2\frac{1}{2}$ as halves; $\frac{5}{2}$ as a mixed number.

2. In \$5 how many half dollars? Express 5 as halves. Express $\frac{10}{2}$ as an integer; also $\frac{15}{5}$, $\frac{20}{4}$, $\frac{30}{3}$, $\frac{40}{10}$.

3. Express as integers: $\frac{12}{6}$, $\frac{15}{3}$, $\frac{16}{2}$, $\frac{81}{9}$, $\frac{25}{5}$, $\frac{125}{25}$.

4. Express as mixed numbers: $\frac{15}{7}$, $\frac{17}{8}$, $\frac{20}{3}$, $\frac{32}{10}$, $\frac{37}{7}$, $\frac{73}{12}$.

94. Reduction to an integer or a mixed number. Reduce $\frac{124}{5}$ to an integer or a mixed number.

Since $\frac{1}{5}$ means that 1 has been divided into 5 equal parts, $\frac{124}{5}$ means that 124×1 has been divided into 5 equal parts. That is, $\frac{124}{5}$ means the same as $124 \div 5$.

Therefore $\frac{124}{5} = 124 \div 5 = 24\frac{4}{5}$.

95. Therefore *a fraction is an indicated division.*

To reduce an improper fraction to an integer or a mixed number, divide the numerator by the denominator.

WRITTEN EXERCISE

Reduce to integers or mixed numbers:

1. $\frac{33}{3}$.	2. $\frac{42}{7}$.	3. $\frac{65}{6}$.	4. $\frac{29}{3}$.	5. $\frac{81}{8}$.
6. $\frac{47}{6}$.	7. $\frac{78}{5}$.	8. $\frac{69}{7}$.	9. $\frac{87}{11}$.	10. $\frac{92}{12}$.
11. $\frac{125}{25}$.	12. $\frac{236}{33}$.	13. $\frac{425}{30}$.	14. $\frac{93}{92}$.	15. $\frac{76}{76}$.
16. $\frac{298}{16}$.	17. $\frac{349}{32}$.	18. $\frac{276}{18}$.	19. $\frac{198}{16}$.	20. $\frac{725}{42}$.
21. $\frac{632}{32}$.	22. $\frac{427}{16}$.	23. $\frac{629}{24}$.	24. $\frac{332}{64}$.	25. $\frac{342}{12}$.
26. $\frac{190}{95}$.	27. $\frac{625}{50}$.	28. $\frac{221}{11}$.	29. $\frac{625}{25}$.	30. $\frac{896}{128}$.
31. $\frac{127}{25}$.	32. $\frac{175}{119}$.	33. $\frac{113}{104}$.	34. $\frac{125}{75}$.	35. $\frac{441}{21}$.
36. $\frac{2653}{53}$.	37. $\frac{12221}{121}$.	38. $\frac{10001}{77}$.	39. $\frac{20002}{11}$.	40. $\frac{17288}{144}$.

ORAL EXERCISE

1. Reduce $\frac{48}{80}$ to fortieths. Reduce the result to twentieths. Continue reducing as far as possible.

2. Reduce $\frac{42}{63}$ to twenty-firsts. Reduce the result to thirds. Can you reduce to any lower terms?

3. Write the fraction $\frac{144}{216}$ on the blackboard. Reduce it to as low terms as possible.

96. Lowest terms. A fraction is reduced to its *lowest terms* when the terms are prime to each other (§ 60).

For example, $\frac{44}{48} = \frac{22}{24} = \frac{11}{12}$, and $\frac{44}{48}$ is reduced to its lowest terms when reduced to $\frac{11}{12}$, for 11 and 12 are prime to each other (§ 60).

97. Reduction to lowest terms. *To reduce a fraction to lowest terms, cancel all common factors from both terms.*

We might cancel the greatest common divisor, but to find this we usually factor the terms, and it is easier to cancel the factors as we find them. Cancel as large factors as possible.

98. Practically we do not meet fractions whose terms are large numbers, like $\frac{345604}{590604}$, decimal fractions having taken their place. Even such easy fractions as are given below are rarely met.

WRITTEN EXERCISE

Reduce to lowest terms:

1. $\frac{28}{77}$.	2. $\frac{35}{45}$.	3. $\frac{15}{55}$.	4. $\frac{24}{32}$.	5. $\frac{56}{63}$.
6. $\frac{63}{77}$.	7. $\frac{45}{55}$.	8. $\frac{26}{39}$.	9. $\frac{56}{91}$.	10. $\frac{66}{78}$.
11. $\frac{45}{75}$.	12. $\frac{21}{91}$.	13. $\frac{51}{93}$.	14. $\frac{27}{57}$.	15. $\frac{63}{69}$.
16. $\frac{85}{95}$.	17. $\frac{44}{52}$.	18. $\frac{69}{93}$.	19. $\frac{40}{56}$.	20. $\frac{60}{65}$.
21. $\frac{72}{104}$.	22. $\frac{60}{105}$.	23. $\frac{72}{144}$.	24. $\frac{111}{123}$.	25. $\frac{65}{70}$.
26. $\frac{12}{144}$.	27. $\frac{75}{125}$.	28. $\frac{35}{630}$.	29. $\frac{130}{195}$.	30. $\frac{147}{350}$.
31. $\frac{125}{175}$.	32. $\frac{1440}{1728}$.	33. $\frac{18750}{25000}$.	34. $\frac{385}{700}$.	35. $\frac{231}{308}$.

Least Common Denominator

ORAL EXERCISE

1. Express $\frac{3}{4}$ as eighths. Add $\frac{3}{4}$ and $\frac{1}{8}$; $\frac{3}{4}$ and $\frac{7}{8}$.

2. In adding $\frac{3}{4}$ and $\frac{1}{8}$ we first reduce $\frac{3}{4}$ to a fraction with what denominator?

3. In subtracting $\frac{3}{8}$ from $\frac{1}{2}$ we first reduce $\frac{1}{2}$ to a fraction with what denominator?

4. Before we add or subtract fractions, what must be the nature of the denominators?

Express as fractions with the same denominator:

5. $\frac{1}{2}$, $\frac{1}{4}$. 6. $\frac{1}{4}$, $\frac{1}{8}$. 7. $\frac{1}{2}$, $\frac{1}{8}$. 8. $\frac{1}{2}$, $\frac{5}{8}$.

99. Similar fractions. If several fractions have the same denominator, they are said to be *similar fractions* and to have a *common denominator.*

100. Least common denominator. If the common denominator is the smallest possible, the fractions are said to have the *least common denominator* (l.c.d.).

For example, $\frac{1}{8}$ and $\frac{3}{8}$ have a common denominator, but they may be reduced to $\frac{2}{4}$ and $\frac{1}{4}$, 4 being the least common denominator.

101. Reduction to l.c.d. To reduce $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{12}$ to fractions having the least common denominator.

Since the required denominator must contain 4, 8, and 12, it must be their least common multiple.

We see that 4, 8, and 12 all contain 4, and that 8 also contains 2, and 12 contains 3. Therefore the least common multiple is $4 \times 2 \times 3$, or 24.

Since $24 \div 4 = 6$, the terms of $\frac{1}{4}$ must be multiplied by 6. In the same way the other multipliers are found.

$$4)\overline{4,\ 8,\ 12}$$
$$1,\ 2,\ 3$$

$$\frac{6 \times 1}{6 \times 4} = \frac{6}{24}$$

$$\frac{3 \times 3}{3 \times 8} = \frac{9}{24}$$

$$\frac{2 \times 5}{2 \times 12} = \frac{10}{24}$$

ORAL EXERCISE

Reduce to similar fractions, in Exs. 1–12:

1. $\frac{2}{3}, \frac{1}{2}$.
2. $\frac{3}{5}, \frac{2}{3}$.
3. $\frac{4}{16}, \frac{2}{8}$.
4. $\frac{2}{5}, \frac{7}{10}$.
5. $\frac{5}{15}, \frac{2}{30}$.
6. $\frac{1}{7}, \frac{2}{14}$.
7. $\frac{3}{5}, \frac{4}{10}$.
8. $\frac{2}{8}, \frac{3}{4}$.
9. $\frac{10}{20}, \frac{15}{30}$.
10. $\frac{6}{8}, \frac{9}{10}$.
11. $\frac{2}{3}, \frac{3}{30}$.
12. $\frac{5}{7}, \frac{6}{14}$.

Reduce to fractions having the least common denominator:

13. $\frac{1}{2}, \frac{2}{3}$.
14. $\frac{3}{5}, \frac{4}{7}$.
15. $\frac{4}{10}, \frac{3}{5}$.
16. $\frac{3}{7}, \frac{2}{3}$.
17. $\frac{7}{8}, \frac{1}{16}$.
18. $\frac{3}{16}, \frac{5}{32}$.
19. $\frac{7}{12}, \frac{1}{4}$.
20. $\frac{2}{7}, \frac{4}{5}$.
21. $\frac{2}{8}, \frac{3}{4}, \frac{1}{6}$.
22. $\frac{2}{9}, \frac{2}{3}, \frac{5}{6}$.
23. $\frac{3}{8}, \frac{3}{5}, \frac{3}{10}$.
24. $\frac{7}{8}, \frac{1}{5}, \frac{1}{40}$.

WRITTEN EXERCISE

Reduce to fractions having the least common denominator, in Exs. 1–30:

1. $\frac{3}{25}, \frac{6}{35}$.
2. $\frac{2}{49}, \frac{5}{63}$.
3. $\frac{8}{27}, \frac{5}{36}$.
4. $\frac{6}{25}, \frac{7}{55}$.
5. $\frac{3}{62}, \frac{4}{93}$.
6. $\frac{5}{36}, \frac{7}{72}$.
7. $\frac{8}{121}, \frac{13}{143}$.
8. $\frac{2}{7}, \frac{8}{9}, \frac{3}{8}$.
9. $\frac{7}{150}, \frac{11}{180}$.
10. $\frac{4}{625}, \frac{6}{1225}$.
11. $\frac{12}{144}, \frac{11}{132}$.
12. $\frac{3}{130}, \frac{4}{143}$.
13. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.
14. $\frac{5}{222}, \frac{11}{420}$.
15. $\frac{5}{8}, \frac{3}{16}, \frac{4}{32}$.
16. $\frac{3}{25}, \frac{4}{15}, \frac{3}{10}$.
17. $\frac{4}{9}, \frac{8}{15}, \frac{6}{25}$.
18. $\frac{5}{32}, \frac{3}{16}, \frac{7}{8}$.
19. $\frac{3}{20}, \frac{7}{30}, \frac{2}{15}$.
20. $\frac{3}{50}, \frac{2}{25}, \frac{4}{5}$.
21. $\frac{2}{27}, \frac{4}{9}, \frac{5}{81}$.
22. $\frac{3}{8}, \frac{2}{13}, \frac{7}{130}$.
23. $\frac{2}{3}, \frac{3}{7}, \frac{4}{5}, \frac{1}{10}$.
24. $\frac{1}{16}, \frac{5}{8}, \frac{9}{112}$.
25. $\frac{4}{37}, \frac{7}{74}, \frac{5}{111}$.
26. $\frac{3}{26}, \frac{7}{39}, \frac{5}{52}$.
27. $\frac{2}{21}, \frac{5}{49}, \frac{4}{63}$.
28. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$.
29. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{5}{16}, \frac{3}{32}$.
30. $\frac{1}{9}, \frac{2}{3}, \frac{5}{6}, \frac{7}{12}, \frac{5}{36}$.

Reduce the fractional parts as directed above:

31. $31\frac{4}{27}, 42\frac{7}{99}$.
32. $29\frac{5}{11}, 36\frac{13}{77}$.
33. $42\frac{17}{18}, 61\frac{25}{81}$.
34. $23\frac{4}{21}, 67\frac{9}{28}$.
35. $32\frac{4}{63}, 77\frac{6}{77}$.
36. $81\frac{7}{32}, 27\frac{7}{48}$.
37. $71\frac{7}{15}, 62\frac{11}{50}$.
38. $68\frac{2}{37}, 41\frac{11}{74}$.
39. $18\frac{7}{65}, 21\frac{11}{52}$.

Addition

ORAL EXERCISE

1. How many ninths are $\frac{2}{9} + \frac{4}{9}$? How many thirds?
2. How many elevenths are $\frac{7}{11} + \frac{4}{11}$? How many ones?
3. How many tenths are $\frac{9}{10} + \frac{9}{10} + \frac{2}{10}$? How many ones?
4. How much is $\frac{1}{2} + \frac{1}{4}$? $\frac{1}{2} + \frac{3}{4}$? $\frac{1}{2} + \frac{1}{8}$? $\frac{1}{2} + \frac{7}{8}$? $\frac{2}{3} + \frac{5}{6}$?

102. Adding fractions. Therefore,

To add fractions, reduce them to fractions having the least common denominator, and add the numerators for the numerator of the sum. Then reduce to lowest terms.

103. For example, add $\frac{3}{5}$, $\frac{9}{10}$, $\frac{7}{12}$.

The l.c.d. is evidently $5 \times 2 \times 2 \times 3 = 60$.

$$60 \div 5 = 12, \text{ and } \frac{12 \times 3}{12 \times 5} = \frac{36}{60}.$$

$$60 \div 10 = 6, \text{ and } \frac{6 \times 9}{6 \times 10} = \frac{54}{60}.$$

$$60 \div 12 = 5, \text{ and } \frac{5 \times 7}{5 \times 12} = \frac{35}{60}.$$

$$\tfrac{36}{60} + \tfrac{54}{60} + \tfrac{35}{60} = \tfrac{125}{60} = 2\tfrac{5}{60} = 2\tfrac{1}{12}.$$

104. Adding mixed numbers. To add $2\frac{3}{4}$ and $6\frac{7}{8}$.

$$2\tfrac{3}{4} + 6\tfrac{7}{8} = 2\tfrac{6}{8} + 6\tfrac{7}{8}$$
$$= 8\tfrac{13}{8} \text{ or } 9\tfrac{5}{8}.$$

$$\begin{array}{r} 2\frac{6}{8} \\ 6\frac{7}{8} \\ \hline 9\frac{5}{8} \end{array}$$

WRITTEN EXERCISE

1. $\frac{1}{2} + \frac{2}{3}$.
2. $\frac{3}{4} + \frac{1}{8}$.
3. $\frac{4}{5} + \frac{1}{2}$.
4. $\frac{1}{8} + \frac{3}{4} + \frac{1}{2}$.
5. $\frac{1}{10} + \frac{2}{5} + \frac{1}{5}$.
6. $\frac{1}{16} + \frac{3}{8} + \frac{1}{4}$.
7. $\frac{3}{16} + \frac{3}{8} + \frac{3}{4}$.
8. $\frac{1}{6} + \frac{1}{3} + \frac{1}{12}$.
9. $\frac{5}{6} + \frac{7}{12} + \frac{5}{24}$.
10. $2\frac{1}{2} + 7\frac{1}{4} + 9\frac{3}{4}$.
11. $6\frac{7}{8} + 2\frac{1}{8} + 9\frac{1}{2}$.
12. $2\frac{3}{4} + 1\frac{7}{8} + 3\frac{5}{56}$.

Subtraction

ORAL EXERCISE

1. $\frac{1}{2} + \frac{1}{4}$. 2. $\frac{3}{4} - \frac{1}{2}$. 3. $\frac{3}{4} - \frac{1}{8}$. 4. $1\frac{3}{4} - \frac{1}{8}$.
5. $\frac{5}{6} + \frac{1}{3}$. 6. $1\frac{1}{6} - \frac{5}{6}$. 7. $1\frac{1}{3} - \frac{5}{6}$. 8. $5\frac{1}{3} - \frac{5}{6}$.
9. $\frac{2}{5} - \frac{1}{10}$. 10. $\frac{1}{3} - \frac{1}{6}$. 11. $\frac{1}{4} - \frac{1}{8}$. 12. $4\frac{1}{4} - 1\frac{1}{8}$.
13. $\frac{1}{8} - \frac{1}{12}$. 14. $\frac{5}{6} - \frac{1}{12}$. 15. $\frac{5}{6} - \frac{5}{12}$. 16. $3\frac{5}{6} - 1\frac{5}{12}$.

Tell the number that n *stands for in the following:*

17. $\frac{1}{2} + n = \frac{3}{4}$. 18. $\frac{3}{4} - n = \frac{1}{2}$. 19. $\frac{3}{4} - \frac{1}{2} = n$.
20. $\frac{2}{3} + n = 1\frac{1}{3}$. 21. $1\frac{1}{3} - n = \frac{2}{3}$. 22. $1\frac{1}{3} - \frac{2}{3} = n$.

105. Subtracting fractions. From our work in adding fractions we learn how to subtract.

To subtract fractions, reduce them to fractions having the least common denominator, and subtract the numerators for the numerator of the difference. Then reduce to lowest terms.

For example, $\frac{1}{2} - \frac{3}{10} = \frac{5}{10} - \frac{3}{10} = \frac{2}{10} = \frac{1}{5}$.

WRITTEN EXERCISE

1. $\frac{1}{2} - \frac{5}{12}$. 2. $\frac{7}{8} - \frac{1}{16}$. 3. $\frac{3}{8} - \frac{1}{24}$. 4. $\frac{2}{3} - \frac{5}{12}$.
5. $\frac{3}{4} - \frac{3}{16}$. 6. $\frac{5}{8} - \frac{5}{24}$. 7. $\frac{7}{8} - \frac{5}{24}$. 8. $\frac{3}{4} - \frac{7}{12}$.
9. $\frac{5}{12} - \frac{1}{36}$. 10. $\frac{5}{24} - \frac{1}{12}$. 11. $\frac{7}{16} - \frac{7}{32}$. 12. $\frac{8}{9} - \frac{4}{21}$.
13. $\frac{6}{7} - \frac{2}{3}$. 14. $\frac{8}{9} - \frac{1}{6}$. 15. $\frac{7}{11} - \frac{1}{2}$. 16. $\frac{5}{12} - \frac{3}{8}$.
17. $\frac{9}{10} - \frac{3}{5}$. 18. $\frac{5}{9} - \frac{1}{12}$. 19. $\frac{8}{21} - \frac{3}{14}$. 20. $\frac{7}{9} - \frac{2}{21}$.
21. $\frac{2}{3} - \frac{4}{27}$. 22. $\frac{6}{7} - \frac{2}{35}$. 23. $\frac{5}{8} - \frac{3}{32}$. 24. $\frac{7}{11} - \frac{5}{33}$.
25. $1\frac{1}{14} - \frac{4}{21}$. 26. $1\frac{5}{16} - \frac{3}{64}$. 27. $\frac{9}{25} - \frac{9}{75}$. 28. $1\frac{1}{12} - 1\frac{1}{84}$.
29. $1\frac{5}{17} - \frac{2}{51}$. 30. $1\frac{2}{13} - \frac{7}{39}$. 31. $\frac{8}{9} - \frac{7}{15}$. 32. $\frac{6}{7} - \frac{4}{5}$.

33. If you live $\frac{7}{8}$ mi. from school, how far have you still to go when you have gone $\frac{3}{4}$ mi.?

106. Subtracting mixed numbers. From $17\frac{1}{3}$ to subtract $8\frac{5}{6}$.

$$17\tfrac{1}{3} - 8\tfrac{5}{6} = 17\tfrac{2}{6} - 8\tfrac{5}{6}.$$

We may think of $17\frac{2}{6}$ as $16\frac{8}{6}$, and $16\frac{8}{6} - 8\frac{5}{6} = 8\frac{3}{6} = 8\frac{1}{2}$.

$$\begin{array}{r} 17\frac{2}{6} \\ -\,8\frac{5}{6} \\ \hline 8\frac{1}{2} \end{array}$$

WRITTEN EXERCISE

Add or subtract as indicated:

1. $493\frac{1}{3} - 127\frac{5}{9}$.
2. $200\frac{1}{9} - 77\frac{5}{6}$.
3. $426\frac{1}{5} - 263\frac{3}{10}$.
4. $396\frac{2}{3} - 129\frac{2}{3}$.
5. $620\frac{5}{8} - 531\frac{1}{12}$.
6. $426\frac{2}{7} - 329\frac{5}{21}$.
7. $500\frac{4}{7} - 287\frac{3}{14}$.
8. $531\frac{3}{5} - 148\frac{4}{15}$.
9. $\frac{3}{7} + \frac{5}{14} + \frac{6}{35}$.
10. $\frac{4}{5} + \frac{3}{8} + \frac{1}{10}$.
11. $\frac{3}{8} + \frac{5}{12} + \frac{7}{16}$.
12. $\frac{4}{5} + \frac{3}{4} + \frac{7}{10}$.
13. $\frac{7}{12} + \frac{7}{20} + \frac{7}{10}$.
14. $\frac{6}{7} + \frac{3}{14} + \frac{2}{21}$.
15. $\frac{9}{20} + \frac{7}{30} + \frac{3}{10}$.
16. $\frac{3}{5} + \frac{7}{10} + \frac{4}{15}$.
17. $\frac{3}{14} + \frac{4}{21} + \frac{9}{35}$.
18. $\frac{7}{8} + \frac{3}{4} + \frac{1}{2} + \frac{5}{32}$.
19. $\frac{5}{12} + \frac{11}{72} + \frac{13}{144}$.
20. $4\frac{2}{3} + 9\frac{7}{8} + 23\frac{5}{12}$.
21. $\frac{2}{3} + \frac{3}{4} + \frac{5}{24} + \frac{7}{12}$.
22. $16\frac{4}{5} + 21\frac{1}{8} + 32\frac{1}{20}$.
23. $\frac{6}{7} + \frac{3}{5} + \frac{9}{35} + \frac{9}{14}$.
24. $\frac{3}{16}$ in. $+ \frac{7}{8}$ in. $- \frac{1}{4}$ in.

25. If Carrie has a strip of cloth $5\frac{1}{8}$ in. wide, how much must she cut off to make it $3\frac{1}{2}$ in. wide?

26. If Jack has a kite that is $14\frac{3}{4}$ in. on each of two of its sides, and $19\frac{7}{8}$ in. on each of the other two, how many inches around the kite?

27. If you are building a playhouse and need a stick $15\frac{7}{8}$ in. long for a window sill, and have a stick $17\frac{1}{4}$ in. long, how much must you saw off?

28. A farmer owns a triangular piece of land that is $175\frac{2}{3}$ rd. on one side, $167\frac{1}{2}$ rd. on another, and $37\frac{1}{8}$ rd. on the third. How many rods of fence will he need for it?

WRITTEN EXERCISE

1. Frank is $4\frac{1}{8}$ ft. tall, and Rob lacks 4 in. of being as tall as Frank. How tall is Rob?

2. John is $11\frac{2}{3}$ yr. old, Mary is $\frac{1}{2}$ yr. older, and Tom is $1\frac{1}{4}$ yr. younger than Mary. How old is Tom?

3. If my finger nail is $\frac{7}{16}$ in. long, and the rest of my finger is $3\frac{3}{8}$ in. long, what is the total length?

4. If you weigh $56\frac{3}{4}$ lb., and your dog weighs $18\frac{2}{3}$ lb., and your books weigh $3\frac{1}{2}$ lb., what is the total weight?

5. Harry had a kite string $198\frac{2}{3}$ yd. long. His kite was up in a shower and the string shrank $1\frac{7}{8}$ yd. How long was it then?

6. A plate of glass $37\frac{1}{8}$ in. by $23\frac{1}{4}$ in. was set in a picture frame that covered $\frac{3}{8}$ in. from each edge. What are the dimensions of the glass not covered by the frame?

7. In making a dress ruffle $3\frac{7}{8}$ in. wide, enough cloth must be allowed to turn in $\frac{7}{8}$ in. on one side and $1\frac{1}{4}$ in. on the other. What is the total width of cloth needed?

8. In running a gas pipe into a schoolroom the gas fitter found he needed four pieces, 6 ft. $2\frac{1}{2}$ in., 5 ft. $3\frac{7}{8}$ in., 10 ft. 2 in., and a piece half as long as the third. What was the total length required?

9. A tenpenny nail is $2\frac{3}{4}$ in. long, and a fourpenny nail is $1\frac{3}{8}$ in. shorter; how long is a fourpenny nail? A sixpenny nail is $\frac{5}{8}$ in. longer than a fourpenny; how long is a sixpenny nail?

107. Work with fractions can be made much more interesting and real by supplying the class with rulers and with paper for folding. Let the pupils fold a piece 1 ft. square, fold a border $\frac{7}{8}$ in., fold again $1\frac{1}{2}$ in. farther from the edge, and so on, introducing reduction, addition, and subtraction.

ORAL EXERCISE

Add as indicated in Exs. 1–12:

1. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.
2. $\frac{1}{2} + \frac{3}{4} + \frac{7}{8}$.
3. $\frac{1}{4} + \frac{3}{8} + \frac{1}{2}$.
4. $\frac{1}{3} + \frac{1}{6} + \frac{1}{12}$.
5. $\frac{2}{3} + \frac{5}{6} + \frac{7}{12}$.
6. $\frac{1}{6} + \frac{2}{3} + \frac{5}{12}$.
7. $\frac{1}{4} + \frac{1}{8} + \frac{1}{16}$.
8. $\frac{3}{4} + \frac{1}{8} + \frac{1}{16}$.
9. $\frac{3}{4} + \frac{7}{8} + \frac{1}{16}$.
10. $\frac{1}{6} + 1\frac{1}{2} + 2\frac{1}{3}$.
11. $\frac{5}{6} + 3\frac{1}{2} + 5\frac{2}{3}$.
12. $\frac{1}{8} + 5\frac{3}{4} + 7\frac{7}{8}$.

Subtract as indicated in Exs. 13–28:

13. $\frac{3}{4} - \frac{1}{2}$.
14. $\frac{3}{4} - \frac{3}{8}$.
15. $\frac{15}{16} - \frac{3}{8}$.
16. $\frac{7}{20} - \frac{1}{10}$.
17. $\frac{1}{3} - \frac{1}{6}$.
18. $\frac{2}{3} - \frac{1}{6}$.
19. $\frac{5}{12} - \frac{1}{6}$.
20. $\frac{3}{40} - \frac{1}{20}$.
21. $1\frac{1}{2} - \frac{1}{2}$.
22. $\frac{7}{12} - \frac{1}{4}$.
23. $\frac{2}{3} - \frac{5}{12}$.
24. $\frac{5}{7} - \frac{3}{14}$.
25. $\frac{5}{16} - \frac{1}{8}$.
26. $\frac{25}{32} - \frac{1}{16}$.
27. $\frac{1}{8} - \frac{1}{64}$.
28. $\frac{7}{11} - \frac{1}{22}$.

WRITTEN EXERCISE

1. $5\frac{2}{9} + 3\frac{1}{12} + 1\frac{4}{27}$.
2. $\frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{7}{8}$.
3. $6\frac{3}{5} + 2\frac{1}{4} + \frac{3}{10} + 2\frac{1}{2}$.
4. $4\frac{3}{16} + \frac{2}{9} + 2\frac{5}{12} + \frac{1}{2}$.
5. $3\frac{5}{12} + \frac{3}{25} + 2\frac{1}{6} + 4\frac{7}{10}$.
6. $2\frac{1}{16} + 5\frac{4}{5} + \frac{2}{3} + 6\frac{3}{8}$.
7. $20\frac{1}{8} - 2\frac{1}{4}$.
8. $5\frac{3}{16} - 2\frac{7}{12}$.
9. $11\frac{1}{40} - 6\frac{5}{8}$.
10. $20\frac{3}{10} - 7\frac{5}{6}$.
11. $15\frac{3}{8} - 5\frac{23}{32}$.
12. $42\frac{1}{16} - 38\frac{7}{12}$.
13. $37\frac{3}{5} - 19\frac{7}{8}$.
14. $41\frac{6}{7} - 19\frac{4}{5}$.
15. $28\frac{5}{8} - 19\frac{31}{32}$.
16. $121\frac{3}{8} - 36\frac{7}{8}$.
17. $42\frac{5}{8} - 29\frac{3}{4}$.
18. $31\frac{1}{2} - 18\frac{7}{8}$.

19. What is the difference in length between a tack $\frac{1}{8}$ in. long and one $\frac{1}{4}$ in. long? Also between one $\frac{1}{4}$ in. long and one $\frac{3}{8}$ in. long? between one $\frac{3}{8}$ in. long and one $\frac{7}{16}$ in. long? Also between one $\frac{7}{16}$ in. long and one $\frac{9}{16}$ in. long? between one $\frac{9}{16}$ in. long and one $\frac{5}{8}$ in. long?

108. It should be remembered that the common fractions most frequently met are those with denominators 2, 4, 8, 16, 32, 64; 3, 6, 9, 12, 24, 36, 48; 5, 10.

WRITTEN EXERCISE

1. A farmer has 57 hens. In 5 weeks in the spring they laid 855 eggs. How many eggs did they all average per week? How many did each average per week? for the 5 weeks?

2. He fed these hens 112 lb. of corn in the 5 weeks. At 56 lb. to the bushel, how many bushels did he feed? How much was it worth at 75 ct. a bushel?

3. He also fed them 96 lb. of wheat. At 60 lb. to the bushel, how many bushels did he feed? How much was it worth at 90 ct. a bushel?

4. He also fed them 40 lb. of oats. At 32 lb. to the bushel, how many bushels did he feed? How much was it worth at 48 ct. a bushel?

5. He also fed them 16 lb. of buckwheat. At 48 lb. to the bushel, what part of a bushel did he feed? How much was it worth at 60 ct. a bushel?

6. The corn, wheat, oats, and buckwheat fed to these hens amounted to how many pounds? bushels? How many pounds did each hen eat?

7. The rest of the feed cost $1.96. What was the total cost of feed for the 57 hens for 5 weeks? the average cost per hen? the average cost per egg?

8. The eggs sold at 20 ct. a dozen. How much did the farmer make per dozen above the cost of feed? How much on all the 855 eggs?

9. Allowing $4.25 for the use of the property, and $3 for the time spent in caring for the hens, how much did the farmer make on eggs during these 5 weeks? How much did he make per hen?

Multiplication of Fractions

ORAL EXERCISE

1. How much is $\frac{1}{2}$ of \$4? $\frac{1}{2}$ of 4 fifths? $\frac{1}{2}$ of $\frac{4}{7}$?

2. How much is $\frac{1}{3}$ of 6 bu.? $\frac{1}{3}$ of 6 sevenths? $\frac{1}{3}$ of $\frac{6}{11}$?

3. How much is $\frac{1}{4}$ of $\frac{16}{17}$? $\frac{1}{4}$ of $\frac{8}{15}$? $\frac{1}{4}$ of $\frac{24}{25}$? $\frac{1}{3}$ of $\frac{6}{5}$?

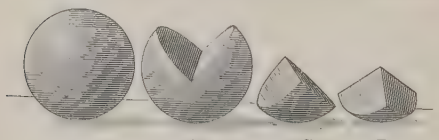
4. In each of these cases in which you found a fraction of a fraction, how did you proceed?

State results in Exs. 5–12:

5. $\frac{1}{7}$ of $\frac{14}{15}$. 6. $\frac{1}{5}$ of $\frac{15}{16}$. 7. $\frac{1}{6}$ of $\frac{12}{13}$. 8. $\frac{1}{2}$ of $\frac{8}{9}$.

9. $\frac{1}{8}$ of $\frac{16}{21}$. 10. $\frac{1}{9}$ of $\frac{18}{25}$. 11. $\frac{1}{10}$ of $\frac{20}{21}$. 12. $\frac{1}{12}$ of $\frac{24}{25}$.

13. Point to $\frac{1}{3}$ of $\frac{3}{4}$ of the sphere. What part of the sphere is it?



14. Why can you not find $\frac{1}{2}$ of $\frac{1}{4}$ in the same way?

15. Point to $\frac{1}{2}$ of $\frac{1}{4}$ of this sphere. What part of the sphere is it?

16. Draw on the blackboard a rectangle $2\frac{1}{2}$ ft. long and 1 ft. high; mark it off into fifths; mark off each fifth into halves. Then how much is $\frac{1}{2}$ of $\frac{1}{5}$? Also $\frac{1}{5}$ of $\frac{1}{2}$?

17. Draw another rectangle as in Ex. 16, marking off each fifth into thirds. How much is $\frac{1}{3}$ of $\frac{1}{5}$?

18. State the value of $\frac{1}{3}$ of $\frac{1}{5}$, $\frac{1}{5}$ of $\frac{1}{3}$; $\frac{1}{5}$ of $\frac{1}{7}$, $\frac{1}{7}$ of $\frac{1}{5}$; $\frac{1}{5}$ of $\frac{1}{9}$, $\frac{1}{9}$ of $\frac{1}{5}$; $\frac{1}{3}$ of $\frac{1}{10}$, $\frac{1}{10}$ of $\frac{1}{3}$; $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{3}$ of $\frac{1}{2}$.

19. In each case in Ex. 18 what did you do to the two denominators?

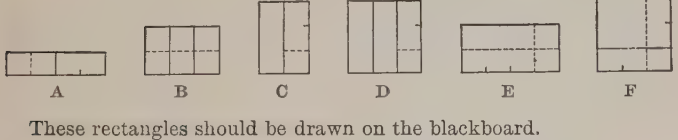
20. Because $\frac{1}{4}$ of \$1 is 25 ct., $\frac{3}{4}$ of \$1 is how much?

21. Because $\frac{1}{4}$ of $\frac{1}{5}$ is $\frac{1}{20}$, $\frac{3}{4}$ of $\frac{1}{5}$ is how many times $\frac{1}{20}$?

22. Then how much is $\frac{3}{4}$ of $\frac{1}{5}$? $\frac{3}{4}$ of $\frac{1}{7}$? $\frac{3}{4}$ of $\frac{1}{10}$? $\frac{3}{4}$ of $\frac{1}{11}$?

ORAL EXERCISE

1. What part of A is $\frac{1}{2}$ of $\frac{1}{2}$ of A?



These rectangles should be drawn on the blackboard.

2. What part of B is $\frac{1}{2}$ of $\frac{1}{3}$ of B?

3. Show that $\frac{1}{2}$ of $\frac{1}{3}$ of B is the same as $\frac{1}{3}$ of $\frac{1}{2}$ of B.

4. What part of C is $\frac{1}{3}$ of $\frac{1}{2}$ of C?

5. What part of D is $\frac{1}{3}$ of $\frac{1}{3}$ of D?

6. Each third of D equals how many ninths of D?

7. Point to $\frac{1}{2}$ of E; to $\frac{1}{4}$ of $\frac{1}{2}$ of E. Also point to $\frac{1}{2}$ of $\frac{1}{4}$ of E. What part of E is $\frac{1}{2}$ of $\frac{1}{4}$ of E, or $\frac{1}{4}$ of $\frac{1}{2}$ of E?

8. Point to $\frac{1}{2}$ of $\frac{3}{4}$ of E; to $\frac{3}{4}$ of $\frac{1}{2}$ of E. What part of E is $\frac{3}{4}$ of $\frac{1}{2}$?

9. Point to $\frac{1}{3}$ of F; to $\frac{1}{4}$ of $\frac{1}{3}$ of F. Also point to $\frac{1}{4}$ of F, and to $\frac{1}{3}$ of $\frac{1}{4}$ of F. Each is what part of F?

10. Point to $\frac{3}{4}$ of F; to $\frac{1}{3}$ of $\frac{3}{4}$ of F. Also point to $\frac{1}{3}$ of F, and to $\frac{3}{4}$ of $\frac{1}{3}$ of F. Each is what part of F?

WRITTEN EXERCISE

Draw rectangles to show:

1. $\frac{1}{5}$ of $\frac{1}{3}$, or $\frac{1}{3}$ of $\frac{1}{5}$.
2. $\frac{1}{2}$ of $\frac{1}{6}$, or $\frac{1}{6}$ of $\frac{1}{2}$.
3. $\frac{2}{3}$ of $\frac{1}{5}$, or $\frac{1}{5}$ of $\frac{2}{3}$.
4. $\frac{1}{2}$ of $\frac{5}{6}$, or $\frac{5}{6}$ of $\frac{1}{2}$.
5. $\frac{3}{5}$ of $\frac{2}{3}$, or $\frac{2}{3}$ of $\frac{3}{5}$.
6. $\frac{4}{5}$ of $\frac{1}{6}$, or $\frac{1}{6}$ of $\frac{4}{5}$.
7. $\frac{1}{8}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{8}$.
8. $\frac{3}{8}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{3}{8}$.
9. $\frac{5}{8}$ of $\frac{1}{5}$, or $\frac{1}{5}$ of $\frac{5}{8}$.
10. $\frac{2}{6}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{2}{5}$.
11. $\frac{1}{10}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{10}$.
12. $\frac{3}{10}$ of $\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{3}{10}$.

ORAL EXERCISE

1. How much is 2 times 3 in.? 2 times $\frac{3}{7}$? 5 times $\frac{2}{11}$? How did you multiply these fractions by an integer?

2. How much is 4 times $\frac{1}{4}$? 4 times $\frac{3}{4}$? 2 times $\frac{3}{4}$? 5 times $\frac{7}{40}$? 7 times $\frac{4}{49}$? How did you multiply these fractions by an integer?

109. How to multiply a fraction by an integer. Therefore,

1. *To multiply a fraction by an integer, multiply the numerator by the multiplier.*

2. *Before actually multiplying, indicate the multiplication and cancel if possible.*

For example, to multiply $\frac{11}{63}$ by 18. Since we have 11 sixty-thirds, if we have 18 times as much we shall have

$$\frac{\cancel{18}^{2} \text{ times } 11}{\cancel{63}_{7}}, \text{ or } \frac{22}{7}, \text{ or } 3\tfrac{1}{7}.$$

ORAL EXERCISE

Multiply:

1. $\frac{5}{9}$ by 9.	2. $\frac{6}{7}$ by 7.	3. $\frac{4}{5}$ by 5.	4. $\frac{3}{8}$ by 8.
5. $\frac{3}{4}$ by 2.	6. $\frac{9}{10}$ by 5.	7. $\frac{7}{8}$ by 4.	8. $\frac{3}{10}$ by 5.
9. $\frac{3}{8}$ by 16.	10. $\frac{4}{5}$ by 10.	11. $\frac{5}{7}$ by 21.	12. $\frac{3}{11}$ by 44.
13. $\frac{5}{8}$ by 40.	14. $\frac{6}{7}$ by 28.	15. $\frac{5}{9}$ by 36.	16. $\frac{7}{12}$ by 24.
17. $\frac{3}{17}$ by 34.	18. $\frac{4}{13}$ by 26.	19. $\frac{5}{11}$ by 99.	20. $\frac{6}{25}$ by 100.

WRITTEN EXERCISE

Multiply:

1. $\frac{4}{21}$ by 35.	2. $\frac{6}{35}$ by 28.	3. $\frac{7}{36}$ by 27.	4. $\frac{5}{81}$ by 63.
5. $\frac{3}{41}$ by 82.	6. $\frac{4}{11}$ by 121.	7. $\frac{5}{49}$ by 63.	8. $\frac{7}{64}$ by 96.
9. $\frac{7}{72}$ by 16.	10. $\frac{11}{54}$ by 36.	11. $\frac{9}{56}$ by 49.	12. $\frac{13}{88}$ by 121.
13. $\frac{7}{108}$ by 24.	14. $\frac{6}{21}$ by 99.	15. $\frac{11}{128}$ by 192.	16. $\frac{11}{144}$ by 132.

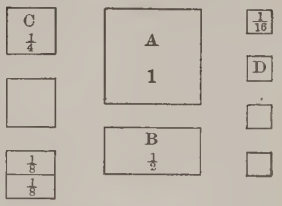
ORAL EXERCISE

1. In the picture, call A *one*. Point to $\frac{1}{2}$ of $\frac{1}{2}$; to $\frac{1}{2}$ of $\frac{1}{4}$.

2. Point to $\frac{3}{4}$ (three squares); to $\frac{2}{3}$ of $\frac{3}{4}$.

3. Point to $\frac{1}{3}$ of $\frac{3}{4}$ and tell what fraction it equals.

4. Point to $\frac{1}{3}$ of $1\frac{1}{2}$ and tell what fraction it equals.



Point to the following and tell what each equals:

5. $\frac{1}{5}$ of $1\frac{1}{4}$. 6. $\frac{1}{7}$ of $1\frac{3}{4}$. 7. $\frac{3}{4}$ of $\frac{1}{4}$. 8. $\frac{1}{2}$ of $\frac{4}{16}$.

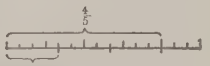
9. $\frac{1}{4}$ of $\frac{4}{16}$. 10. $\frac{1}{4}$ of $\frac{1}{4}$. 11. $\frac{1}{8}$ of $\frac{1}{2}$. 12. $\frac{1}{3}$ of $\frac{3}{16}$.

13. How much is $\frac{1}{2}$ of 4 ft.? $\frac{1}{2}$ of $\frac{4}{5}$? $\frac{1}{2}$ of $\frac{6}{7}$? $\frac{1}{4}$ of $\frac{4}{5}$? How did you take these fractional parts of fractions?

110. Multiplication by a fraction. To take a fractional part of a number is called *multiplying by a fraction.*

For example, to take $\frac{1}{2}$ of 4 is called multiplying 4 by $\frac{1}{2}$. It is indicated either by "$\frac{1}{2}$ of 4" or by "$\frac{1}{2} \times 4$," the $\times$ being read "of."

111. How to multiply a fraction by a fraction.

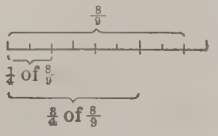


Since $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$, $\frac{2}{3}$ of $\frac{4}{5}$ is twice as much, or $\frac{8}{15}$. Therefore,

1. *To multiply a fraction by a fraction, multiply the numerators for a new numerator and the denominators for a new denominator.*

2. *Instead of actually multiplying, it is better to indicate the operation and cancel if possible.*

For example, $\frac{3}{4}$ of $\frac{8}{9} = \frac{\overset{2}{\cancel{3}} \times \cancel{8}}{\underset{3}{\cancel{4}} \times \cancel{9}} = \frac{2}{3}$.



ORAL EXERCISE

1. $\frac{1}{2}$ of $\frac{3}{5}$. 2. $\frac{1}{3}$ of $\frac{2}{7}$. 3. $\frac{1}{14}$ of $\frac{7}{8}$. 4. $\frac{2}{3}$ of $\frac{7}{9}$.

5. $\frac{1}{15}$ of $\frac{5}{7}$. 6. $\frac{1}{7}$ of $\frac{42}{50}$. 7. $\frac{1}{8}$ of $\frac{32}{35}$. 8. $\frac{1}{6}$ of $\frac{7}{8}$.

9. $\frac{1}{6}$ of $\frac{4}{11}$. 10. $\frac{1}{3}$ of $\frac{12}{13}$. 11. $\frac{1}{12}$ of $\frac{8}{15}$. 12. $\frac{1}{10}$ of $\frac{5}{6}$.

13. If a man owns a garden plot of $\frac{4}{5}$ acre, and sells $\frac{1}{3}$ of it, what part of an acre does he sell?

14. If one of you lives $1\frac{2}{3}$ miles from here, and I live $\frac{1}{5}$ as far away, how far from here do I live?

15. If you live $\frac{4}{5}$ of a mile from school, and Jack lives a quarter as far, how far does he live from school?

16. If Kate lives $1\frac{1}{3}$ miles away, and Mary lives $\frac{1}{3}$ as far, how far away does Mary live? If Jennie lives $\frac{1}{2}$ as far as Mary, how far away does Jennie live?

WRITTEN EXERCISE

1. $\frac{2}{3}$ of $\frac{4}{5}$. 2. $\frac{3}{7}$ of $\frac{5}{6}$. 3. $\frac{3}{4}$ of $\frac{6}{7}$. 4. $\frac{5}{6}$ of $1\frac{2}{5}$.

5. $\frac{7}{8}$ of $1\frac{2}{21}$. 6. $\frac{5}{7}$ of $1\frac{4}{15}$. 7. $\frac{3}{5}$ of $\frac{6}{7}$. 8. $\frac{4}{7}$ of $1\frac{1}{2}$.

9. $2\frac{1}{2}$ of $1\frac{1}{7}$. 10. $\frac{6}{5}$ of $1\frac{5}{16}$. 11. $\frac{2}{3}$ of $\frac{9}{4}$. 12. $\frac{6}{5}$ of $1\frac{5}{2}$.

13. $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{10}{11}$. 14. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$. 15. $\frac{3}{7}$ of $\frac{1}{6}$ of 14.

16. If a yard of cloth costs $\$\frac{3}{4}$, what will $\frac{5}{8}$ yd. cost?

17. If an express train runs 1 mi. in $\frac{3}{4}$ min., how long will it take it to run $\frac{7}{8}$ mi. at the same rate?

18. If a train runs $\frac{1}{3}$ of a certain distance in 1 hr., what part of the distance can it run in 45 min.?

19. Draw a rectangle 2 in. wide and 4 in. long. Divide it into square inches. How many are there? Show that you have either 2 times 4 sq. in. or 4 times 2 sq. in.

20. Find the area of a rectangle 17 ft. by 19 ft.; of one $\frac{3}{4}$ in. by 16 in.; of one 35 in. by $\frac{1}{5}$ in.

ORAL EXERCISE

1. How much is 4 times 5? 4 times $\frac{2}{9}$? 4 times $5\frac{2}{9}$?
2. How much is 2×8? $\frac{3}{4}$ of 8? $2\frac{3}{4}$ times 8?
3. How much is $\frac{1}{3}$ of $\frac{6}{5}$? Then how much is $3\frac{1}{3} \times 1\frac{1}{5}$?

112. Mixed numbers.

1. *To multiply a mixed number by an integer, multiply the whole and the fractional parts separately and add the products.*

For example, 6 times $3\frac{2}{5}$ equals $18\frac{12}{5}$, or $20\frac{2}{5}$.

2. *To multiply an integer by a mixed number, multiply by the whole and the fractional parts separately and add the products.*

For example, $3\frac{2}{5}$ times 6 equals 3×6, and $\frac{2}{5}$ of 6, or $18\frac{12}{5}$, or $20\frac{2}{5}$.

3. *To multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply the results.*

For example, $5\frac{2}{5} \times 3\frac{3}{4} = \frac{27}{\cancel{5}} \text{ of } \frac{\overset{3}{\cancel{15}}}{4} = \frac{81}{4} = 20\frac{1}{4}$.

WRITTEN EXERCISE

1. $17 \times 2\frac{1}{3}\frac{2}{4}$.	2. $27 \times 41\frac{5}{9}$.	3. $49 \times 81\frac{6}{7}$.
4. $81 \times 62\frac{5}{9}$.	5. $48 \times 37\frac{3}{10}$.	6. $26\frac{3}{4} \times 32$.
7. $37\frac{4}{5} \times 45$.	8. $29\frac{4}{5} \times 35$.	9. $49\frac{7}{8} \times 68$.
10. $25\frac{2}{3} \times 19$.	11. $47\frac{1}{10} \times 65$.	12. $26\frac{1}{2} \times 63$.
13. $35\frac{7}{8} \times 36$.	14. $6\frac{7}{8} \times 7\frac{7}{8}$.	15. $9\frac{2}{3} \times 15\frac{7}{8}$.
16. $16\frac{3}{4} \times 14\frac{1}{3}$.	17. $15\frac{7}{9} \times 14\frac{3}{7}$.	18. $14\frac{3}{4} \times 16\frac{4}{5}$.
19. $18\frac{4}{7} \times 23\frac{7}{8}$.	20. $19\frac{2}{3} \times 27\frac{4}{5}$.	21. $16\frac{3}{4} \times 28\frac{4}{9}$.

22. At $\$2\frac{3}{4}$ a yard, how much will $1\frac{1}{2}$ yd. of cloth cost?

Cancellation

ORAL EXERCISE

1. What is the value of $\frac{2 \times 3}{5 \times 3}$? What factor may you cancel?

2. State the value of: $\frac{2 \times 7}{2 \times 8}$, $\frac{3 \times 11}{4 \times 11}$, $\frac{5 \times 6}{6 \times 7}$.

3. If there is the same factor in both numerator and denominator, what do you do with it?

113. We found, in § 86, that we may cancel equal factors from both terms of a fraction.

114. Cancellation. Since a fraction is an indicated division, *equal factors may be canceled from both dividend and divisor without changing the quotient.*

Divide $2 \times 7 \times 15$ by $21 \times 5 \times 6$.

Writing this as a fraction (§ 95), 2 is canceled from 2 and 6 (§ 86); 7 is canceled from 7 and 21; 3 is canceled from 3 (left from the 21) and 15; 5 is canceled from 5 (left from the 15) and 5. Therefore the quotient is $\frac{1}{3}$, for 1 is a factor of the numerator.

$$\frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{7}} \times \overset{\cancel{5}}{15}}{\underset{\cancel{3}}{21} \times \cancel{5} \times \underset{3}{\cancel{6}}} = \frac{1}{3}$$

WRITTEN EXERCISE

Divide, using cancellation:

1. $2 \times 3 \times 6$ by $5 \times 9 \times 4$.
2. $3 \times 14 \times 20$ by $15 \times 21 \times 7$.
3. $4 \times 15 \times 27$ by $9 \times 5 \times 24$.
4. $7 \times 21 \times 77 \times 6$ by $49 \times 33 \times 42$.
5. $20 \times 25 \times 32 \times 48$ by $27 \times 40 \times 15 \times 12$.
6. $32 \times 27 \times 44 \times 39$ by $55 \times 72 \times 26 \times 8 \times 3$.
7. $125 \times 108 \times 51 \times 8$ by $40 \times 45 \times 84 \times 12 \times 3$.

115. Practical uses of cancellation. We found in §§ 109–112 that cancellation is useful in the multiplication of fractions and mixed numbers. In this it is of much practical importance, as will be seen on page 60.

What is the product of $\frac{1}{15}$, $45\frac{1}{2}$, $33\frac{1}{3}$, and $16\frac{7}{8}$?

Reducing to improper fractions, $45\frac{1}{2} = \frac{9}{2}\frac{1}{0}$, $33\frac{1}{3} = \frac{1}{3}\frac{0}{0}$, $16\frac{7}{8} = \frac{1}{8}\frac{3}{5}$. Proceeding, as in § 111, and canceling, the product is $1706\frac{1}{4}$.

$$\frac{\cancel{1} \times 91 \times \cancel{25} \times \cancel{27} \times \cancel{3}}{\cancel{15} \times 2 \times \cancel{3} \times \cancel{8}} = \frac{6825}{4} = 1706\frac{1}{4}$$

$$\cancel{3} \qquad\qquad \cancel{2}$$

WRITTEN EXERCISE

1. $20\frac{1}{2} \times 32\frac{1}{5} \times 16\frac{7}{8}$.
2. $15\frac{2}{3} \times 16\frac{7}{8} \times 33\frac{1}{3}$.
3. $27\frac{1}{5} \times 24\frac{1}{2} \times 37\frac{1}{2}$.
4. $42\frac{2}{3} \times 66\frac{2}{3} \times 75\frac{1}{16}$.
5. $35\frac{1}{16} \times 22\frac{1}{32} \times 12\frac{4}{5}$.
6. $7\frac{3}{5} \times 2\frac{1}{7} \times 5\frac{1}{4} \times 21\frac{1}{3}$.
7. $2\frac{1}{2} \times 3\frac{3}{4} \times 4\frac{4}{5} \times 5\frac{5}{6}$.
8. $3\frac{1}{2} \times 4\frac{2}{3} \times 5\frac{3}{4} \times 6\frac{4}{5} \times 7\frac{5}{6}$.
9. $8\frac{2}{3} \times 7\frac{1}{2} \times 5\frac{7}{16} \times 1\frac{3}{13}$.
10. $2\frac{2}{3} \times 3\frac{4}{5} \times 5\frac{6}{7} \times 6\frac{7}{8} \times 7\frac{1}{7}$.

11. Divide the product of 4, 7, 19, 21, and 64 by the product of 14, 38, 28, and 3.

12. Divide the product of 3, 9, 15, 38, 42, and 75 by the product of 19, 7, 3, 5, 27, 60, and 8.

13. Divide the product of 2, 3, 4, 5, 6, 7, 8, and 9 by the product 10, 12, 14, 15, and 16.

14. How many times as much will it cost to transport 12 boxes weighing 300 lb. each 200 mi., as to transport 6 boxes weighing 400 lb. each 75 mi., at the same rate per pound for each mile?

15. How many times as much will it cost to transport 3 boxes of merchandise weighing 400 lb. each a given distance, as to transport 2 boxes weighing 300 lb. each the same distance, at the same rate per pound?

116. Area of rectangle. If each of these orange boxes is 20 in. long and 10 in. wide, how many square inches in the area of the base?

If the base were 1 in. long and 1 in. wide, the area would be 1 sq. in. But it is 20 times as long as this, and 10 times as wide. Therefore the area is 20×10 times 1 sq. in. $= 200$ sq. in.

In all such examples the *inside measure* is meant.

117. Volume of a box. How many cubic inches in a box $3\frac{1}{2}$ in. long, $2\frac{1}{2}$ in. wide, and $1\frac{1}{3}$ in. deep?

If it were 1 in. long and 1 in. wide and 1 in. thick, it would contain 1 cu. in. But it is $3\frac{1}{2}$ times as long as this, $2\frac{1}{2}$ times as wide, and $1\frac{1}{3}$ times as deep. Therefore it contains $3\frac{1}{2} \times 2\frac{1}{2} \times 1\frac{1}{3}$ times 1 cu. in., or

$$\frac{7}{2} \times \frac{5}{2} \text{ times } \frac{4}{3} \text{ cu. in.} = \frac{7 \times 5 \times \cancel{4}}{\cancel{2} \times \cancel{2} \times 3} \text{ cu. in.} = \frac{35}{3} \text{ cu. in.} = 11\tfrac{2}{3} \text{ cu. in.}$$

WRITTEN EXERCISE

1. Figs are shipped in a box 20 in. by 2 in. by $3\frac{1}{2}$ in. How many cubic inches in a box?

2. California cherries are shipped in a box $15\frac{1}{2}$ in. by $8\frac{1}{2}$ in. by $3\frac{1}{2}$ in. How many cubic inches in a box?

3. The standard Florida orange box for shipping is 12 in. by 12 in. by $26\frac{5}{8}$ in. How many cubic inches does it contain?

4. A bushel contains $2150\frac{1}{2}$ cu. in. The capacity of one of the Florida orange boxes is how many cubic inches more than $1\frac{3}{4}$ bu.?

5. In shipping Georgia peaches a crate 8 in. by $12\frac{1}{2}$ in. by 22 in. is used. This contains how many cubic inches more than 1 bu.?

6. In New Jersey a bushel cranberry box is 12 in. by $8\frac{3}{4}$ in. by 22 in. How does this differ in capacity from the standard bushel?

7. California apples and pears for local use are shipped in a 50-pound box 22 in. by 10 in. by 12 in. How does this differ in capacity from the standard bushel?

8. A California cantaloupe crate is 38 in. long, 16 in. wide, and 15 in. deep. How many cubic inches does it contain?

9. The California orange box is sometimes $26\frac{1}{2}$ in. by $11\frac{1}{4}$ in. by $11\frac{1}{4}$ in., and sometimes 22 in. by $7\frac{3}{4}$ in. by $17\frac{1}{2}$ in. Which holds the more, and how many cubic inches more?

10. Dried fruits are shipped in 50-lb. or 25-lb. boxes. The 50-lb. box is $15\frac{1}{4}$ in. by 9 in. by 9 in. How many cubic inches in the box?

11. The 25-lb. box mentioned in Ex. 10 is sometimes $13\frac{3}{4}$ in. by $9\frac{3}{4}$ in. by $5\frac{3}{4}$ in., and sometimes 16 in. by 9 in. by 5 in. Which box holds the more, and how much more?

12. For shipping fruit out of the state, the sizes of some of the boxes used in California are as follows:

Plums, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by 4 in.
Cherries, $16\frac{1}{2}$ in. by $10\frac{3}{8}$ in. by $2\frac{1}{2}$ in.
Pears, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by $8\frac{1}{2}$ in.
Peaches, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by $4\frac{7}{8}$ in.

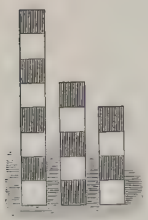
Find the number of cubic inches in each box.

Division of Fractions

ORAL EXERCISE

1. How much is 10 ft. $\div 5$? $\frac{10}{11} \div 5$? $\frac{10}{13} \div 5$? $\frac{14}{15} \div 7$? What is the effect of dividing the numerator?

2. How much is $\frac{1}{2}$ of $\frac{1}{5}$? Then how much is $\frac{1}{5} \div 2$? $\frac{1}{5} \div 3$? What is the effect of multiplying the denominator?



A B C

3. In this picture, if A is called 1, point to $\frac{5}{8}$. How much is $\frac{5}{8} \div 5$? Point to $\frac{1}{2}$. How much is $\frac{1}{2} \div 4$?

4. If B is called 1, point to $1\frac{3}{5}$. Point to that part which shows $1\frac{3}{5} \div 8$. Point to $\frac{4}{5}$; to $\frac{4}{5} \div 4$.

118. Dividing a fraction by an integer. We see that,

1. *Dividing the numerator by an integer divides the fraction by the integer.*

2. *Multiplying the denominator by an integer divides the fraction by the integer.*

119. It is sometimes easier to divide mixed numbers than improper fractions.

While $10\frac{1}{7} \div 3 = \frac{71}{7} \div 3 = \frac{71}{21} = 3\frac{8}{21}$, with a number like $244\frac{1}{2}$ to be divided by 3 we would not do this, for it would be too long. Here we should say, "$244\frac{1}{2} \div 3 = 81 + \frac{1}{3}$ of $1\frac{1}{2} = 81\frac{1}{3}$."

WRITTEN EXERCISE

1. $2\frac{5}{36} \div 5$.	2. $\frac{3}{17} \div 5$.	3. $\frac{5}{21} \div 15$.	4. $1\frac{13}{36} \div 7$.
5. $2\frac{3}{16} \div 5$.	6. $3\frac{5}{12} \div 41$.	7. $2\frac{7}{24} \div 11$.	8. $306\frac{1}{2} \div 9$.
9. $225\frac{1}{3} \div 5$.	10. $175\frac{1}{3} \div 6$.	11. $273\frac{1}{8} \div 2$.	12. $826\frac{1}{5} \div 5$.

ORAL EXERCISE

1. How many thirds of an inch in 1 in.? Then $1 \div \frac{1}{3}$ = how many?

2. How many fifths of an inch in 1 in.? Then $1 \div \frac{1}{5}$ = how many?

3. Tell the values of

$1 \div \frac{1}{8}$ $\quad 1 \div \frac{1}{9}$ $\quad 1 \div \frac{1}{7}$ $\quad 1 \div \frac{1}{10}$ $\quad 1 \div \frac{1}{20}$

4. If there are five fifths in 1, how many fifths are there in 2? in 3? Then $2 \div \frac{1}{5}$ = how many? $3 \div \frac{1}{5}$ = how many? $4 \div \frac{1}{5}$ = how many? $5 \div \frac{1}{7}$ = how many?

5. Tell the values of

$2 \div \frac{1}{4}$ $\quad 3 \div \frac{1}{7}$ $\quad 2 \div \frac{1}{10}$ $\quad 4 \div \frac{1}{6}$ $\quad 9 \div \frac{1}{8}$

6. Instead of dividing 2 by $\frac{1}{4}$, you will get the same result by multiplying 2 by what number?

7. Then $5 \div \frac{1}{7} = 5$ multiplied by what number?
$7 \div \frac{1}{8} = 7$ multiplied by what number?
$9 \div \frac{1}{12} = 9$ multiplied by what number?

120. Dividing by a unit fraction. There is, therefore, this easy way to divide by a unit fraction:

To divide by a unit fraction, multiply by the denominator.

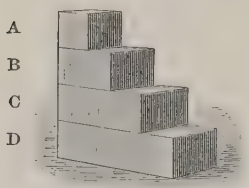
For example, $13 \div \frac{1}{3} = 13 \times 3 = 39$.

WRITTEN EXERCISE

1. $81 \div \frac{1}{8}$.	2. $16 \div \frac{1}{9}$.	3. $23 \div \frac{1}{7}$.
4. $32 \div \frac{1}{6}$.	5. $54 \div \frac{1}{4}$.	6. $29 \div \frac{1}{3}$.
7. $48 \div \frac{1}{5}$.	8. $63 \div \frac{1}{7}$.	9. $27 \div \frac{1}{9}$.
10. $17 \div \frac{1}{12}$.	11. $97 \div \frac{1}{12}$.	12. $39 \div \frac{1}{20}$.
13. $125 \div \frac{1}{16}$.	14. $144 \div \frac{1}{32}$.	15. $240 \div \frac{1}{64}$.

ORAL EXERCISE

1. If B is 1, which block is $\frac{1}{2}$? $1\frac{1}{2}$? 2? How many $\frac{1}{2}$'s in 1? in $1\frac{1}{2}$, or $\frac{3}{2}$? in 2?



2. If C is 1, which block is $\frac{1}{3}$? $1\frac{1}{3}$? $\frac{2}{3}$? How many $\frac{1}{3}$'s in 1? in $\frac{2}{3}$? in $1\frac{1}{3}$?

3. If C is 1, what is B? D? What is $1\frac{1}{3} \div \frac{2}{3}$?

4. If C is called 1, how much is C ÷ B, or $1 \div \frac{2}{3}$? If D is called 1, how much is D ÷ C, or $1 \div \frac{3}{4}$?

5. If $1 \div \frac{2}{3} = \frac{3}{2}$, how much is $2 \div \frac{2}{3}$? $3 \div \frac{2}{3}$? $\frac{3}{5} \div \frac{2}{3}$?

6. If $1 \div \frac{2}{3} = \frac{3}{2}$, then $\frac{7}{8} \div \frac{2}{3} = \frac{7}{8}$ of $\frac{3}{2}$ = how much?

121. Reciprocal. The fraction $\frac{3}{2}$ is called the *reciprocal* of $\frac{2}{3}$.

122. How to divide by a fraction. We see that *to divide by a fraction we may multiply by its reciprocal.*

Thus, $$\frac{5}{8} \div \frac{3}{4} = \frac{5}{\cancel{8}_{2}} \times \frac{\cancel{4}}{3} = \frac{5}{6}.$$

Also, $$4\frac{2}{5} \div 2\frac{4}{5} = \frac{\cancel{14}}{3} \div \frac{\cancel{14}}{5} = \frac{\cancel{14}}{3} \times \frac{5}{\cancel{14}} = \frac{5}{3}.$$

WRITTEN EXERCISE

1. $\frac{7}{8} \div \frac{2}{3}$.
2. $\frac{5}{6} \div \frac{3}{4}$.
3. $\frac{9}{10} \div \frac{4}{5}$.
4. $\frac{5}{12} \div \frac{7}{8}$.
5. $\frac{3}{8} \div \frac{5}{12}$.
6. $\frac{7}{16} \div \frac{3}{32}$.
7. $\frac{5}{64} \div \frac{7}{32}$.
8. $\frac{9}{24} \div \frac{5}{16}$.
9. $\frac{3}{40} \div \frac{6}{25}$.
10. $\frac{7}{48} \div 1\frac{7}{12}$.
11. $\frac{3}{4} \div 1\frac{1}{16}$.
12. $2\frac{7}{16} \div 3\frac{1}{8}$.
13. $10\frac{7}{36} \div 9\frac{1}{4}$.
14. $5\frac{1}{4} \div 19\frac{3}{8}$.
15. $3\frac{9}{20} \div 2\frac{3}{10}$.
16. $4\frac{5}{9} \div 12\frac{1}{3}$.
17. $7\frac{3}{10} \div 1\frac{4}{5}$.
18. $4\frac{9}{10} \div 12\frac{5}{6}$.

Kitchen Problems

WRITTEN EXERCISE

1. Butter is worth 24 ct. a pound, and I bought \$1.86 worth to-day. How many pounds did I buy?

2. A recipe calls for dividing 6 qt. 1 pt. of milk into 4 equal parts. How many quarts and pints in each part?

3. I have 11 qt. $\frac{3}{4}$ pt. of cream, and a recipe calls for $1\frac{5}{8}$ pt. to each person. How many portions can I make?

4. How much does a purchaser gain by buying a barrel of flour (196 lb.) for \$5.75, over buying the same amount @ $3\frac{1}{2}$ ct. a pound?

5. We ordered from the grocer this morning, 10 lb. of sugar @ $5\frac{1}{2}$ ct., $2\frac{1}{2}$ lb. of tea @ 56 ct., and 8 lb. of coffee @ $32\frac{1}{2}$ ct. How much was the bill?

6. The weight of materials for a recipe for brown bread is as follows: corn meal 6 oz., rye meal 6 oz., flour 8 oz., sour milk $1\frac{1}{2}$ lb., molasses 12 oz., and $\frac{1}{2}$ oz. each of soda and salt. Required the total weight.

7. The cost is as follows: corn meal $1\frac{1}{4}$ ct., rye meal 1 ct., flour $1\frac{1}{2}$ ct., sour milk 4 ct., molasses $3\frac{1}{4}$ ct., and the soda and salt together $\frac{1}{4}$ ct. Required the total cost.

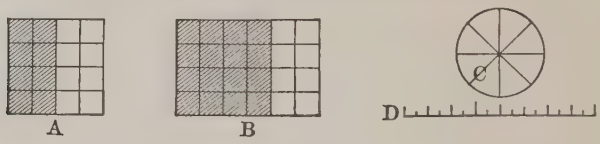
8. The weight of materials for a recipe for blueberry pudding for a family of 6 is as follows: bread crumbs 12 oz., blueberries $1\frac{1}{2}$ lb., sugar 8 oz., butter 2 oz. Required the total weight of these materials. What would be the weight of each of the materials for a family of 12?

9. The cost of the materials in Ex. 8 is as follows: bread crumbs 5 ct., blueberries 12 ct., sugar $2\frac{1}{2}$ ct., butter $2\frac{1}{2}$ ct., and the sauce for the pudding 3 ct. This being served to 6 persons, how much was the cost apiece?

Summary of Principles of Fractions

ORAL EXERCISE

1. In line D, point to $\frac{3}{16}$ of the line; also to $\frac{6}{16}$. We see that $\frac{6}{16}$ is how many times $\frac{3}{16}$? How does multiplying the numerator by 2 change the fraction? Then how does dividing the numerator by 2 change the fraction?



These figures can be used more easily if drawn on the board.

2. Point to $\frac{1}{2}$ of square A; also to $\frac{1}{4}$. We see that $\frac{1}{2}$ is how many times $\frac{1}{4}$? How does multiplying the denominator by 2 change the fraction? Then how does dividing the denominator by 2 change the fraction?

3. In rectangle B, show that $\frac{2}{3} = \frac{4}{6}$. Then does it change the value of a fraction to multiply or divide both terms by the same number? Show this with the circle C.

123. Multiplying and dividing the terms of a fraction.

Multiplying the numerator by any number multiplies the fraction by that number.

Dividing the numerator by any number divides the fraction by that number.

Multiplying the denominator by any number divides the fraction by that number.

Dividing the denominator by any number multiplies the fraction by that number.

Dividing both terms of a fraction by the same number does not change the value of the fraction.

ORAL EXERCISE

1. How many cents in $\$\frac{1}{8}$? Then 16 times $\$\frac{1}{8}$ is the same as 16 times how many cents?

2. How many cents in $\$\frac{1}{3}$? Then 9 times $\$\frac{1}{3}$ is the same as 9 times how many cents?

3. How many cents in $\$\frac{1}{6}$? Then 24 times $\$\frac{1}{6}$ is the same as 24 times how many cents?

124. Multiplying by aliquot parts. It is easier to multiply $\$\frac{1}{8}$, $\$\frac{1}{6}$, and $\$\frac{1}{3}$, than it is to multiply $12\frac{1}{2}$ ct., $16\frac{2}{3}$ ct., and $33\frac{1}{3}$ ct.

At $33\frac{1}{3}$ ct. each, 9 neckties cost 9 times $\$\frac{1}{3}$, or \$3.
At $16\frac{2}{3}$ ct. each, 12 bottles of ink cost 12 times $\$\frac{1}{6}$, or \$2.
At $12\frac{1}{2}$ ct. each, 9 balls cost 9 times $\$\frac{1}{8}$, or $\$1\frac{1}{8}$, or $\$1.12\frac{1}{2}$.
While goods are not marked $16\frac{2}{3}$ ct., they are often marked 6 for \$1, which is the same thing.

4. At $12\frac{1}{2}$ ct. a yard, what will 16 yd. of cloth cost?

5. At $16\frac{2}{3}$ ct. a yard, what will 18 yd. of cloth cost?

6. At $33\frac{1}{3}$ ct. a yard, what will 21 yd. of cloth cost?

7. At 8 doz. marbles for \$1, what will 6 doz. cost?

8. At \$1 per half dozen, what will 3 doz. bottles of mucilage cost?

9. At the rate of 3 for \$1, what must the school pay for 16 arithmetics?

10. At $16\frac{2}{3}$ ct. each, find the cost of 9 water pitchers; of a dozen plants for the school; of 6 yards of ribbon.

11. At $12\frac{1}{2}$ ct. each, find the cost of 10 glasses; of 4 lamp chimneys; of 20 cans of soup; of 32 yards of calico.

12. How many jackets can be cut from $17\frac{1}{2}$ yd. of cloth, allowing $2\frac{1}{2}$ yd. to a jacket?

ORAL EXERCISE

1. How much is 8 times $\$\frac{1}{8}$? $\$1\frac{1}{8}$? $\$1.12\frac{1}{2}$?
2. How much is 6 times $\$\frac{1}{6}$? $\$1\frac{1}{6}$? $\$1.16\frac{2}{3}$?
3. How much is 3 times $\$\frac{1}{3}$? $\$1\frac{1}{3}$? $\$1.33\frac{1}{3}$?
4. At $\$1.33\frac{1}{3}$ a yard, how much will 15 yd. of lace cost?
5. At $\$1.16\frac{2}{3}$ a yard, how much will 9 yd. of silk cost?
6. At $\$1.12\frac{1}{2}$ a yard, how much will 6 yd. of velvet cost?
7. Find the cost of 12 books at $\$1.12\frac{1}{2}$ each; at $\$1.16\frac{2}{3}$.
8. At $\$1.33\frac{1}{3}$ each, find the cost of 3 chairs; of 18 chairs; of 33 chairs; of 36 chairs.
9. At $\$1.16\frac{2}{3}$ each, find the cost of 6 maps; of 12 maps; of 30 maps; of 54 maps.
10. At $\$1.12\frac{1}{2}$ each, find the cost of 8 books; of 40 books; of 32 books; of 20 books.

125. Therefore, at $\$1.12\frac{1}{2}$ each, 7 books cost $\$7 + \$\frac{7}{8}$, or $\$7.87\frac{1}{2}$ ($\$7.88$).

At $\$1.16\frac{2}{3}$ each, 5 books cost $\$5 + \$\frac{5}{6}$, or $\$5.83\frac{1}{3}$ ($\$5.83$).

At $\$1.33\frac{1}{3}$ each, 8 articles cost $\$8 + \$\frac{8}{3}$, or $\$10\frac{2}{3}$ ($\$10.67$).

WRITTEN EXERCISE

1. At $12\frac{1}{2}$ ct. each, find the cost of 147 balls.
2. At $16\frac{2}{3}$ ct. each, find the cost of 113 notebooks.
3. At $33\frac{1}{3}$ ct. each, find the cost of 101 handkerchiefs.
4. At $\$1.33\frac{1}{3}$ a volume, find the cost of 165 books.
5. At $\$1.16\frac{2}{3}$ a yard, find the cost of 142 yd. of lace.
6. At $\$1.12\frac{1}{2}$ each, find the cost of 112 yd. of velvet.
7. At $\$2.16\frac{2}{3}$ each, find the cost of 72 umbrellas.
8. What is the difference in cost of 144 books at $\$1.16\frac{2}{3}$ a volume, and the same number at $\$1.12\frac{1}{2}$ a volume?

Review Drill

1. $47\frac{1}{2} + 93\frac{1}{3}$.
2. $72\frac{1}{4} + 83\frac{1}{6}$.
3. $29\frac{3}{4} + 47\frac{1}{5}$.
4. $62\frac{1}{5} + 35\frac{1}{3}$.
5. $85\frac{4}{5} + 16\frac{2}{3}$.
6. $87\frac{5}{6} + 19\frac{2}{3}$.
7. $67\frac{4}{3} + 29\frac{1}{6}$.
8. $89\frac{3}{7} + 23\frac{1}{2}$.
9. $67\frac{4}{5} + 49\frac{1}{2}$.
10. $39\frac{9}{10} + 28\frac{3}{4}$.
11. $87\frac{3}{5} + 62\frac{7}{10}$.
12. $37\frac{4}{7} + 18\frac{1}{14}$.
13. $79\frac{4}{5} - 22\frac{1}{2}$.
14. $137\frac{1}{2} - 86\frac{2}{3}$.
15. $48\frac{3}{4} - 23\frac{7}{8}$.
16. $92\frac{1}{4} - 37\frac{2}{5}$.
17. $30\frac{2}{3} - 19\frac{7}{8}$.
18. $82\frac{1}{2} - 68\frac{7}{8}$.
19. $72\frac{2}{5} - 56\frac{1}{8}$.
20. $41\frac{3}{8} - 16\frac{5}{6}$.
21. $72 - 46\frac{2}{3}$.
22. $27 \times 14\frac{2}{3}$.
23. $32 \times 92\frac{1}{2}$.
24. $67 \times 74\frac{2}{5}$.
25. $18 \times 96\frac{3}{4}$.
26. $30\frac{1}{2} \times 68$.
27. $40\frac{1}{3} \times 96$.
28. $32\frac{2}{5} \times 85$.
29. $41\frac{3}{4} \times 96$.
30. $\frac{2}{3}$ of $\frac{7}{8}$ of $1\frac{5}{7}$.
31. $10\frac{1}{2} \times 12\frac{1}{2}$.
32. $6\frac{1}{3} \times 7\frac{1}{5}$.
33. $4\frac{1}{4} \times 8\frac{1}{5}$.
34. $5\frac{3}{4} \times 9\frac{4}{5}$.
35. $29 \times 635\frac{1}{4}$.
36. $42 \times 408\frac{3}{4}$.
37. $52 \times 607\frac{3}{4}$.
38. $73 \times 429\frac{3}{7}$.
39. $87 \times 602\frac{4}{5}$.
40. $\frac{4}{5} \div \frac{2}{3}$.
41. $\frac{2}{3} \div \frac{4}{5}$.
42. $\frac{3}{7} \div \frac{2}{3}$.
43. $\frac{2}{3} \div \frac{3}{7}$.
44. $\frac{7}{8} \div \frac{4}{5}$.
45. $\frac{4}{5} \div \frac{7}{8}$.
46. $\frac{6}{7} \div \frac{3}{4}$.
47. $\frac{3}{4} \div \frac{6}{7}$.
48. $\frac{3}{4} \div 5\frac{1}{2}$.
49. $\frac{5}{7} \div 2\frac{1}{5}$.
50. $\frac{2}{3} \div 4\frac{2}{3}$.
51. $7\frac{6}{7} \div 1\frac{3}{4}$.
52. $9\frac{2}{3} \div 3\frac{1}{3}$.
53. $10\frac{4}{5} \div 3\frac{1}{2}$.
54. $5\frac{5}{7} \div 1\frac{1}{7}$.
55. $29\frac{1}{5} \div \frac{1}{3}$.
56. $3\frac{9}{10} \div 2\frac{1}{3}$.
57. $2\frac{4}{5} \div 3\frac{1}{2}$.
58. $4\frac{7}{8} \div 2\frac{2}{3}$.
59. $4\frac{6}{7} \div 1\frac{3}{14}$.
60. $16\frac{2}{7} \div 2\frac{1}{5}$.
61. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.
62. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of 1 cu. ft.
63. $\frac{3}{5}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of 1 cu. ft.
64. $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of 1 cu. in.
65. $\frac{1}{2}$ of 5 times 8 sq. ft.
66. $\frac{3}{5}$ of 15 times 48 sq. ft.
67. $\frac{7}{8}$ of 4 times 16 sq. in.
68. $\frac{3}{7}$ of 13 times 14 sq. in.

Examples like the above should be given, a few at a time, with concrete problems like those on page 65, or should be assigned in groups for seat work, a certain number for ten minutes.

Compound and Complex Fractions

126. Compound fraction. An indicated multiplication of fractions is sometimes called a *compound fraction.*

127. Complex fraction. An indicated division of fractions is sometimes called a *complex fraction.*

For example, $\frac{2}{3}$ of $\frac{4}{5}$ is a compound fraction, and $\frac{\frac{2}{3}}{\frac{4}{5}}$ and $\frac{\frac{3}{7}}{5}$ are complex fractions.

These terms are not of great value, but they are used in certain courses of study.

128. Such forms are treated the same as ordinary multiplications and division of fractions.

For example, $\frac{\frac{2}{3}}{5} = \frac{2}{3} \div 5 = \frac{2}{15}$.

WRITTEN EXERCISE

Reduce to the simpler forms in Exs. 1–24:

1. $\frac{\frac{2}{3}}{4}$. 2. $\frac{\frac{3}{8}}{3}$. 3. $\frac{\frac{4}{7}}{14}$. 4. $\frac{\frac{5}{8}}{15}$. 5. $\frac{\frac{5}{8}}{\frac{7}{8}}$.

6. $\frac{1\frac{6}{11}}{2\frac{3}{22}}$. 7. $\frac{\frac{5}{9}}{1\frac{3}{18}}$. 8. $\frac{2\frac{1}{2}}{3\frac{1}{3}}$. 9. $\frac{\frac{3}{5}}{\frac{2}{3}}$. 10. $\frac{3\frac{4}{5}}{6\frac{7}{8}}$.

11. $\frac{8\frac{2}{3}}{16\frac{1}{4}}$. 12. $\frac{2}{3}$ of $\frac{3\frac{1}{3}}{4}$. 13. $\frac{3}{5}$ of $\frac{4\frac{1}{2}}{6}$. 14. $\frac{7}{8}$ of $\frac{2}{3\frac{1}{2}}$.

15. $\frac{5}{56\frac{1}{4}}$. 16. $\frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{5}+\frac{1}{3}}$. 17. $\frac{2\frac{1}{2}+\frac{2}{3}}{3\frac{1}{2}-\frac{1}{3}}$. 18. $\frac{4\frac{1}{2}+\frac{2}{3}}{4\frac{1}{2}-\frac{2}{3}}$.

19. $\frac{3}{5}$ of $1\frac{5}{16}$ of $\frac{8}{9}$. 20. $\frac{2}{7}$ of $\frac{3}{4}$ of $2\frac{1}{2}$. 21. $\frac{3}{8}$ of $\frac{4}{5}$ of $\frac{9}{10}$.

22. $1\frac{6}{11}$ of $\frac{2}{7}$ of $\frac{2}{2}\frac{1}{2}$. 23. $\frac{5}{21}$ of $\frac{7}{15}$ of $\frac{3}{8}$. 24. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $2\frac{1}{2}$.

25. If Mr. A buys $\frac{3}{4}$ of the share of Mr. B who owns $\frac{4}{16}$ of a farm, and then sells $\frac{2}{3}$ of what he buys to Mr. C, what part of the farm does Mr. C own?

Fractional Relations

ORAL EXERCISE

1. What part of 4 is 2? What part of 10 is 8? How do you find what part one number is of another?

2. What part of 7 is 5? of 9 is 4? of 7 is 6? of 16 is 12? of 25 is $12\frac{1}{2}$? of 100 is $33\frac{1}{3}$? of 100 is $16\frac{2}{3}$?

129. Fractional parts. It therefore appears that

To find what part the second of two numbers is of the first, divide the second by the first.

For example, what part of 51 is 17? 17 is $\frac{17}{51}$, or $\frac{1}{3}$ of 51.

WRITTEN EXERCISE

In Exs. 1–24 the second number is what part of the first?

1. 14, 8.	2. 18, 14.	3. $\frac{7}{8}$, $\frac{2}{3}$.
4. $\frac{2}{7}$, $\frac{6}{11}$.	5. $3\frac{1}{3}$, $1\frac{2}{3}$.	6. 17, $8\frac{1}{2}$.
7. 82, $9\frac{1}{9}$.	8. $41\frac{3}{5}$, $5\frac{1}{5}$.	9. $66\frac{2}{3}$, $16\frac{2}{3}$.
10. 243, 27.	11. 100, $37\frac{1}{2}$.	12. $45\frac{3}{5}$, $7\frac{3}{5}$.
13. $16\frac{5}{8}$, $2\frac{3}{8}$.	14. $42\frac{1}{2}$, $14\frac{1}{6}$.	15. $62\frac{1}{8}$, $7\frac{7}{8}$.
16. $66\frac{2}{3}$, $33\frac{1}{3}$.	17. 225, $37\frac{1}{2}$.	18. $22\frac{1}{2}$, $16\frac{2}{3}$.
19. $41\frac{7}{8}$, $21\frac{2}{5}$.	20. $37\frac{1}{2}$, $33\frac{1}{3}$.	21. 421, $60\frac{1}{7}$.
22. $33\frac{1}{3}$, $16\frac{2}{3}$.	23. 427, $47\frac{4}{9}$.	24. 325, $29\frac{6}{11}$.

25. Frank is 12 yr. old. His age is what part of his brother's, who is 18?

26. The age of his brother, who is 18, is what part of his uncle's, who is 54?

27. If Frank is 12 and his brother is 18, Frank's age was what part of his brother's 6 yr. ago? It will be what part of his brother's 6 yr. from now?

ORAL EXERCISE

1. 10 is $\frac{1}{2}$ of what number? $\frac{1}{3}$ of what number?

2. 12 is $\frac{1}{3}$ of what number? If 12 is $\frac{2}{3}$ of some number, what is $\frac{1}{3}$ of the number? What is $\frac{3}{3}$ of it?

3. 15 is $\frac{3}{7}$ of some number. What is $\frac{1}{7}$ of it? all of it?

4. 21 is $\frac{7}{8}$ of what number? $\frac{3}{5}$ of what number?

130. Illustrative problem. If $33\frac{1}{3}$ is $\frac{2}{3}$ of some number, what is the number?

Because $\frac{2}{3}$ of some number $= 33\frac{1}{3}$, or $\frac{100}{3}$,
therefore $\frac{1}{3}$ of the number $= \frac{1}{2}$ of $\frac{100}{3} = \frac{50}{3}$,
and $\frac{3}{3}$ of the number $= 3$ times $\frac{50}{3} = 50$.
Therefore $33\frac{1}{3}$ is $\frac{2}{3}$ of 50.

WRITTEN EXERCISE

Find the number of which

1. 23 is $\frac{1}{3}$.	2. 38 is $\frac{2}{3}$.	3. 63 is $\frac{7}{8}$.
4. 75 is $\frac{3}{8}$.	5. 45 is $\frac{9}{10}$.	6. 150 is $\frac{3}{5}$.
7. 222 is $\frac{6}{7}$.	8. 125 is $\frac{25}{32}$.	9. 144 is $\frac{9}{16}$.
10. \$150 is $\frac{3}{5}$.	11. \$2.25 is $\frac{5}{8}$.	12. \$10.25 is $\frac{5}{9}$.

13. If there are 85 girls in a school, and they are $\frac{5}{9}$ of the total number of pupils, how many pupils are there?

14. In my library are 164 books on history, and these are $\frac{2}{17}$ of all my books. How many books have I?

15. If a man spends \$1687.50 this year, and this is $\frac{3}{4}$ of his income, how much is his income?

16. A man left to his son $\frac{3}{5}$ of his estate, this share amounting to \$3210. To his daughter he left $\frac{1}{10}$ of his estate, and the rest to his wife. How much did the wife and daughter each receive?

ORAL EXERCISE

1. What will $2\frac{1}{2}$ lb. of coffee cost at 24¢ a pound?

2. What will $\frac{3}{4}$ lb. of pepper cost at 24¢ a pound?

3. What will $2\frac{1}{2}$ lb. of butter cost at 26¢ a pound?

4. What will 5 oz. of cinnamon cost at 32¢ a pound?

5. What will 20 lb. of loaf sugar cost at $6\frac{1}{4}$¢ a pound?

6. What will $\frac{3}{4}$ doz. hats cost at $36 a dozen?

7. At 32¢ a gallon, what will $12\frac{1}{2}$ gal. of vinegar cost?

8. What will $1\frac{1}{2}$ lb. of cocoa cost at 22¢ per half-pound can?

9. At 32¢ a pound, what will $1\frac{1}{4}$ lb. of ground ginger cost?

10. At the rate of 80¢ for 10 lb. of rice, how much must I pay for 15 lb.?

11. At the rate of $1.64 for $\frac{1}{2}$ gal. of olive oil, how much will $2\frac{1}{2}$ gal. cost?

12. I can buy a 14-lb. box of oatmeal for $1.54. At this rate, what do I pay for 5 lb.?

13. At 30¢ a pound, how much will $\frac{3}{4}$ lb. of mustard cost? (Always count the half cent as a cent.)

14. I can buy a box of 60 cakes of soap for $2.94, or the same soap at $5\frac{1}{2}$¢ a cake. What shall I gain on 60 cakes by buying the whole box at once?

15. I can buy a 5-lb. can of baking powder for $1.95. How much cheaper is this than buying the same amount in half-pound cans at 23¢ each?

16. A hotel needs a pint and a half of vanilla extract this week. How much cheaper is it to buy a pint bottle for 98¢ and a half-pint bottle for 54¢, than to buy it in half-pint bottles?

WRITTEN EXERCISE

1. At \$1.38 a yard, what will 37 yd. of carpet cost?

2. In buying 2 doz. cans of peaches, how much is gained by buying @ \$3.73 a dozen over buying at 3 cans for a dollar?

3. What is the cost of $6\frac{1}{4}$ lb. of mackerel @ $16\not{c}$, $3\frac{1}{2}$ lb. of codfish @ $12\not{c}$, and 4 cans of salmon at the rate of 2 for a quarter?

4. I bought $\frac{1}{2}$ gal. of olive oil @ \$3.30 a gallon, but $\frac{1}{4}$ of what I bought leaked out. What rate per gallon do I pay for what is left?

5. In buying 48 cans of tomatoes, how much is gained by buying two cases of 2 doz. each, @ \$2.89 a case, over buying at the rate of 3 cans for $40\not{c}$?

6. I bought 3 gal. of best maple sirup @ $\$1.12\frac{1}{2}$ a gallon, $2\frac{1}{2}$ gal. New Orleans molasses @ $62\not{c}$ a gallon, and gave the grocer a \$5 bill. How much change did I get?

7. I bought 6 lb. of prunes @ $12\frac{1}{2}\not{c}$, 8 lb. of dried apricots @ $12\frac{1}{2}\not{c}$, $2\frac{1}{2}$ lb. of raisins @ $16\not{c}$, $1\frac{1}{2}$ lb. of figs @ $22\not{c}$, and $\frac{1}{2}$ lb. English walnuts @ $18\not{c}$, and gave the grocer a \$5 bill. How much change did I get?

8. I bought 8 cans of peaches @ \$3.75 a dozen, 15 cans of tomatoes @ $\$1.37\frac{1}{2}$ a dozen, 9 cans of soup @ \$1.50 a dozen, $3\frac{1}{2}$ lb. of raisins @ $18\not{c}$, $2\frac{1}{2}$ lb. of nuts @ $17\not{c}$, $1\frac{1}{2}$ gal. of sirup @ $60\not{c}$, $1\frac{1}{2}$ doz. cakes of soap @ $42\not{c}$ a dozen. How much was my bill?

9. I bought 10 lb. of white beans @ $6\frac{1}{4}\not{c}$, $3\frac{1}{3}$ lb. split peas @ $7\not{c}$, 3 cans of soup @ $12\frac{1}{2}\not{c}$, and $2\frac{1}{2}$ lb. of paraffin candles @ $12\frac{1}{2}\not{c}$. How much was my bill? (Neglect fractions less than $\frac{1}{2}\not{c}$, although in a bill, as on page 75, a fraction of a cent is usually called a cent.)

Cash Checks in Large Stores

131. Cash checks. In large stores the clerk fills out a small cash check like the following:

Sold by No. 127		Am't rec'd	$15.00	
½ yd.	Silk	.57		29
4⅜ "	Summer Silk	.38	1	67
11½ "	Silk	.99	11	39
	Total		13	35
	Change due		1	65
			$15	00

WRITTEN EXERCISE

Make out cash checks for the following sales:

1. 14½ yd. gingham @ 25¢, 9 yd. cotton @ 12½¢, 10½ yd. madras @ 35¢. Amount received, $10.

2. 6½ yd. cheviot @ $1.10, 5⅜ yd. silk @ $1.25, 9½ yd. velveteen @ 98¢. Amount received, $25.

3. 11½ yd. Persian lawn @ $1.95, 6⅝ yd. dimity @ 25¢, 12⅛ yd. linen suiting @ 75¢. Amount received, $40.

4. 9½ yd. Persian lawn @ $1.37½, 5⅛ yd. cheviot @ $1.25, 15 yd. cotton @ 12½¢. Amount received, $30.

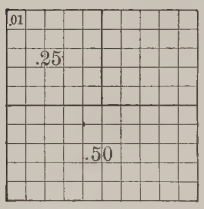
5. 7 yd. muslin @ 25¢, 12½ yd. lining @ 11¢, 6⅝ yd. lawn @ $1.50, 7 yd. suiting @ 75¢. Amount received, $20.

6. 16 yd. velvet @ $2.25, 14½ yd. suiting @ 48¢, 23 yd. cotton @ 15¢, 6⅜ yd. dimity @ 24¢, 7¼ yd. ribbon @ 25¢. Amount received, $50.

DECIMAL FRACTIONS

ORAL EXERCISE

1. In the picture each column of squares is what part of the large square? Point to $\frac{5}{10}$ of the large square.



2. Each small square is what part of a column? of the large square? Point to $\frac{50}{100}$ of the large square; to $\frac{25}{100}$.

3. If the large square represents \$1, what part represents 1 dime? 1 cent?

132. Meaning of decimals. Just as \$3.25 means $\$3 + \$\frac{25}{100}$, so 3.25 ft. means 3 ft. + $\frac{25}{100}$ ft., or $3\frac{1}{4}$ ft.; 4.75 in. means 4 in. + $\frac{75}{100}$ in., or $4\frac{3}{4}$ in.; 2.5 means $2 + \frac{5}{10}$, or $2\frac{1}{2}$; 0.5, or .5, means $\frac{5}{10}$, or $\frac{50}{100}$, or .50; 0.125, or .125, means $\frac{125}{1000}$, or $\frac{1}{8}$.

133. Decimal fractions. A fraction whose denominator is not written, but is some power of 10, is called a *decimal fraction.*

134. Decimals. Decimal fractions are usually called *decimals.*

135. Decimal point. The period placed at the left of tenths is called the *decimal point.*

Thus, $0.2 = \frac{2}{10}$, or $\frac{1}{5}$; $0.7 = \frac{7}{10}$; $0.8 = \frac{8}{10}$, or $\frac{4}{5}$; $.06 = \frac{0}{10} + \frac{6}{100}$, or $\frac{3}{50}$; $.08 = \frac{8}{100} = \frac{4}{50} = \frac{2}{25}$; $.007 = \frac{7}{1000}$; $.125 = \frac{125}{1000} = \frac{1}{8}$; $.1231 = \frac{1231}{10000}$.

136. The expression 0.5 means the same as .5. The 0 may be written or not as we please.

137. Reading decimals. 125.375 is read, "one hundred twenty-five *and* three hundred seventy-five thousandths."

Pupils should be given much exercise in reading such numbers from the blackboard, avoiding the use of "and" except between the units and tenths, and the vulgarism of "aught" for "naught."

ORAL EXERCISE

Read the numbers in Exs. 1–12:

1. 1.25.	2. 3.78.	3. 2.95.	4. 6.33.
5. 0.125.	6. 48.675.	7. 0.375.	8. 67.005.
9. 32.375.	10. 100.025.	11. 91.075.	12. 300.075.

13. What is the name of the place to the right of tenths? to the right of that? What is the name of the fourth decimal place? the fifth? the sixth?

138. Since we rarely read aloud the names of numbers beyond three decimal places, even when we write them, the chief drill should be on reading decimal fractions like those in Exs. 1–12.

Read the following:

14. 16.004.	15. 82.3745.	16. 91.8273.	17. 68.9982.
18. 75.1375.	19. 12.31478.	20. 16.42398.	21. 14.763458.

WRITTEN EXERCISE

Write decimally the fractions in Exs. 1–5:

1. Three tenths; five tenths; nine tenths.

2. Four and four tenths; seven and eight tenths.

3. Seventy-five hundredths; thirty-three and a third hundredths; three hundred and three hundredths.

4. One hundred forty-four thousandths; six hundred seventy-five thousandths; four ten-thousandths.

5. Five and seven thousandths; eight and forty-two thousandths; twenty-five thousandths; one millionth.

6. Reduce to lowest terms the fractions $\frac{30}{100}$ and $\frac{300}{10000}$. What does this tell you of the values of 0.30 and 0.300?

7. As in Ex. 6, show that $0.250 = 0.25 = \frac{1}{4}$; that $0.500 = 0.50 = 0.5 = \frac{1}{2}$; that $0.7500 = 0.750 = 0.75 = \frac{3}{4}$.

Reduction of Decimal Fractions

ORAL EXERCISE

1. Express $\frac{5}{10}$ as hundredths; as thousandths. Write the results as decimal fractions on the blackboard.

2. Express as tenths: $\frac{1}{2}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{3}{2}$, $\frac{4}{2}$ or 2, $\frac{5}{2}$ or $2\frac{1}{2}$.

3. Express as hundredths: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{1}{50}$, $\frac{3}{25}$, $\frac{11}{50}$.

139. Annexing zeros to decimals. Because $\frac{5}{10} = \frac{50}{100}$, or $.5 = .50$, we see that

Annexing a zero to a decimal fraction does not change its value.

140. Reducing decimals to common fractions. Because $.25 = \frac{25}{100}$, which equals $\frac{1}{4}$,

To reduce a decimal to a common fraction, write the denominator of the decimal and then reduce the fraction to lowest terms.

WRITTEN EXERCISE

Reduce to common fractions in lowest terms, or to mixed numbers, in Exs. 1–16:

1. .4.	2. .6.	3. .2.	4. .8.
5. .25.	6. .75.	7. .45.	8. .55.
9. .125.	10. .375.	11. .625.	12. .875.
13. 1.25.	14. 2.75.	15. 5.125.	16. 10.375.

17. Express as hundredths: 0.7, 0.6, 0.1, 0.8, 4, 63.

18. Express as thousandths: 0.1, 0.25, 0.37, 0.09, 31.

19. Express as hundredths: .470, .680, .100, .040, 7.8.

20. Express 0.125 as eighths. Find 0.125 of \$1600.

ORAL EXERCISE

1. Express 0.25 as a common fraction; $\frac{3}{5}\frac{0}{0}$ as a decimal.
2. Express 0.40 as a common fraction; $\frac{3}{2}\frac{5}{5}$ as a decimal.

Express the numbers in Exs. 3–12 as common fractions:

3. 0.2. 4. 0.4. 5. 1.5. 6. 0.6. 7. 0.8.
8. 0.20. 9. 0.40. 10. 0.75. 11. 1.125. 12. 0.250.

Express as decimals:

13. $\frac{1}{5}$. 14. $\frac{2}{5}$. 15. $\frac{1}{2}$. 16. $\frac{3}{8}$. 17. $\frac{4}{5}$.
18. $\frac{1}{4}$. 19. $\frac{3}{4}$. 20. $\frac{1}{8}$. 21. $\frac{3}{5}\frac{0}{0}$. 22. $\frac{7}{2}\frac{0}{0}$.

141. Reducing common fractions to decimals. Because

$$\frac{1}{3} = \frac{1.0}{3} = 0.3\frac{1}{3}, \quad \text{and} \quad \frac{1}{3} = \frac{1.00}{3} = 0.33\frac{1}{3},$$

To reduce a common fraction to a decimal fraction, divide the numerator by the denominator.

Write the decimal point, and annex zeros as necessary.

142. Exact decimals not always possible. A common fraction cannot always be reduced to an exact decimal.

Thus, $\frac{1}{3} = 0.3\frac{1}{3} = 0.33\frac{1}{3} = 0.333\frac{1}{3}$, and so on, or 0.333+.

No common fraction equals an exact decimal if the denominator contains other prime factors than 2 and 5, when in lowest terms.

WRITTEN EXERCISE

Express as decimals, carrying the reduction only to thousandths in case of inexact decimals:

1. $\frac{7}{8}$. 2. $\frac{3}{2}\frac{3}{6}$. 3. $\frac{4}{1}\frac{4}{5}$. 4. $\frac{5}{6}$. 5. $\frac{6}{7}$.
6. $\frac{5}{9}$. 7. $\frac{7}{1}\frac{7}{1}$. 8. $\frac{3}{2}\frac{3}{8}$. 9. $\frac{5}{1}\frac{5}{9}$. 10. $\frac{1}{3}\frac{9}{5}$.
11. $\frac{1}{3}\frac{2}{0}\frac{5}{0}$. 12. $\frac{1}{1}\frac{2}{5}\frac{1}{0}$. 13. $\frac{3}{1}\frac{2}{1}\frac{2}{1}$. 14. $\frac{4}{6}\frac{1}{7}\frac{2}{5}$. 15. $3\frac{4}{6}\frac{7}{1}$.
16. $3.07\frac{1}{2}$. 17. $4.05\frac{1}{5}$. 18. $6.75\frac{2}{5}$. 19. $4.00\frac{1}{2}$. 20. $7.25\frac{3}{5}$.

Addition and Subtraction

ORAL EXERCISE

Write on the blackboard and add aloud the numbers in Exs. 1–10:

1. 2 ft.	2. $2	3. $0.02	4. $2.22	5. .02 ft.
6	6	.06	6.66	.06
6. .02	**7.** .04	**8.** .3	**9.** .6	**10.** .60
.06	.05	.4	.7	.70

An abbreviation like ft., at the top of a column, refers to all numbers in the column.

Write on the blackboard and subtract aloud the following:

11. 7 ft.	12. $0.07	13. .07	14. .7	15. 7.77
2	.02	.02	.2	2.22
16. $0.11	**17.** .115	**18.** 1.1	**19.** 1.512	**20.** 1.251
.04	.041	.4	.602	.851

143. Adding and subtracting decimals. We see that to add or subtract numbers containing decimal fractions we

Write like orders under one another and add or subtract as with integers.

144. To add 2.5, 3.06, 9.475, and .1.

If the decimal points are in a column, units will come under units, tenths under tenths, and so on. Read the columns like a word, saying, "5, 13, 11, 15."

```
 2.5
 3.06
 9.475
  .1
15.135
```

145. To subtract .26 from 4.1.

Arranging the decimal points in a column and writing 4.1 as 4.10 (§ 139), the subtraction is the same as with United States money.

```
4.10
 .26
3.84
```

ORAL EXERCISE

1. Add: $\frac{5}{10} + \frac{6}{10}$, $\frac{5}{100} + \frac{6}{100}$, $0.5 + 0.6$, $0.05 + 0.06$.
2. Add: $1\frac{6}{10} + 3\frac{9}{10}$, $1\frac{6}{100} + 3\frac{9}{100}$, $1.6 + 3.9$, $1.06 + 3.09$.
3. Subtract: $\frac{16}{25} - \frac{9}{25}$, $\frac{16}{10} - \frac{9}{10}$, $1\frac{6}{10} - \frac{9}{10}$, $1.6 - 0.9$.
4. Subtract: $125 - 50$, $\$1.25 - \$0.50$, $1.25 - 0.50$.

WRITTEN EXERCISE

See how many of these examples you can solve in ten minutes.

Add the numbers in Exs. 1–8:

1.	2.	3.	4.	5.
\$2.47	\$15.25½	28.92 ft.	68.	432.
3.62	7.42½	6.87	0.09¼	41.025
4.81	19.00	9.09	7.4	7.875
9.	6.04	125.	321.	.905
28.06	7.87	7.75	40.08¾	.095

6. $132 + 0.96 + 8.04 + 19.6 + 0.05 + 98.03$.
7. $42.06 + 98.03 + 925 + 0.6 + 0.85 + 2.5 + 3.4$.
8. $243 + 489 + 0.09 + 0.16 + 0.49 + 0.98 + 27.62$.

Subtract as indicated in Exs. 9–29:

9.	10.	11.	12.
\$0.25½	\$7.25	\$9.63	\$5.70
0.07½	1.50	2.78	2.95

13.	14.	15.	16.
67.025	42.57	48.054	540.
9.8	6.64	6.97	0.675

17.	18.	19.	20.
\$128.42	\$142.	37.204	203.
7.06	4.92	9.32	94.602

21. $78.2 - 9.3$.
22. $80 - 0.07$.
23. $68.04 - 7.6$.
24. $40.04 - 0.4$.
25. $628 - 519.1$.
26. $78.1 - 8.01$.
27. $600.06 - 66.6$.
28. $625 - 500.25$.
29. $37.27 - 28.19$.

ORAL EXERCISE

1. A small geography cost \$0.60 and a music book \$0.35. How much did the two cost?

2. An arithmetic cost \$0.65 and a paint box \$0.35. How much did the two cost?

3. A history cost \$0.60. I gave the dealer a 50-ct. piece and a quarter of a dollar. How much change did I receive?

4. I bought a large geography for \$1.25, and gave the dealer a dollar and a half. How much change did I receive?

5. How much more did the large geography cost than the small one? How much more than the arithmetic?

WRITTEN EXERCISE

1. What was the total cost of all the purchases mentioned in the oral examples?

2. There are 15 girls in the class, and each bought an arithmetic. How much did these books cost them?

3. There are 14 boys in the class, and each bought a large geography. How much did these books cost them?

4. If each of the boys and each of the girls bought a paint box, how much was paid for paint boxes?

5. John paid during the term \$0.08 for pencils, \$0.24 for paper, and \$0.15 for a drawing book. Required the cost of these materials.

6. Make a list of the materials used by you this year in school, with the cost of each book, pencil, and other article, whether paid by you or the school. Add and find the total cost.

The Farmer's Potato Crop

WRITTEN EXERCISE

1. The greatest recent average state yield of potatoes per acre was in Nevada, where 1753 acres produced 190 bu. to the acre. What was the total yield?

2. When the average value of potatoes per acre in Michigan was \$25.11, and 171,453 acres were devoted to this product, what was the value of the total crop?

3. When New York had 339,000 acres in potatoes, and produced 27,628,500 bu. a year, what was the average number of bushels per acre?

4. If these New York potatoes sold at an average price of \$0.48 a bushel, what was the value of the total crop?

5. The state having the highest recent average yield of potatoes per acre in a year is Nevada, 190 bu. Montana averaged 170 bu., Wyoming 167 bu., and Maine 165 bu. What is the average of these amounts?

6. There are a few records of 500 bu. to the acre. If a man had 10 acres of such land, and the potatoes sold for 47 ct. a bushel, and it cost the farmer \$46.25 an acre to plant and cultivate the crop, what would he make a year?

7. A bushel of potatoes weighs 60 lb. How much would the potatoes raised on a 16-acre lot weigh, the average yield being 213.5 bu. per acre?

Multiplication

ORAL EXERCISE

1. Multiply by 7: 2 ft., .2, $30, .030, .30, .003.
2. Multiply by 2: 7 in., $\frac{7}{10}$, .8, .9, .07, .007, .006.
3. Multiply by 5: $11, .11, $\frac{11}{10}$, 1.1, .011, .01, .002.
4. Multiply by 10: $\frac{1}{10}$, $\frac{11}{10}$, 1.1, 1.25, .25, .2, .03, .07.

146. Multiplying by integers. Multiplying decimal fractions by integers is already familiar, especially in working with United States money. Thus, to multiply 24.125 by 35,

$$\begin{array}{r} 24.125 \\ \underline{\quad\quad 35} \\ 120\ 625 \\ 723\ 75 \\ \hline 844.375 \end{array}$$

We know where to put the decimal point, because thousandths multiplied by a whole number must be thousandths.

This subject is considered more fully on page 106.

WRITTEN EXERCISE

Multiply in Exs. 1–12:

1. 27.25 by 25.	2. 65.23 by 32.	3. 13.45 by 72.
4. 61.82 by 37.	5. 4.375 by 81.	6. 6.925 by 93.
7. 42.125 by 45.	8. 68.325 by 50.	9. 20.005 by 75.
10. 102.002 by 35.	11. 42.175 by 62.	12. 29.407 by 87.

13. If a 12-story city building is 10.62 ft. to a story, how tall is it?

14. If one side of a square is 21.33 in., how long is the perimeter (sum of the sides)?

15. If a triangle has three equal sides, each 14.72 in., how long is the perimeter?

16. The measure called the meter, studied later, is 39.37 in. long. How many inches in 62 meters?

Multiplying by Powers of 10

ORAL EXERCISE

1. How much is $2 \times \frac{1}{2}$? $3 \times \frac{1}{3}$? $5 \times \frac{1}{5}$? $10 \times \frac{1}{10}$? 10×0.1?

2. How much is 10×0.2? 10×0.6? $10 \times \frac{1}{100}$? 10×0.01?

3. How much is $100 \times \frac{1}{100}$? 100×0.01? 100×0.02?

4. How much is $10 \times \frac{25}{100}$? $100 \times \frac{25}{100}$? 10×0.25? 100×0.38? 10×0.50? 100×0.75? $1000 \times \frac{25}{1000}$? 1000×0.025?

5. We have found that to multiply a decimal fraction by 10, we need only move the decimal point one place in which direction? How is it when we multiply by 100? by 1000?

6. Multiply by 10: 9.6, 98.4, 0.96; also by 100.

147. Multiplying by powers of 10. We found on page 17 a rule for multiplying by powers of 10. What was it?

148. Multiplying by multiples of 10. Therefore, to multiply 487.7 by 600 is the same as to multiply by 6, moving the decimal point 2 places to the right (see page 17).

In the same way, to multiply 89.07 by 740 is the same as to multiply by 74, moving the decimal point 1 place to the right.

WRITTEN EXERCISE

Multiply:

1. 40×6.73.	2. 30×4.87.	3. 60×9.81.
4. 90×86.4.	5. 80×36.5.	6. 70×625.3.
7. 800×65.42.	8. 900×37.08.	9. 400×61.8.
10. 1000×1.25.	11. 2000×6.725.	12. 6970×432.
13. 9250×67.25.	14. 8220×350.4.	15. 6700×495.4.
16. 2500×374.65.	17. 3500×408.08.	18. $35{,}000 \times 1.25$.

Division

ORAL EXERCISE

1. Divide by 4: 16 ft., $\frac{16}{25}$, .16, .016, 1.6, .160.
2. Divide by 5: 55 in., $\frac{55}{64}$, .55, .055, 5.5, .550.
3. Divide by 7: \$63, $\frac{63}{64}$, .63, .063, $\frac{63}{10}$, 6.3, .630.
4. Divide by 10: \$450, $\frac{450}{100}$, 4.50, 4.5, .450, .45, .75.

149. Dividing by integers. We have seen that *we write United States money in decimal fractions*, \$2.50 meaning merely $\$2+\$\frac{50}{100}$, or \$2 + \$0.50. And since we have learned how to divide a number representing United States money (page 22), we know how to divide a decimal fraction. For example, to divide 844.375 by 35.

This subject is taken up again on page 110, this being only an introduction.

```
  24.125
35)844.375
   70
   144
   140
    43
    35
    87
    70
    175
    175
```

WRITTEN EXERCISE

1. $1795.5 \div 42$.
2. $48.255 \div 15$.
3. $1583.25 \div 75$.
4. $691.53 \div 89$.
5. $1961.96 \div 98$.
6. $1693.44 \div 36$.
7. $4416.93 \div 63$.
8. $23{,}340.75 \div 75$.

9. The perimeter of a square is 851 in. What is the length of each side?

10. The perimeter of a triangle whose sides are all equal is 1237.9 in. What is the length of each side?

11. The perimeter of a six-sided figure, with equal sides, is 7.62 in. What is the length of each side?

Dividing by Powers of 10

ORAL EXERCISE

State the results in Exs. 1–8:

1. 10×0.4,	$4 \div 10$.	2. 10×1.4,	$14 \div 10$.
3. $10 \times \$0.04$,	$\$0.40 \div 10$.	4. 10×0.04,	$0.4 \div 10$.
5. $10 \times \$0.25$,	$\$2.50 \div 10$.	6. 10×0.25,	$2.5 \div 10$.
7. 10×12.5,	$125 \div 10$.	8. 100×0.04,	$4 \div 100$.

150. Dividing by powers of 10. From the above exercise and from page 23, the following is found:

1. *To divide a number by 10, move the decimal point one place to the left.*

Any integer may be thought of as having a decimal point after units. That is, 6 is the same as 6., or 6.0, or 6.00.

2. *To divide a number by 100, move the decimal point two places to the left.*

Thus, $7 \div 100 = .07$.

3. *To divide a number by 1000, move the decimal point three places to the left.*

Therefore, to divide 216 by 600 is the same as to divide 2.16 by 6.

$$\begin{array}{r} 6\cancel{0}\cancel{0}\,)\,2\,|\,16 \\ \hline .36 \end{array}$$

WRITTEN EXERCISE

1. $133 \div 70$.	2. $10.8 \div 60$.	3. $32.9 \div 70$.
4. $3745 \div 500$.	5. $2291 \div 290$.	6. $296.1 \div 470$.
7. $273.6 \div 120$.	8. $2984 \div 4000$.	9. $7344 \div 9000$.
10. $625 \div 25{,}000$.	11. $961 \div 31{,}000$.	12. $882 \div 21{,}000$.
13. $1728 \div 1200$.	14. $144 \div 60{,}000$.	15. $872 \div 22{,}000$.
16. $7260 \div 2500$.	17. $154 \div 12{,}000$.	18. $145.7 \div 4700$.

WRITTEN EXERCISE

In this exercise pupils should state the approximate answers orally, and these should be written down. The problems should then be solved on paper and the real result compared with the approximate one. This is one of the best checks against absurd results, and should frequently be made a subject of class drill with other written exercises. Teachers will not expect the same approximations from all pupils. $3\frac{1}{2}$ times 22¢ is a natural approximation for Ex. 1.

1. At 22¢ a pound, what will $3\frac{5}{8}$ lb. of steak cost?

2. At 18¢ a gallon, what will 6.25 gal. of oil cost?

3. At 26¢ a pound, what will $6\frac{7}{8}$ lb. of butter cost?

4. At $12\frac{1}{2}$¢ a yard, what will $8\frac{3}{4}$ yd. of ribbon cost?

5. At $2.50 a yard, what will 2.8 yd. of velvet cost?

6. If 6 yd. of silk cost $6.75, what is the price per yard?

7. If 8 doz. men's hose cost $55, what is the price per dozen?

8. A man drives 39.6 mi. in $5\frac{1}{2}$ hr. What is the rate per hour?

9. If 48 doz. boys' hose cost $57.60, what is the price per dozen?

10. At 14¢ a gross, how many gross of buttons can be bought for $3.22?

11. A dealer buys 200 lb. of raisins for $19. How much do they cost a pound?

12. A dealer pays $18.83 for 7 rocking-chairs. How much does he pay for each?

13. A dealer sells 17 pails of mackerel for $40.80. What is the price per pail?

14. A man pays $1110 a year for the rent of a city house. How much does he pay a month?

15. Traveling at the rate of 47 mi. an hour, how far will a train go between 8.45 A.M. and 11.30 A.M.?

16. A grocer buys a 50-gal. barrel of Porto Rico molasses for $11.50. How much does it cost per gallon?

17. A grocer sells a half chest of tea, 140 lb. to the chest, for $19.60. What is the price per pound?

18. A merchant sold a 40-yd. piece of cloth for $45, gaining $12\frac{1}{2}¢$ a yard. How much did the piece cost him?

19. A grocer buys 12 doz. 1-lb. cans of cocoa at $43¢$ each, and sells them at $58¢$ each. How much does he gain?

20. A dry goods dealer sells 75 yd. of cloth for $112.50, thereby gaining $28.50. How much did it cost him per yard?

21. A grocer buys six 50-lb boxes of dried apples at $4\frac{1}{2}¢$ a pound, and sells them at $6¢$ a pound. How much does he gain?

22. A salesman receives $15 a week. He works 49 weeks a year. His expenses are $396 a year. How much does he save a year?

23. If 15 men can do a piece of work in 4 da. 7 hr., a working day being 8 hr., how long will it take 3 men, working at the same rate?

24. Four places, A, B, C, D, are in a straight line. It is 17.42 mi. from A to B, 13.74 mi. from B to C, and 52.11 mi. from A to D. How far is it from C to D?

25. A merchant pays $38.40 for a piece of cloth. If there had been 3 yd. more in the piece, it would have cost him $42. How many yards were there in the piece?

26. Mr. A and Mr. B are in partnership, Mr. A contributing $\frac{3}{5}$ of the money and labor to run the business. If they make $6725 this year, what is the share of each?

DENOMINATE NUMBERS

Review of the Common Tables

ORAL EXERCISE

1. How many ounces in a pound? An ounce is what part of a pound?

2. Tell the number of gills in a pint, pints in a quart, and quarts in a gallon.

3. Tell the number of inches in a foot and feet in a yard. An inch is what part of a foot?

4. Tell the number of quarts in a peck and pecks in a bushel. A peck is what part of a bushel?

5. How many seconds in a minute? minutes in an hour? hours in a day? days in a week? days in a common year? days in a leap year? What months have 30 days?

Express the following as indicated:

6. 6 pt. as quarts. 7. 8 qt. as gallons. 8. 3 yd. as feet.
9. 14 da. as weeks. 10. 8 pk. as bushels. 11. 12 ft. as yards.
12. $1\frac{1}{2}$ lb. as ounces. 13. 15 pt. as quarts. 14. 4 wk. as days.
15. 48 hr. as days. 16. 32 oz. as pounds. 17. $2\frac{1}{2}$ qt. as pints.

WRITTEN EXERCISE

1. Write the table of weight, — ounces, pounds, tons.

2. Write the table of length, — inches, feet, yards, rods, miles.

3. Write the table of dry measure, — quarts, pecks, bushels.

4. Write the table of liquid measure, — gills, pints, quarts, gallons.

151. Tables for reference. While these tables have been learned and used in primary arithmetic, they are here inserted for reference and for such review as may be necessary.

152. Table of length:

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
5½ yards, or 16½ feet = 1 rod (rd.).
320 rods, or 5280 feet = 1 mile (mi.).

The *hand* (4 in.) is used in measuring the height of horses at the shoulder. Sailors use the fathom (6 ft.) and cable length (120 fathoms) for measuring depths, and the knot (nautical mile, 1.15 common or statute miles, or 6080.27 ft.) for distances at sea.

153. Table of square measure:

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
30¼ square yards = 1 square rod (sq. rd.).
160 square rods = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).

Carpenters, architects, and mechanics often write 8″ for 8 in., and 5′ for 5 ft. They also use sq.″ and sq.′ for square inches and square feet. In this book both of these forms are used.

154. Table of cubic measure:

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
128 cubic feet = 1 cord (cd.).

A perch of stone or masonry is usually considered as 1 rd. long, 1 ft. high, and 1½ ft. thick, and it contains 24¾ cu. ft. It varies, however, in different parts of the country. A cubic yard of earth is considered a load.

155. Table of weight:

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).

The ton of 2000 lb. is sometimes called the *short ton*, there being a *long ton* of 2240 lb. which is used in some wholesale transactions in mining products.

Goldsmiths still use an old table of *Troy weight*, but it is not of enough importance to justify teaching. In this table

24 grains (gr.) = 1 pennyweight (pwt. or dwt.).
20 pennyweights = 1 ounce (oz.).
12 oz. = 1 pound (lb.).

The avoirdupois pound contains 7000 gr., the Troy pound 5760 gr. Therefore 1 lb. of iron is heavier than 1 lb. of gold.

A *carat* weight, used in weighing diamonds, varies, but is commonly taken in the United States as 3.2 Troy grains. The word *carat* is also used in speaking of the purity of gold, meaning $\frac{1}{24}$, "16 carats fine" meaning $\frac{16}{24}$ pure gold.

There is also a table of *Apothecaries' weight*, used by physicians and druggists in prescriptions. Its importance is too slight to justify teaching, although it is here inserted for those who may care to use it.

20 grains (gr.) = 1 scruple (sc. or ℈).
3 scruples = 1 dram (dr. or ℥).
8 drams = 1 ounce (oz. or ℨ).
12 ounces = 1 pound (lb.).

156. Table of liquid measure:

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

A gallon contains 231 cu. in. Barrels vary in size, although in estimating the capacity of tanks and cisterns 31.5 gal. are considered a barrel, and 2 bbl. a hogshead.

There is also a table of Apothecaries' liquid measure, in which 16 fluid ounces make 1 pint.

157. Table of dry measure:

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

A bushel contains 2150.42 cu. in. The dry quart is more than the liquid quart, containing 67.2 cu. in., while the liquid quart contains only 57.75 cu. in.

158. Table of time:

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
12 months (mo.) = 1 year (yr.).

"Thirty days hath September,
April, June, and November."

The other months have 31 days, except February, which has 28 days in common years and 29 days in leap years. The common year has 365 days, or 52 weeks and 1 day; the leap year 366 days. A century is 100 years.

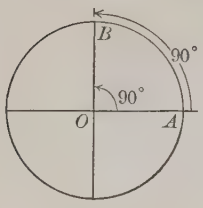
Years whose numbers are divisible by 4 but not by 100, and those divisible by both 4 and 100, are leap years. Thus, 1904, 1908, 2000, but not 2100, are leap years.

159. Table of value:

10 mills = 1 cent (ct. or ¢).
10 cents = 1 dime (d.).
10 dimes = 1 dollar ($).
10 dollars = 1 eagle (E.).

The term *eagle* is not used, although it commonly appears in the table of United States money. The mill is not coined, although the name is frequently used in speaking of fractional parts of a cent.

160. Circular measure. The circumference of a circle is measured in *degrees*.



There are 360 degrees (written 360°) in a whole circumference.

Because the angle at O, the center, increases just as fast as the part of the circumference between its arms increases, an angle is also measured in degrees.

ORAL EXERCISE

1. How many degrees in a circumference? in $\frac{1}{2}$ of a circumference? in $\frac{1}{4}$ of a circumference?

2. If one line is *perpendicular* to another, what angle do the two form? How many degrees in the angle?

3. How many degrees in a right angle? Is an angle of 100° acute, right, or obtuse? an angle of 60°? an angle of 90°? an angle of 120°? an angle of 1°?

161. Perpendicular lines. A line that makes a right angle with another is said to be *perpendicular* to it.

162. Arc. A portion of a circumference is called an *arc*.

163. Arc and angle measure. In measuring arcs

360° = a circumference.

In measuring angles

360° = 4 right angles.

In both cases

60 seconds (60″) = 1 minute (1′).
60 minutes = 1 degree (1°).

4. How many minutes in 5°? 7°? 11°? 25°?

5. How many seconds in 3′? in 5′? in 20′? in 1°?

6. Draw on the board an angle of about 30°; 45°; 135°.

Reduction

164. Denominate numbers. Concrete numbers denoting measure (including weight) are called *denominate numbers.*

For example, \$2, 3 ft., 7 lb. 4 oz.

165. Compound numbers. Denominate numbers in which two or more units are expressed are called *compound numbers.*

For example, 2 ft. 3 in., 4 mi. 3 rd. 2 ft.

166. Reduction. We may express 2 ft. 3 in. as 27 in., or as 2.25 ft., or as $2\frac{1}{4}$ ft. This changing from one form to another, without changing the value, is called *reduction.*

167. Reduction descending. Reduction to a lower denomination is called *reduction descending.*

For example, 3 ft. may be reduced to 36 in.

168. Reduction ascending. Reduction to a higher denomination is called *reduction ascending.*

For example, 24 oz. may be reduced to 1 lb. 8 oz.

Definitions like the above have little value, the subject of compound numbers having lost most of its former importance. We speak of 2 ft. 3 in. instead of $2\frac{1}{4}$ ft., but we rarely use the pound and ounce *together*, speaking of 2 lb. 4 oz. as "2 pounds and a quarter." Hence but little time should be given to work involving more than two denominations.

ORAL EXERCISE

1. Express 10 bu. as pecks; as quarts.
2. Express 64 qt. as pecks; as bushels.
3. Express 40 qt. as gallons; as pints.
4. Express 40 pt. as quarts; as gallons.
5. Express 20 ft. as inches; 21 ft. as yards.

Reduction Descending

169. Reduction descending. Required to reduce 160 rd. to inches.

Explanation:

Since 1 rd. = $16\frac{1}{2}$ ft.,

160 rd. = 160 times $16\frac{1}{2}$ ft. = 2640 ft.

Since 1 ft. = 12 in.,

2640 ft. = 2640 times 12 in. = 31,680 in.

While we think of multiplying $16\frac{1}{2}$ ft. by 160, we actually multiply 160 by $16\frac{1}{2}$ because this is shorter, and so for 2640 times 12 in.

Actual work:

160
$16\frac{1}{2}$
80
960
160
2640 no. of ft.
12
31680 no. of in.

170. Required to reduce 3 T. 42 lb. to ounces.

Since 1 T. = 2000 lb.,

3 T. = 3 times 2000 lb. = 6000 lb.

6000 lb. + 42 lb. = 6042 lb.

Since 1 lb. = 16 oz.,

6042 lb. = 6042 times 16 oz. = 96,672 oz.

2000
3
6000
42
6042 no. of lb.
16
36252
6042
96672 no. of oz.

WRITTEN EXERCISE

Reduce:

1. 8 bu. 2 pk. to pecks.
2. 3 mi. 75 rd. to rods.
3. 10 rd. 3 ft. to feet.
4. 2 A. to square rods.
5. 175 rd. 9 ft. to feet.
6. 5′ 6″ to inches.
7. 2 lb. 2 oz. to ounces.
8. 5 yd. 1 ft. to feet.
9. 2 gal. 3 qt. to quarts.
10. $2\frac{3}{4}$ A. to square rods.
11. 10 cd. to cubic feet.
12. $23\frac{1}{3}$ cu. yd. to cubic feet.
13. 6 cu. yd. 7 cu. ft. to cubic feet; to cubic inches.
14. 4 sq. rd. 3 sq. ft. to square feet; to square inches.

Reduction Ascending

171. Reduction ascending. Required to reduce 200 pt. to gallons.

Explanation:	*Actual work:*
Since 1 pt. $= \frac{1}{2}$ qt.,	$2\overline{)200}$ no. of pt.
200 pt. $= 200$ times $\frac{1}{2}$ qt. $= 100$ qt.	$4\overline{)100}$ " " qt.
Since 1 qt. $= \frac{1}{4}$ gal.,	25 " " gal.
100 qt. $= 100$ times $\frac{1}{4}$ gal. $= 25$ gal.	

172. Required to reduce 175 in. to feet and inches.

Since 1 in. $= \frac{1}{12}$ ft.,	$12\overline{)175}$ no. of in.
175 in. $= 175$ times $\frac{1}{12}$ ft.	14 " " ft.
$= 14\frac{7}{12}$ ft. $= 14$ ft. 7 in.	7 in. rem.

WRITTEN EXERCISE

Reduce:

1. 960 in. to feet.
2. 492 in. to feet.
3. 480 oz. to pounds.
4. 396 in. to yards.
5. 176 oz. to pounds.
6. 7200 sec. to hours.
7. 3600 min. to hours.
8. 275 ft. to rods and feet.
9. 272 pk. to bushels.
10. 3255 lb. to tons and pounds.
11. 527 pt. to bushels, quarts, and pints.
12. 500 sq. in. to square feet and square inches.

173. Reduce $\frac{7}{8}$ in. to a fraction of a yard.

Since 1 in. $= \frac{1}{36}$ yd.,

$\frac{7}{8}$ in. $= \frac{7}{8}$ of $\frac{1}{36}$ yd. $= \frac{7}{288}$ yd.

If required to reduce to a decimal, $\frac{7}{288}$ yd. $= 0.024\frac{11}{36}$ yd.

13. 475 yd. 0.6 ft. to a decimal of a mile.
14. $\frac{3}{4}$ in. to a decimal of a foot; of a yard.
15. $266\frac{2}{3}$ rd. to a common fraction of a mile.

Addition and Subtraction

ORAL EXERCISE

Add in Exs. 1–6:

1. 13 ft. 2 in. 24 8	2. 36 lb. 9 oz. 15 3	3. 10 rd. 4 ft. 25 11
4. 35 mi. 72 rd. 25 26	5. 25 yd. 15 in. 42 9	6. 42 ft. 8 in. 32 4

174. Adding compound numbers. Required the sum of 3 ft. 9 in., 6 ft. 7 in., and 5 ft. 10 in.

Here the sum of the inches is 26 in. = 2 ft. 2 in. The sum of the feet is 14 ft., which with the 2 ft. equals 16 ft.

3 ft. 9 in.
6 7
5 10
16 ft. 2 in.

WRITTEN EXERCISE

1. 25 lb. 12 oz. 24 4	2. 73 rd. $4\frac{1}{2}$ ft. 26 12	3. 26 mi. 300 rd. 23 20
4. 41 yd. 25 in. 41 11	5. 60 ft. 9 in. 10 4	6. 30 lb. 10 oz. 20 8
7. 30 rd. $6\frac{1}{2}$ ft. 40 12	8. 70 mi. 300 rd. 80 25	9. 30 yd. 30 in. 20 8
10. 3 mo. 27 da. 11 16	11. 2 wk. 6 da. 3 5	12. 44 bu. 2 pk. 17 3
13. 32 yd. 1 ft. 49 2 72 2 10 1	14. 94 gal. 2 qt. 68 1 29 3 13 2	15. 27 mi. 68 rd. 37 300 16 200 22 160

ORAL EXERCISE

Subtract in Exs. 1–9:

1. 58 ft. 10 in.	2. 25 lb. 14 oz.	3. 35 rd. 10 ft.
9 6	17 6	26 7
4. 85 mi. 75 rd.	5. 27 yd. 10 in.	6. 49 ft. 7 in.
66 50	19 5	36 7
7. 37 lb. 12 oz.	8. 42 rd. 15 ft.	9. 23 mi. 7 rd.
37 7	27 15	16

175. Subtracting compound numbers. In subtracting 17 ft. 10 in. from 23 ft. 8 in., we have: 8 in. − 10 in., impossible; 1 ft. 8 in., or 20 in. − 10 in. = 10 in.; 22 ft. − 17 ft. = 5 ft.

23 ft. 8 in.
17 10
5 ft. 10 in.

WRITTEN EXERCISE

1. 49 yd. 8 in.	2. 3 ft. 4 in.	3. 10 lb. 3 oz.
30 8	8	7
4. 6 rd. $6\frac{1}{2}$ ft.	5. 10 mi.	6. 10 yd. 10 in.
13	20 rd.	30
7. $47\frac{1}{4}$ yd.	8. 49 ft. $4\frac{1}{2}$ in.	9. 37 lb. $6\frac{1}{3}$ oz.
$38\frac{1}{2}$	39 $8\frac{3}{4}$	28 $7\frac{2}{3}$
10. 26 rd. $8\frac{1}{2}$ ft.	11. 76 mi. $7\frac{3}{4}$ rd.	12. 53 yd. $28\frac{1}{2}$ in.
18 $9\frac{3}{4}$	29 8	47 $32\frac{3}{4}$

176. We rarely need to add or subtract compound numbers of more than two denominations. One case where three denominations are used is in finding the difference between two dates, as in Exs. 21, 22, page 100.

WRITTEN EXERCISE

Add in Exs. 1–11:

1. 37 ft. 9 in.
   42 7
   67 6
   31 4

2. 23 lb. 7 oz.
   48 9
   61 4
   37 5

3. 82 rd. 5 ft.
   75 7
   62 12
   9 $3\frac{1}{2}$

4. 24 qt. 1 pt.
   3 0
   45 1
   12 1

5. 91 yd. 27 in.
   61 14
   33 31
   42 9

6. 42 bu. 3 pk.
   27 1
   63 2
   48 3

7. 23 gal. 2 qt.
   41 1
   6 0
   83 3

8. 23 mi. 57 rd.
   46 120
   29 230
   80 175

9. 32 mi. 4260 ft.
   27 2027
   33 45
   241 605

10. 92 cu. ft. 246 cu. in.
    73 487
    34 995

11. 75 sq. ft. 92 sq. in.
    86 49
    93 135

Subtract in Exs. 12–22:

12. 93 ft. 7 in.
    61 8

13. 47 lb. 8 oz.
    26 9

14. 32 rd. 4 ft.
    26 5

15. 81 mi. 7 rd.
    65 98

16. 87 yd. 4 in.
    69 8

17. $65\frac{1}{2}$ ft.
    48 9 in.

18. 85 lb. 6 oz.
    $36\frac{1}{2}$

19. 67 rd. $8\frac{1}{4}$ ft.
    $39\frac{1}{2}$

20. 89 mi. 80 rd.
    $69\frac{1}{2}$

21. 1910 yr. 6 mo. 8 da.
    1905 7 9

22. 1909 yr. 7 mo. 3 da.
    1905 2 15

Multiplication

ORAL EXERCISE

Multiply:

1. 3 ft. 4 in. by 2.
2. 8 lb. 2 oz. by 6.
3. 7 ft. 2 in. by 4.
4. 7 rd. 1 ft. by 8.
5. 8 ft. 1 in. by 9.
6. 11 lb. 3 oz. by 4.
7. 9 yd. 6 in. by 3.
8. 9 gal. 1 qt. by 3.
9. 8 yd. 3 in. by 8.
10. 12 gal. 1 qt. by 2.
11. 6 yd. 2 in. by 7.
12. 10 mi. 200 ft. by 5.

177. Multiplying compound numbers. Required to multiply 4 lb. 9 oz. by 6.

We might multiply as with other numbers and obtain 24 lb. 54 oz., and reduce this to 27 lb. 6 oz. But it is easier to say: "6 times 9 oz. = 54 oz. = 3 lb. 6 oz.; 6 times 4 lb. = 24 lb., which added to 3 lb. = 27 lb."

4 lb. 9 oz.
6
27 lb. 6 oz.

178. Required to multiply 243 ft. 7 in. by 27.

Multiplying as with other numbers, the result is 6561 ft. 189 in., which is reduced to 6576 ft. 9 in.

243 ft. 7 in.
27
1701 49
486 14
6561 ft. 189 in. =
6576 ft. 9 in.

WRITTEN EXERCISE

Multiply in Exs. 1–10:

1. 47 ft. 6 in. by 25.
2. 27 lb. 3 oz. by 84.
3. 49 yd. 2 in. by 96.
4. 44 mi. 3 ft. by 89.
5. 109 ft. 3 in. by 47.
6. 273 ft. 2 in. by 35.
7. 121 lb. 4 oz. by 92.
8. 309 lb. 5 oz. by 67.
9. 401 yd. 3 in. by 68.
10. 370 yd. 1 in. by 83.

Division

ORAL EXERCISE

1. 24 ft. 6 in. ÷ 2.
2. 48 ft. 8 in. ÷ 4.
3. 36 ft. 6 in. ÷ 6.
4. 21 ft. 7 in. ÷ 7.
5. 81 lb. 9 oz. ÷ 9.
6. 72 ft. 8 in. ÷ 8.
7. 50 ft. 10 in. ÷ 5.
8. 27 lb. 12 oz. ÷ 3.
9. 56 lb. 14 oz. ÷ 7.
10. 72 in. ÷ 9 in.
11. 64 ft. ÷ 8 ft.
12. 54 rd. ÷ 6 rd.
13. 108 yd. ÷ 12 yd.
14. 225 ft. ÷ 25 ft.

179. Two kinds of division. As shown on page 20, there are two kinds of division. Because 9 times 8 ft. = 72 ft., therefore,

(1) 72 ft. ÷ 9 = 8 ft., (2) 72 ft. ÷ 8 ft. = 9.

180. Abstract divisor. To consider the first kind, divide 371 ft. 6 in. by 15.

Dividing as usual, 371 ft. ÷ 15 = 24 ft., and 11 ft. remainder. 11 ft. = 11 times 12 in. = 132 in., and 132 in. + 6 in. = 138 in., to be divided. 138 in. ÷ 15 = $9\frac{1}{5}$ in. Therefore the quotient is 24 ft. $9\frac{1}{5}$ in.

$$\begin{array}{r} 24 \text{ ft.} \\ 15\overline{)371 \text{ ft.}} \\ 30 \\ \hline 71 \\ 60 \\ \hline 11 \text{ ft.} = 132 \text{ in.} \\ \hline 138 \text{ in.} \end{array} \qquad \begin{array}{r} 6 \text{ in.} \\ \\ \\ \\ \\ \hline 9\frac{1}{5} \text{ in.} \\ 15\overline{)138 \text{ in.}} \\ 135 \\ \hline 3 \end{array}$$

181. Concrete divisor. To consider the second kind, divide 22 ft. 9 in. by 3 ft. 3 in.

In such cases reduce dividend and divisor to the same denomination, thus:

22 ft. 9 in. = 273 in.
3 ft. 3 in. = 39 in.
Therefore 273 in. ÷ 39 in. = 7.

$$\begin{array}{r} 7 \\ 39 \text{ in.}\overline{)273 \text{ in.}} \\ 273 \\ \hline \end{array}$$

WRITTEN EXERCISE

1. 241 lb. 3 oz. ÷ 8.
2. 472 yd. 2 ft. ÷ 3.
3. 123 ft. 8 in. ÷ 11.
4. 82 yd. 18 in. ÷ 27.
5. 955 ft. 6 in. ÷ 13.
6. 423 lb. 15 oz. ÷ 21.
7. 10 yd. 20 in. ÷ 19 in.
8. 30 lb. ÷ 7 lb. 8 oz.
9. 12 ft. 10 in. ÷ 4 ft. 2 in.
10. 32 gal. 2 qt. ÷ 3 gal. 1 qt.
11. 1 cu. yd. 3 cu. ft. ÷ 10 cu. ft.
12. 10 sq. yd. 6 sq. ft. ÷ 3 sq. yd. 5 sq. ft.
13. 24 yr. 11 mo. 6 da. ÷ 2 yr. 3 mo. 6 da.
14. 2 da. 7 hr. 9 min. 18 sec. ÷ 3 hr. 27 min. 42 sec.

Multiply in Exs. 15–20:

15. 15 hr. 27 min. by 6.
16. 18 bu. 3 pk. by 8.
17. 621 mi. 20 ft. by 97.
18. 9 T. 1650 lb. by 4.
19. 32 min. 48 sec. by 12.
20. 404 mi. 70 ft. by 73.

21. If the average weight of the members of your class is 57 lb. 4 oz., what is the total weight of the class?

22. If the average length of your steps is 2 ft. 2 in., how far will you go in 425 steps?

23. If a wheel is 9 ft. 2 in. in circumference, how far will it travel in making 125 revolutions?

24. If one side of a square is 8 ft. 9 in., how far is it around the square?

25. If it is 18 ft. 8 in. around a square, how long is each side?

26. It is 54 ft. around a certain figure each of whose sides is 6 ft. 9 in. How many sides has the figure?

27. If a wheel is 8 ft. $9\frac{3}{8}$ in. in circumference, how many revolutions will it make in going a mile?

182. Areas. What is the area of a flower bed 8 ft. wide and 1 rd. long?

If it were $16\frac{1}{2}$ ft. long and 1 ft. wide, the area would be $16\frac{1}{2}$ sq. ft. Since it is 8 ft. wide, the area is 8 times $16\frac{1}{2}$ sq. ft., or 132 sq. ft.

WRITTEN EXERCISE

1. At the rate of a mile in 4 min., how far will an automobile go in 3 hr.?

2. How far will a horse go in the same time, traveling at the rate of a mile in 9 min.? How far will a train go in the same time, at the rate of 45 mi. an hour?

3. If the large wheels of an automobile are $7\frac{2}{5}\frac{3}{5}$ ft. in circumference, how many times will each one turn in going 25 mi.?

4. At \$65 an acre, what is a rectangular farm worth that is 150 rd. wide and 160 rd. long?

5. At 21 ct. a foot, how much would it cost to inclose the farm of Ex. 4 by a wire fence?

6. If the man wishes to enrich the soil of a 75-acre meadow, how many tons of fertilizer would he use at 75 lb. to the acre?

7. If a farmer has cut 35 cords of 4-ft. wood from his property and piled it 4 ft. high, how long is the pile?

8. If a lot 8 rd. by 40 rd. is planted to corn, and produces $27\frac{1}{2}$ bu. to the acre, how many bushels does the owner get in all?

9. A farmer has a barn 60 ft. long, 30 ft. wide, and 15 ft. high to the eaves. The triangular gables on the ends, above the line of the eaves, have each an area of 150 sq. ft. How much will it cost to paint the barn at 20 ct. a square yard? (Illustrate the four sides by drawings.)

II. DECIMAL FRACTIONS AND PERCENTAGE

ADDITION AND SUBTRACTION OF DECIMALS

Some of our Great Railways

WRITTEN EXERCISE

1. The lengths of the various divisions of the Atchison, Topeka, and Santa Fé road were recently as follows: 1177.81 mi., 487.37 mi., 887.99 mi., 372.48 mi., 129 mi., 60 mi., and 4818.08 mi. What was the total length?

2. On the Baltimore and Ohio the divisions were: 5.3 mi., 129.42 mi., 949.98 mi., 403.08 mi., 323 mi., 790.19 mi. What was the total length?

3. On the Boston and Maine the divisions were: 115.31 mi., 108.29 mi., 73.37 mi., 148.34 mi., 222.32 mi., 69.5 mi., 142¼ mi., 98.77 mi., 74 mi., 250.98 mi., 53.85 mi., 907.71 mi. What was the total length?

4. How much longer was the Chicago and Northwestern, when it had 8791.29 mi., than the Canadian Pacific with 8356.1 mi.?

5. How much longer was the Burlington Route, when it had 8219.87 mi., than the Denver and Rio Grande with 1708.19 mi.? than the Chicago, Milwaukee, and St. Paul with 6582.58 mi.?

6. How much shorter was the Great Northern, when it had 5240.47 mi., than the combined lengths of the Erie, 2170 mi., and the Illinois Central, 4265.5 mi.?

7. How much longer was the Pennsylvania railroad, when it had 9970 mi., than the Southern Pacific, 8206.75 mi.? than the New York Central, 3313.81 mi.? than the Wabash, 2367.3 mi.? than the Plant System, 3589 mi.?

MULTIPLICATION OF DECIMALS

ORAL EXERCISE

1. Multiply:

2 ft.	\$2	2 tenths	0.2	2 hundredths	0.02
3	3	3	3	3	3

2. What is the product of $\frac{2}{3}$ and $\frac{1}{7}$? What is the product of thirds and sevenths? What is the product of halves and fifths? What is the product of tenths and tenths?

3. What is the product of $\frac{7}{10}$ and $\frac{3}{10}$? of 0.7 and 0.3? of 0.3 and 0.3? of $\frac{3}{10}$ and $\frac{9}{10}$? of 0.3 and 0.9?

4. Multiply: $\frac{2}{10} \times \frac{5}{10}$ $\frac{6}{10} \times \frac{7}{10}$ $\frac{5}{10} \times \frac{9}{10}$

0.4×0.7 0.6×0.8 0.9×0.9

183. Multiplying by decimals. Required to multiply 0.67 by 4.

Since 4 times hundredths is hundredths, the 268 is hundredths, and the decimal point goes before the 6. That is, 4 times $\frac{67}{100} = \frac{268}{100} = 2.68$.

$$\begin{array}{r} 0.67 \\ 4 \\ \hline 2.68 \end{array}$$

To multiply 52 by 0.03.

The product of 52 and hundredths is hundredths. Therefore the answer is hundredths and the decimal point goes before the 5. That is, $\frac{3}{100}$ of $52 = \frac{156}{100} = 1.56$.

$$\begin{array}{r} 52 \\ 0.03 \\ \hline 1.56 \end{array}$$

§§ 184 and 185 may be introduced here if desired.

WRITTEN EXERCISE

1. 2×0.79. 2. 3×0.96. 3. 4×0.78. 4. 7×0.93.

5. 0.82×8. 6. 0.67×9. 7. 0.83×4. 8. 0.27×7.

9. 3×1.75. 10. 5×2.42. 11. 6×7.23. 12. 8×9.23.

13. At 6.95 in. to a step, how high do some stairs of 20 steps reach?

184. Multiply .42 by .35.

If we were to multiply 42 by 35, the result would be 1470. And since hundredths multiplied by hundredths equals ten-thousandths, the result is .1470. That is, $\frac{42}{100} \times \frac{35}{100} = \frac{1470}{10000} = .1470$ or .147.

$$\begin{array}{r} .42 \\ .35 \\ \hline 210 \\ 126 \\ \hline .1470 \end{array}$$

185. Decimal places in the product. Therefore, in multiplying decimals,

1. *Multiply as with integers.*

2. *Point off as many decimal places in the product as there are decimal places in all the factors.*

It is unnecessary to remember this rule, since it is easy to see that the product of tenths and tenths is hundredths, of tenths and hundredths is thousandths, and so on. It is often helpful, however.

186. Multiply 6.2 by 3.5.

If we were to multiply 62 by 35, the result would be 2170. And since tenths multiplied by tenths equals hundredths, the result is 21.70 (that is, $\frac{2170}{100}$ or $21\frac{70}{100}$), or 21.7.

$$\begin{array}{r} 6.2 \\ 3.5 \\ \hline 310 \\ 186 \\ \hline 21.70 \\ \text{or } 21.7 \end{array}$$

ORAL EXERCISE

1. 3×0.6.	2. 4×0.7.	3. 5×0.61.	4. 6×0.11.
5. .5 of 2.	6. .6 of 7.	7. .8 of 8.	8. .7 of 10.
9. .2 of .2.	10. .3 of .4.	11. .5 of .7.	12. .8 of .9.

WRITTEN EXERCISE

1. .7 of 96.	2. .8 of 74.	3. .6 of 48.	4. .9 of 84.
5. .8 of 9.8.	6. .7 of 4.9.	7. .9 of 4.7.	8. .3 of 9.9.
9. 1.5×67.	10. 1.4×75.	11. 3.8×46.	12. 4.7×35.
13. 2.7×3.4.	14. 1.9×2.6.	15. 4.7×3.6.	16. 8.1×8.1.

17. A meter being 39.37 in. long, how many inches in 2.8 meters? in 17.6 meters?

ORAL EXERCISE

1. Why will there be three decimal places in the product of 0.75 multiplied by 0.3? of 2.325 multiplied by 7?

2. How do you know the number of decimal places in the product of 0.62 and 2.79? of 2.756 and 3.123?

Without multiplying, tell the order of the decimal part of each of the following:

3. 2.8×7.9. 4. 2.007×6.09. 5. 1.203×2.67.
6. 1.7×1.42. 7. 1.234×3.456. 8. 3.421×0.713.

WRITTEN EXERCISE

1. 43.2×98.6. 2. 89.1×49.3. 3. 68.7×40.5.
4. $.823 \times 2946$. 5. 72.9×87.09. 6. 53.6×2.889.
7. 3.49×80.23. 8. 36.4×49.87. 9. 98.9×69.89.
10. 2.99×124.63. 11. 48.7×14.689. 12. 52.9×68.023.

Multiply the following:

13. 234.5 by 8.001. 14. 0.500 by 0.600.
15. \$683.45 by 0.2. 16. \$281.42 by 6.50.
17. \$283.75 by 42.8. 18. \$333.33$\frac{1}{3}$ by 0.3.
19. 25×9.2. 20. 37×4.6. 21. 48×0.8. 22. 96×9.6.
23. 5.9×9.5. 24. 7.3×4.8. 25. 6.7×1.8. 26. 5.4×3.5.
27. 48.1×32.5. 28. 20.5×60.7. 29. 43.7×74.1.
30. 26.8×80.8. 31. 20.2×20.2. 32. 30.5×57.2.
33. 60.9×90.6. 34. 73.7×73.7. 35. 8.25×6.37.
36. 4.72×6.93. 37. 42.3×5.37. 38. 17.2×26.35.
39. 2.87×5.492. 40. 6.81×7.92. 41. 2.175×6.73.
42. 16.4×17.25. 43. 8.35×6.423. 44. 0.273×0.625.
45. 0.352×6.725. 46. $3.33\frac{1}{3} \times 6.963$. 47. $0.16\frac{2}{3} \times 0.41\frac{1}{2}$.

WRITTEN EXERCISE

1. At $1.75 each, what will 17 books cost?

2. What is the perimeter of a square 34.7 ft. on a side?

3. At $75.50 an acre, what will 25.5 acres of land cost?

4. At $0.75 a yard, what will 32.2 yd. of China silk cost?

5. At $125 a front foot, how much will a city lot cost having a frontage of 72.3 ft.?

6. At $65.50 an acre, how much must a man pay for 200 acres of land? for 25.4 acres? for 3.75 acres?

7. At $17.52 a front foot, what will a piece of land 28.33⅓ ft. front cost?

8. A tank when full holds 42.8 gal. How much does it contain when .37 full?

9. A certain rectangle is .78 as wide as it is long. It is 4.9 in. long. What is the perimeter?

10. A certain rectangle is 3.42 times as long as it is wide. It is 4.8 ft. wide. What is the perimeter?

11. A city lot is 45.8 ft. wide and 76.3 ft. deep. What is the area of the lot? What is it worth at $1.50 a square foot?

12. The circumference of a circle is 3.1416 times the diameter. If the diameter is 42.27 ft., what is the circumference?

13. How many square rods of land are there in a piece of land 80 rd. wide and 114.1 rd. long? How many acres? What is it worth at $75 an acre?

14. There are three towns, A, B, C, forming the three corners of a triangle. From A to B is 27.32 mi., from B to C it is 1.4 times as far, and the perimeter of the triangle is 72.8 mi. How far is it from C to A?

DIVISION OF DECIMALS

ORAL EXERCISE

1. Divide 10 by 2; 100 by 20. Compare the results.

2. Divide \$2.50 by 5; \$25 by 50. Compare the results.

3. If multiplying both dividend and divisor by the same number does not change the quotient, how does the quotient of $360 \div 4$ compare with that of $36 \div 0.4$?

187. Division of decimals. Since both dividend and divisor may be multiplied by the same number without changing the quotient, therefore,

In the division of decimal fractions, multiply both dividend and divisor by the power of 10 that shall make the divisor an integer, and divide as in United States money.

188. Divide 6.25 by 2.5.

The result is the same if we divide 62.5 by 25, thus making the divisor an integer.

Dividing as in United States money, $62.5 \div 25 = 2$, with 12.5 still to be divided. $12.5 \div 25 = 0.5$. Therefore the quotient is 2.5.

$$
\begin{array}{r}
2.5 \\
25\overline{)62.5} \\
50 \\
\hline
12\,5 \\
12\,5 \\
\hline
\end{array}
$$

In developing this before a class, the decimal point in 12.5 should be preserved at first, and finally dropped as unnecessary.

WRITTEN EXERCISE

1. $6.25 \div 0.25$, or $625 \div 25$.
2. $625 \div 0.25$.
3. $60 \div 0.6$, or $600 \div 6$.
4. $600 \div 0.06$.
5. $62.9 \div 3.7$.
6. $4.62 \div 2.2$.
7. $629 \div .37$.
8. $8.05 \div 3.5$.
9. $.462 \div .21$.
10. $7.14 \div 4.2$.
11. $3050 \div .61$.
12. $11.56 \div 3.4$.
13. $30.5 \div 0.61$.
14. $6.232 \div 0.82$.
15. $17.466 \div 7.1$.
16. $62.32 \div 8.2$.

WRITTEN EXERCISE

Add in Exs. 1–8:

1.	2.	3.	4.
240.4	4.28	3.21	169.28
802.2	85.00	150.5	3.7
79.8	6.32	7.32	45.94
388.7	108.51	46.48	2.33
96.3	7.23	2.87	25.4
434.1	.75	18.95	8.86
625.4	34.87	9.26	10.92

5.	6.	7	8.
4.028	.387	.1.043	.2909
.396	.0426	.427	.0628
.285	.0539	6.091	.0703
4.72	.2064	.5	.5076
3.942	.0708	.709	.0289
.409	.5029	6.273	.0079
6.029	.0627	.108	.0403

Subtract in Exs. 9–20:

9.	10.	11.	12.
90.32	209.8	6.273	81.04
4.86	49.2	.992	6.98

13.	14.	15.	16.
1.035	.2809	82.481	64.035
.876	.1589	2.909	37.029

17.	18.	19.	20.
86.431	102.98	62.035	80.289
28.902	72.89	51.975	79.939

21. 1.4×6.64. 22. 1.5×553.35. 23. 1.6×140.64.
24. 3.6×1972.08. 25. 5.8×439.002. 26. 6.2×53.134.
27. $888.16 \div 2.8$. 28. $825.468 \div 4.2$. 29. $20.720 \div 0.28$.
30. $103.60 \div 0.56$. 31. $1131.264 \div 4.8$. 32. $38.8692 \div 5.4$.

189. Illustrative problems. 1. Divide 16.35 by 500.

Here it is better to divide both dividend and divisor by 100, canceling the zeros and moving the decimal point two places to the left.

$$5\cancel{0}\cancel{0}\overline{)\,.1635}$$
$$.0327$$

2. Divide 3.82 by 2.5, carrying the quotient to one decimal place.

Multiplying both numbers by 10, we have $38.2 \div 25 = 1.5$, with 7 remainder. If we do not write the fraction $\frac{7}{25}$ in the quotient, the quotient is written 1.5+, meaning 1.5 and a fraction less than $\frac{1}{2}$.

3. Divide .049 by .17, carrying the quotient to two decimal places.

Multiplying both numbers by 100, we have $4.9 \div 17 = 0.28$, with 14 remainder. If we do not write the fraction $\frac{14}{17}$ in the quotient, the quotient is written 0.29−, meaning 0.28 and a fraction more than $\frac{1}{2}$.

WRITTEN EXERCISE

1. $17.25 \div 500$
2. $3.248 \div 200$.
3. $817.6 \div 700$.
4. $82.48 \div 8000$.
5. $.7364 \div 7000$.
6. $74.34 \div 9000$.
7. $15.720 \div 12{,}000$.
8. $133.10 \div 11{,}000$.
9. $404.30 \div 13{,}000$.
10. $\$6844.50 \div 900$
11. $\$7337.75 \div 500$.
12. $\$1247.01 \div 300$.

In the following divisions, carry the quotient to three decimal places:

13. $17.78 \div 1.5$.
14. $62.23 \div .23$.
15. $19.26 \div 3.1$.
16. $423.6 \div .27$.
17. $167.8 \div 3.7$.
18. $8.348 \div 2.68$.
19. $976.34 \div .125$.
20. $483.62 \div 42.7$.
21. $826.3 \div 12.5$

190. Illustrative problem. What is the quotient of 17.8 × 45.39 divided by .267?

Multiplying both dividend and divisor by 1000, to make the divisor integral, we have for the dividend 178×4539 (i.e. 10 times 17.8 and 100 times 45.39). Factoring and canceling, we have $2 \times 1513 = 3026$.

$$\frac{2 \times 89 \quad 1513}{178 \times 4539}$$
$$\overline{267}$$
$$3 \times 89$$

WRITTEN EXERCISE

1. $\frac{3.9 \times 0.84}{15.6}$.
2. $\frac{5.5 \times 0.42}{0.77}$.
3. $\frac{2.1 \times 0.4}{0.015}$.
4. $\frac{1.4 \times 0.6}{0.63}$.
5. $\frac{4.5 \times 6.3}{8.1}$.
6. $\frac{0.21 \times 0.6}{0.126}$
7. $\frac{2.2 \times 2.4}{4 \times 7.7}$.
8. $\frac{5.6 \times 0.3}{0.63 \times 0.8}$.
9. $\frac{1.8 \times 5.5}{9.9 \times 0.1}$.
10. $\frac{0.8 \times 1.5 \times 0.6}{3.6 \times 0.2 \times 10}$.
11. $\frac{0.2 \times 3.6 \times 1.43}{9.9 \times 0.4 \times 2.6}$.
12. $\frac{1.2 \times 0.39 \times 1.4}{2.6 \times 0.42 \times 2}$.
13. $\frac{6.3 \times 4.2 \times .17}{2.7 \times 3.4 \times .49}$.

14. The product of 6.3, 1.4, and 5.7 is how many times as great as the product of 17.1, 0.06, and 7?

15. The product of 0.2, 0.21, and 7.5 is how many times as great as the product of 0.14, 0.3, and 1.5?

16. How many times is the product of 1.6, 0.01, and 1.5 contained in the product of 0.032, 2.25, and 3?

17. The volume of a box whose interior dimensions are 2.4 in., 5 in., 1.7 in., is how many times as great as that of one whose interior dimensions are 3 in., 1.6 in., and .85 in.?

18. A block of marble 3.5 ft. by $4\frac{1}{4}$ ft. by 2.2 ft. weighs how many times as much as one that is $2\frac{1}{2}$ ft. by 7.7 ft. by 0.85 ft.?

Common Business Fractions

ORAL EXERCISE

1. Express as decimal fractions:

$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{3}{8}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{2}{5}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{7}{8}$

2. Express as common fractions:

.5	.25	.6	$.16\frac{2}{3}$	$.12\frac{1}{2}$	.125	$.33\frac{1}{3}$
.50	.75	.8	$.83\frac{1}{3}$	$.37\frac{1}{2}$	.625	$.87\frac{1}{2}$

191. Common business fractions. The following are so commonly used that they should be remembered:

$\frac{1}{4} = .25$	$\frac{1}{16} = .06\frac{1}{4}$	$\frac{1}{12} = .08\frac{1}{3}$	$\frac{1}{5} = .2$
$\frac{1}{2} = .5$	$\frac{1}{8} = .12\frac{1}{2}$	$\frac{1}{6} = .16\frac{2}{3}$	$\frac{2}{5} = .4$
$\frac{1}{2} = .50$	$\frac{3}{8} = .37\frac{1}{2}$	$\frac{1}{3} = .33\frac{1}{3}$	$\frac{3}{5} = .6$
$\frac{3}{4} = .75$	$\frac{5}{8} = .62\frac{1}{2}$	$\frac{2}{3} = .66\frac{2}{3}$	$\frac{4}{5} = .8$
	$\frac{7}{8} = .87\frac{1}{2}$	$\frac{5}{6} = .83\frac{1}{3}$	

Hence to find .5 of 80, take $\frac{1}{2}$ of 80;
" " $.66\frac{2}{3}$ of 90, take $\frac{2}{3}$ of 90;
" " $.37\frac{1}{2}$ of 160, take $\frac{3}{8}$ of 160.

3. .2 of 35.	4. .4 of 50.	5. .75 of 44.
6. $.66\frac{2}{3}$ of 270.	7. $.83\frac{1}{3}$ of 48.	8. $.37\frac{1}{2}$ of 240.
9. $.62\frac{1}{2}$ of 720.	10. $.87\frac{1}{2}$ of 800.	11. $.06\frac{1}{4}$ of 320.
12. $.08\frac{1}{3}$ of 120.	13. .40 of 4500.	14. $.33\frac{1}{3}$ of 333.
15. .50 of 2400.	16. .80 of 5555.	17. $.08\frac{1}{3}$ of 240.
18. 20 times .75.	19. 30 times $.66\frac{2}{3}$.	20. 60 times $.83\frac{1}{3}$.

21. At $6\frac{1}{4}$ ct. a gallon, how much will 16 gal. of oil cost?

22. At $62\frac{1}{2}$ ct. a pound, how much will 8 lb. of tea cost?

23. At $87\frac{1}{2}$ ct. a pair, how much must a dealer pay for 80 pairs of overshoes?

HOW TO SOLVE PROBLEMS

192. Nature of the rest of arithmetic. Having learned how to work with integers and fractions, this work is hereafter to be applied chiefly to problems of life.

193. Step work. In solving a problem, or working an example as we also say, we should not only have the work in neat form on paper, but we should also have a brief statement of each operation, in numbered steps.

194. Illustrative problem. What is the value of a rectangular piece of land 20 rd. wide and 32 rd. long at \$75 an acre?

Work in steps:

1. 20 times 32 sq. rd. = 640 sq. rd.
2. 640 times $\frac{1}{160}$ A. = 4 A.
3. 4 times \$75 = \$300.

Actual work:

$$\frac{\overset{4}{\cancel{20}} \times 32 \times \$75}{\underset{\$}{\cancel{160}}} = \$300$$

Use × for *times*, following the custom of the school as to writing 4 × \$75, or \$75 × 4.

Oral Analysis

Since a strip of land 32 rd. long and 1 rd. wide contains 32 sq. rd., a strip 20 times as wide contains 20 times 32 sq. rd., or 640 sq. rd.

Since 1 sq. rd. = $\frac{1}{160}$ A., 640 sq. rd. = 640 times $\frac{1}{160}$ A. = 4 A.

Since 1 A. is worth \$75, 4 A. are worth 4 times \$75 = \$300.

195. Teachers should not forget the value of each of these three phases of the work. To neglect the form of the actual computation is to place a premium upon slovenly work, always a temptation to inaccuracy; to neglect the step work is to tempt the pupil to looseness of thought as to the meaning of each operation; to neglect the analysis is to be uncertain as to the pupil's understanding of the work as a whole. The teacher's judgment must determine the emphasis to be placed at various times upon these respective phases.

Clothing Our People

GROWING THE COTTON

WRITTEN EXERCISE

1. This cotton field is 70 rd. long and 32 rd. wide. How many acres does it contain? What is it worth at $14.50 an acre?

2. Last year it yielded 245 lb. of cotton to the acre, net lint (without counting the seed). What was the total yield? How many bales at 490 lb. to the bale?

3. What did it cost to gin this cotton at $1.25 a bale? What to compress the bales for shipping, at 40 ct. a bale?

4. What did it cost to store it for 2 months in a warehouse, at 25 ct. a bale per month for storage and insurance?

5. The cotton was compressed in bales 5 ft. by 2 ft. by 1 ft. in size. How many cubic feet in each bale? How many cubic inches in each?

6. The 7 bales were sold by an agent. How much commission did he receive at 50 ct. a bale?

7. When cotton is worth 7 ct. a pound, what is the value of the crop produced on 14 acres averaging 245 lb. to the acre?

8. How long would it take a man to pick 3696 lb. of cotton if he averaged 154 lb. a day? It would take 4 men how long, at the same rate?

9. If this country produced 10,701,453 bales of cotton in a certain year, how many tons did it produce, estimating 500 lb. to a bale?

10. If Texas produced 2,438,555 bales in one year; Georgia, 1,345,699; Mississippi, 1,202,739; and Alabama, 1,005,313, what was the total for the four states?

11. If the Texas crop sold for $92,196,000, the Georgia crop for half as much, the Mississippi crop for $\frac{4}{5}$ as much, and the Alabama crop for $1\frac{7}{8}$ as much, for how much did each sell?

12. If Texas pays $24,136,800 annually to have the cotton picked, and Georgia pays $\frac{1}{4}$ as much, and Alabama 0.9 as much as Georgia, how much does Alabama pay?

13. If there are 6,642,309 acres planted to cotton in Texas, in South Carolina $\frac{1}{3}$ as much as in Texas, and in Louisiana $\frac{1}{2}$ as much as in South Carolina, how many acres in each?

14. If our country raised $334,847,800 worth of cotton in a certain year, and paid $\frac{1}{5}$ of this amount for the labor of harvesting and marketing the crop, how much is left for the growers?

15. If we sold in one year to Great Britain $90,240,000 worth of cotton, $\frac{2}{3}$ as much to Germany, $\frac{1}{2}$ as much to France as to Germany, and $\frac{1}{6}$ as much to Italy as to Great Britain, what is the value of the cotton sold to each?

GROWING THE WOOL

WRITTEN EXERCISE

1. If the 20 sheep on a certain farm are worth on an average \$3.25 apiece, what is the value of a tenant's share who owns $\frac{3}{5}$ of them?

2. How much is the wool of these 20 sheep worth, each sheep yielding $7\frac{1}{4}$ lb., and the wool selling at 35 ct. a pound?

3. In a year when 40,267,818 sheep were reported in the United States, each yielding an average of $6\frac{1}{2}$ lb. of wool, what was the total weight of the wool?

4. If this country produced 259,972,815 lb. of wool in one year, and $\frac{3}{5}$ of the weight was lost in scouring (thoroughly cleaning), how much did the scoured wool weigh?

5. In one year this country produced 302,502,000 lb. of wool, and $\frac{1}{20}$ more in the following year. How many pounds did it produce in the second year?

6. In the year when we produced the 302,502,000 lb., this was $\frac{1}{3}$ of the world's total production. How much did the world produce in that year?

7. In one year Montana, our greatest wool-growing state, produced 9,627,000 lb., Ohio 0.8 as much, and New Mexico and Wyoming each 0.9 as much as Ohio. How many pounds were produced by each of these states?

SHIPPING THE MATERIAL

WRITTEN EXERCISE

1. How much will it cost to ship 100 bales of cotton from New Orleans to New York, the average weight being 490 lb., and the freight being 61 ct. a hundred?

2. How much will it cost to ship a 200-lb. box of men's suits from New York to Chicago, at 75 ct. a hundred, with 75 ct. extra for cartage?

3. It costs 69 ct. a hundred pounds to ship cloth from Philadelphia to Chicago. How much would Chicago save in freight on 1000 tons of cloth, by manufacturing it at home instead of buying it in Philadelphia?

4. If 400 lb. of raw cotton are required for 100 lb. of finished clothing, and the freight on clothing from Mobile to New York is $1.18 a hundred, and on cotton 61 ct. a hundred, how much freight per 100 lb. of clothing would New York save by having clothing made in Mobile instead of at home?

5. It costs 46 ct. a hundred to ship wool in car lots from Cleveland to New York, and 53 ct. a hundred for smaller amounts. The smallest car lot of wool is 10,000 lb. If a man has 9500 lb. to ship, is it better for him to ship it as a car lot or to pay the 53-ct. rate? Give the cost in each case.

PICTURES OF MAGNITUDES

196. Picturing by lines. We get a better idea of differences by illustrating the numbers.

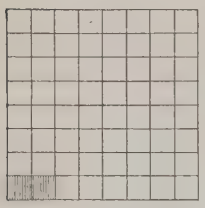
For example, the population of the United States in 1790 was 3,900,000; in 1900 it was 76,303,387. We may let a line 1 in. long represent some number, as 50 millions.

1900, 76,303,387 ———————————————
1790, 3,900,000 —
Increase, 72,403,387 ———————————————

Then the population in 1900, about 76 millions, will be represented by a line a little over $1\frac{1}{2}$ in. long, while that of 1790 will be represented by one a little less than $\frac{1}{12}$ in. long. The picture helps us to realize the growth of our country.

197. Picturing by squares. We may also represent numbers by squares or other figures.

For example, if the population of Idaho, recently about 200,000, is to be compared with that of Pennsylvania, about 6,400,000, we may let some small square, say $\frac{1}{8}$ in. on a side, represent 100,000 people. Then 64 such squares will represent the population of Pennsylvania, while 2 squares will represent that of Idaho. We then have a picture which helps us to realize the great difference in population.



Penn., 6,400,000
Idaho, 200,000
6,200,000

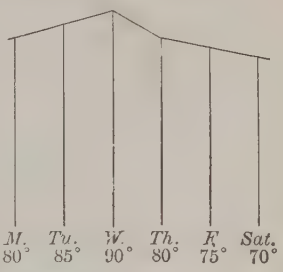
WRITTEN EXERCISE

The Chesapeake and Ohio Canal is 184 mi. long, the Erie 387 mi., the Lehigh 108 mi., and the Ohio 317 mi. Represent these by lines. Let 1 in. = 100 mi.

For further exercises see pages 122, 123.

198. Number changes. Changes in numbers may easily be shown by a picture.

For example, if the temperature at noon on Monday is 80°, and on Tuesday 85°, Wednesday 90°, Thursday 80°, Friday 75°, Saturday 70°, we may draw vertical lines to represent these temperatures. Then a line joining the tops gives us an idea of the changes in temperature.



Here $\frac{1}{8}$ in. has been taken to represent 10°. It makes no difference what distance is taken between the lines, only it must be the same because it represents 24 hours in each case.

WRITTEN EXERCISE

1. In million bushels the production of wheat in this country in 1850 was 100; in 1860 it was 173; in 1870 it was 288; in 1880 it was 459; in 1890 it was 399; and in 1900 it was 522. Represent as above.

2. In hundred thousand bales, the production of cotton in this country for thirty years was as follows: in 1870, 31; in 1875, 38; in 1880, 58; in 1885, 57; in 1890, 73; in 1895, 99; in 1900, 94. Represent as above.

3. In hundred thousand tons, the production of sugar in the world has been as follows: in 1870, 28; in 1880, 39; in 1890, 54; in 1900, 84. Represent as above, and notice how great has been the increase as shown by the picture.

4. In hundred-thousands, the population of Illinois in 1860 was 17; in 1870, 25; in 1880, 30; in 1890, 38; in 1900, 48. Represent as above, and notice the great growth in population.

Some Particulars in which Our Country excels Certain Others

WRITTEN EXERCISE

1. Suppose our country should produce 302,502,328 lb. of wool in a certain year, and Spain should produce 102,600,000 lb., what would be the difference? Represent by a picture.

2. The area of Great Britain is 120,979 sq. mi., and that of the single state of Texas is 265,780 sq. mi. How much larger is Texas? Illustrate by squares.

3. The area of European Russia is 2,095,616 sq. mi., and that of the United States is 3,720,856 sq. mi. How much larger is this country? Illustrate by squares.

4. At a time when we were producing 13,689.173 tons of steel, France was producing 1,465,071 tons. What was the difference? Represent 10,000,000 tons by 1 sq. in.

5. The area of Canada is nearly as much as that of the United States, being 3,048,710 sq. mi. What is the difference in the areas of the countries? Illustrate by squares.

6. At a time when this country had $29.04 of money in circulation to each person, Italy had only $9.68. What was the difference in wealth per person? Illustrate by lines, representing $10 by 1 in.

7. Our country averages about 10,800,000 bales of cotton a year. All the rest of the world averages 2,050,000 bales. What is the difference? Represent by squares, taking a square $\frac{1}{8}$ in. on a side to represent 1,000,000 bales.

8. In a certain year we bought from Germany $101,997,523 worth of goods, and sold to that country $170,222,737 worth. How much was the balance in our favor? Illustrate by lines, representing $100,000,000 by 1 in.

Some Particulars in which Certain Other Countries excel Ours

WRITTEN EXERCISE

1. When the population of London was 4,536,063, that of New York City was 3,437,202. Illustrate the difference.

2. If the United States has 20,870,000 spindles in operation in the manufacture of cotton, and England has 46,100,000, what is the difference? Illustrate.

3. If our country mined 55,214,000 oz. of silver in a certain year, and Mexico mined 57,656,549 oz., how much more did Mexico mine than the United States? Illustrate.

4. The area of European Russia is 2,095,616 sq. mi., of Asiatic Russia 6,564,778 sq. mi., of the United States 3,720,856 sq. mi. How much greater is the total area of Russia than that of the United States? Illustrate.

5. The greatest wool-producing country in the western hemisphere is Argentina, which averages about 370,000,000 lb., the United States coming next with an average of about 304,500,000 lb. What was the difference? Illustrate.

6. The area of Italy is 110,646 sq. mi., and that of Nevada is 110,700 sq. mi. When the population of Italy was 32,045,404, that of Nevada was 42,335. How much greater is the area of Nevada, and how much greater was the population of Italy? Illustrate.

7. When the population of the United States was 76,303,387, that of Porto Rico was 953,243; of Hawaii, 154,001; of the Philippines and Guam, 7,909,000. The population of Russia was then 128,852,076. How much greater was the population of Russia than that of the United States and its island possessions? Illustrate.

WRITTEN EXERCISE

In review drills like the following a time limit should be set upon the work, and checks should be required.

1.	2.	3.	4.
\$476.29	\$663.48	\$298.63	\$348.00
36.82	487.29	42.75	29.02
48.19	23.00	349.62	76.35
293.05	304.09	79.36	298.42
47.26	62.68	82.59	109.83
83.42	73.48	682.09	67.08

5.	6.	7.	8.
\$1000.00	\$824.63	\$209.09	\$802.35
289.00	981.42	32.78	608.72
40.27	127.69	45.67	402.81
6.35	842.38	682.73	96.43
42.07	327.29	102.93	87.29
189.27	420.63	41.52	289.62
96.96	29.00	647.38	43.59

9. \$640.00 − \$298.29.
10. \$702.53 − \$129.65.
11. \$902.63 − \$127.49.
12. \$1000.00 − \$298.75.
13. \$3476.42 − \$1029.29.
14. \$2025.30 − \$1627.92.
15. 27 times \$672.42.
16. 39 times \$486.39.
17. 37 times \$129.46.
18. 43 times \$298.63.
19. 4.2×125.3.
20. 3.1×149.9.
21. 4.25×4864.
22. 3.75×8796.
23. 8.9×9876.
24. 2.9×8971.
25. \$32.48 ÷ 8.
26. \$21.42 ÷ 18.
27. \$82.20 ÷ 12.
28. \$26.25 ÷ 25.
29. \$99.54 ÷ 14.
30. \$43.75 ÷ 35.
31. $\dfrac{12\frac{1}{2} + 7\frac{7}{8}}{15.75 - 5\frac{11}{16}}$.
32. $\dfrac{2.75 \div 2.5}{6.15 - 5\frac{1}{20}}$.
33. $\dfrac{7.5 \times 5.1}{.9 \times 8.5}$.
34. $\dfrac{3.45 \times 6.25}{2.3 \times 2.5}$.
35. $\dfrac{1.2 \times 93.1}{4.2 \times 3.8}$.
36. $\dfrac{2\frac{1}{2} + 3\frac{3}{4}}{2.5 \times 1.25}$.

BILLS AND RECEIPTS

199. Debtor and creditor. When one man owes another he is called a *debtor* (Dr.). The man to whom he owes money is called his *creditor* (Cr.).

200. Debit and credit. The amount owed on a bill is called the *debit* part of the bill, and any payments that have been made form the *credit* part.

201. Bills. Bills usually begin like this:

Mr. A. B. Jefferson

Bought of ROE & DOE, Grocers,

No. 4 Washington Street.

They may also read: "To Roe & Doe, Dr.," this meaning that Mr. Jefferson is debtor to Roe & Doe for the goods mentioned in the bill.

The following is an example of a bill filled out, footed, and receipted.

New Orleans, La.,.....[*Date*].....

Mrs. John Doe, 47 Tenth Avenue,

To RICHARD ROE & CO., Dr.

Dry Goods Dealers, No. 10 Grand Street.

.....[*Year*].....				
.....[*Date*].....	$\frac{1}{2}$ yd. Silk	.75		38
"	$\frac{3}{4}$ " "	1.25		94
"	$7\frac{1}{2}$ " Velours	3.50	26	25
			$27	57

Received payment,

Richard Roe & Co.

WRITTEN EXERCISE

Date, fill, foot, and receipt the following bills, inserting the name and address of some purchaser and of some dealer whom you know:

1. Bill for furniture: 6 chairs @ \$3.00; 2 armchairs @ \$6.25; 1 rocking-chair @ \$5.50; 1 table @ \$22.75.

An item like "6 chairs @ \$3.00" means \$3.00 each.

2. Bill for dry goods: 3½ yd. satin @ \$1.50; 2 yd. flannel @ 50 ct.; 1½ yd. canvas @ 20 ct.; 1½ yd. ribbon @ 23 ct.

3. Bill for meat: 2½ lb. steak @ 22 ct.; 5¼ lb. fish @ 20 ct.; 4¾ lb. roast beef @ 20 ct.; 8 lb. turkey @ 21 ct.

4. Bill for dry goods: 17½ yd. embroidery @ 10 ct.; 3 yd. lawn @ 20 ct.; 7½ yd. percale @ 25 ct.; ⅛ yd. satin @ \$1.50.

5. Bill for crockery: ½ doz. cups and saucers @ \$6.00; 3 doz. plates @ \$4.25; 1½ doz. tumblers @ 90 ct.; 2 doz. individual butter plates @ 70 ct.; ½ doz. bowls @ \$2.30.

An item like "½ doz. cups @ \$6.00" means \$6.00 a dozen.

6. Bill for groceries: 10 lb. sugar @ 8 ct.; 2 qt. berries @ 11 ct.; 4 doz. eggs @ 26 ct.; 5½ lb. butter @ 32 ct.; 1½ lb. cheese @ 22 ct.; ½ doz. bars soap @ 44 ct.

7. Bill for dry goods: 7 yd. muslin @ 18 ct.; 6 handkerchiefs @ 30 ct.; 3 pairs kid gloves @ \$1.50; 12 yd. sheeting @ 12½ ct.; 16 yd. pongee @ 70 ct.

8. Bill for stationery: 500 envelopes @ \$3.50 per M (that is, per 1000); 1 box writing paper @ \$2.25; ½ doz. pencils @ 35 ct.; 3 penholders @ 5 ct.; 1 bottle ink @ 30 ct.

9. Bill for groceries: 5 lb. coffee @ 32 ct.; 2 lb. tea @ 45 ct.; 8 lb. ham @ 15 ct.; 7 lb. lard @ 8 ct.; 3½ lb. butter @ 33 ct.; 3 qt. currants @ 7 ct.; 2 doz. eggs @ 31 ct.; 1¾ lb. cheese @ 16 ct.; ½ gal. sirup @ 65 ct.

ORAL EXERCISE

1. Will is 10 yr. old and his father is 40 yr. old. His father is how many times as old as Will? Will's age is what part of his father's age? How many hundredths?

2. Will's age is how many fortieths less than his father's? How many fourths less? How many hundredths?

3. Will's age is what part of the sum of his age and his father's? How many hundredths? His father's age is what part of the sum? How many hundredths?

4. Will's age is what part of the difference of their ages? How many hundredths?

WRITTEN EXERCISE

Estimate the answers to the following, writing the estimate. Then solve, carrying the results to 2 decimal places.

1. $\frac{22.25 + 32.75}{17.65 - 6.45}$.
2. $\frac{34.67 - 22.47}{8.72 - 2.62}$.
3. $\frac{41.73 - 12.43}{17.81 - 9.61}$.
4. $\frac{2.75 \times 3.25}{16.87 - 13.57}$.
5. $\frac{4.33\frac{1}{3} \times 666}{5680 \div 2}$.
6. $\frac{8.25 \times 9.5}{62.45 - 53.05}$.
7. $\frac{82.75 \div 0.25}{311.25 + 13.75}$.
8. $\frac{26.57 + 73.03}{37\frac{1}{2} - 12\frac{1}{4}}$.
9. $\frac{63.32 + 61.08}{3.27 + 1.73}$.

Express the following in fractional form, like the above, and estimate the answer before solving:

10. If a man earns \$15.50 a week, and spends \$10, how many weeks will it take him to pay for a 14-volume encyclopedia at \$2.75 a volume, with the remainder?

11. If a man earns \$1279.50 a year, on an average, and spends \$304.50, how many years will it take him to save enough to buy a 75-acre farm at \$65 an acre?

ORAL EXERCISE

1. John is 15 yr. old and his sister is 10. His age is how many hundredths more than hers? Hers is how many hundredths less than his?

2. How many hundredths of the sum of their ages is John's age? Mary's age?

3. The difference of their ages is how many hundredths of the sum? The sum is how many hundredths more than the sister's age?

4. If a horse cost \$125 and a carriage \$150, the carriage cost how many hundredths more than the horse? The horse cost how many hundredths less than the carriage?

WRITTEN EXERCISE

1. How many hundredths of \$150 is \$50? \$75?
2. How many fiftieths of \$250 is \$50? \$75? \$125?
3. How many thirds of 300 ft. is 100 ft.? 200 ft.?
4. How many eighths of \$400 is \$50? \$300? \$350?
5. 27.5×32.6.
6. 48.25×51.75.
7. $32.33\frac{1}{3} \times 66.6$.
8. 2.83×3.82.
9. 27.48×26.35.
10. 41.25×12.75.
11. $26.25 \div 2.5$.
12. $38.47 \div 0.33\frac{1}{3}$.
13. $685.5 \div 7.5$.
14. $3.476 \div 1.1$.
15. $1.752 \div 1.2$.
16. $4125 \div 0.15$.
17. $\dfrac{2.75 + 3.25 + 6.33\frac{1}{3}}{1.01\frac{1}{9} + 2.1 + 1}$.
18. $\dfrac{49.82 + 6.08 - 1.2}{60.1\frac{2}{3} + 0.4 - 41}$.
19. $\dfrac{3.25 + 6.25 + 4.25}{2.25 + 4\frac{7}{8} - 0.25}$.
20. $\dfrac{5.2 \times 6.3 \times 2.1}{0.7 \times 0.9 \times 2.6}$.
21. $\dfrac{4.75 + 1.75 - 2.50}{4.85 + 1.25 - 4.10}$.
22. $\dfrac{3.4 \times 4.2 \times 3.5}{0.7 \times 0.7 \times 1.7}$.

PERCENTAGE

ORAL EXERCISE

1. Give other names for the following: 3 ft., $\frac{1}{2}$ lb., 16 oz., $5\frac{1}{2}$ yd., 100 ct., 1 rd., 4 qt., 4 pk., $\frac{1}{2}$ qt.

2. Another name for .5 is $\frac{1}{2}$. Another way of writing $\frac{7}{10}$ is .7. Give another name for, and another way of writing $\frac{3}{4}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{3}{5}$; $\frac{1}{3}$.

202. Per cent. Another name for hundredths is *per cent.*

That is, 1 per cent means 1 one-hundredth, or $\frac{1}{100}$; 2 per cent means 2 one-hundredths, or $\frac{2}{100}$; $\frac{1}{2}$ per cent means $\frac{1}{2}$ of a hundredth, or $\frac{1}{200}$.

203. Per cent sign. There is also another way of writing per cent, or hundredths. It is by using the sign %.

That is, 1% means $\frac{1}{100}$, or .01; 6% means $\frac{6}{100}$, or .06; $\frac{1}{4}$% means $\frac{1}{4}$ of $\frac{1}{100} = \frac{1}{400}$.

204. Reading the sign %. The sign % may be read either *per cent* or *hundredths*, for they mean the same. It is usually read *per cent.*

If asked to write 10 per cent, you may write either 10% or .10.

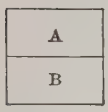
WRITTEN EXERCISE

Write in two other ways the numbers in Exs. 1–18:

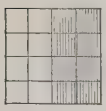
1. Nine per cent.	2. 17 per cent.	3. 28%.
4. 37 per cent.	5. 47 hundredths.	6. 16%.
7. 63 hundredths.	8. 87 per cent.	9. $33\frac{1}{3}$%.
10. 100 per cent.	11. 125 per cent.	12. $16\frac{2}{3}$%.
13. 225 per cent.	14. 300 per cent.	15. 75%.
16. One per cent.	17. 99 per cent.	18. 100%.

ORAL EXERCISE

1. If this square is called 100, how much is A? B? If the square is called 1, how many hundredths is A? B?



2. Read .50, using the words *per cent.* Read it using the word *hundredths.* How many halves are there in 1? Then how many times is 50% contained in 1?

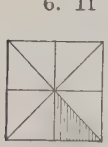


3. If this square is called 16, how much is the shaded part? How many sixteenths of the square are shaded? How many hundredths of it are shaded?

4. If this square is called 1, how much is the shaded part? If the square is called 100, how much is the shaded part? The shaded part is how many hundredths of the square?

5. Read .25, using the words *per cent;* using the word *hundredths.* How many fourths in 1? Then how many times is 25% contained in 1?

6. If this square is called 1, how much is the shaded part? If it is called 100, how much is the shaded part? What per cent of the square is shaded?



7. How many eighths in 1? Then how many times is $12\frac{1}{2}\%$ contained in 1?

8. Draw a picture on the blackboard and shade 10% of it. Draw another and shade 75%.

9. How many times is $16\frac{2}{3}\%$ contained in 1? in 2?

10. How many times is $6\frac{1}{4}\%$ contained in 1? in 3?

11. How many times is 20% contained in 1? in 5?

ORAL EXERCISE

1. Half of a square is what per cent of it?

2. A fourth of a square is what per cent of it?

3. An eighth of a square is what per cent of it?

4. A sixteenth of a square is what per cent of it?

5. How much is $\frac{1}{2}$ of $6\frac{1}{4}$? $\frac{1}{2}$ of $6\frac{1}{4}\%$? $\frac{1}{32}$ is how many per cent?

205. Important per cents. We have found that

$50\% = \frac{1}{2}$, $25\% = \frac{1}{4}$,

$12\frac{1}{2}\% = \frac{1}{8}$, $6\frac{1}{4}\% = \frac{1}{16}$.

6. How much is 50% of 20? of 40? of 50? of 7?

7. How much is 25% of 32? of 60? of 120? of 5?

8. How much is $12\frac{1}{2}\%$ of 40? of 72? of 88? of 100?

9. How much is $6\frac{1}{4}\%$ of 32? of 16? of 160? of 320?

10. If your class numbers 32, and 50% are boys, how many are boys?

WRITTEN EXERCISE

1. 50% of 632.
2. 25% of 592.
3. 50% of 333.
4. 25% of $3416.
5. 50% of $4917.
6. 25% of 432 mi.
7. 50% of 978 bu.
8. 25% of 96 ft. 8. in.
9. $12\frac{1}{2}\%$ of $91.28.
10. $12\frac{1}{2}\%$ of $73.68.
11. $12\frac{1}{2}\%$ of 72 lb. 8 oz.
12. $6\frac{1}{4}\%$ of 17 lb.
13. $6\frac{1}{4}\%$ of $43.20.
14. $6\frac{1}{4}\%$ of $15.68.

15. Of 152 chickens a hawk captured $12\frac{1}{2}\%$. How many were captured? How many were left?

16. Of 1408 sheep owned by a farmer, $6\frac{1}{4}\%$ were sold. How many did he sell? How many remained?

ORAL EXERCISE

1. If this circle is called 100, how much is the shaded part? the unshaded part? If the circle is called 1, what per cent is the shaded part?

2. Read .33$\frac{1}{3}$, using the words *per cent.* Read it using the word *hundredths.* How many thirds are there in 1? Then how many times is 33$\frac{1}{3}$% contained in 1?

3. If this circle is called 100, the shaded part is what part of 33$\frac{1}{3}$? How much is this? Then the shaded part is what per cent of the circle?

4. How much is $\frac{1}{2}$ of 16$\frac{2}{3}$%? The shaded part of this circle is what per cent of the whole circle? Then $\frac{1}{12}$ is what per cent?

206. Important per cents. We have found that

$33\frac{1}{3}\% = \frac{1}{3}$, $66\frac{2}{3}\% = \frac{2}{3}$,

$16\frac{2}{3}\% = \frac{1}{6}$, $8\frac{1}{3}\% = \frac{1}{12}$.

5. How much is 33$\frac{1}{3}$% of 90? of 120? of 45? of 60?
6. How much is 66$\frac{2}{3}$% of 30? of 66? of 90? of 300?
7. How much is 16$\frac{2}{3}$% of 36? of 24? of 66? of 72?
8. How much is 8$\frac{1}{3}$% of 24? of 48? of 60? of 120?

WRITTEN EXERCISE

1. 33$\frac{1}{3}$% of 891.
2. 66$\frac{2}{3}$% of 711.
3. 16$\frac{2}{3}$% of 504.
4. 16$\frac{2}{3}$% of \$41.10.
5. 8$\frac{1}{3}$% of \$15.60.
6. 8$\frac{1}{3}$% of \$159.72.
7. 66$\frac{2}{3}$% of 747 rd.
8. 16$\frac{2}{3}$% of 9774 bu.
9. 33$\frac{1}{3}$% of 222 ft. 9 in.
10. 33$\frac{1}{3}$% of 477 men.
11. 16$\frac{2}{3}$% of 726 bu. 6 qt.
12. 66$\frac{2}{3}$% of 141 lb. 15 oz.

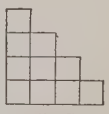
ORAL EXERCISE

1. How much is $\frac{300}{100}$? $\frac{200}{100}$? 200%? $\frac{100}{100}$? 100%?

2. Anything is what per cent of itself? of half itself?

3. If we call column A one, what is B? C? D? If we call A 100%, what per cent is B? C? D?

4. If we call A two, what is B? C? D? If we call A 200%, what per cent is B? C? D?



5. If we call C 100%, what per cent is D? B? A? B + D? A + B + D?

6. If we call D 100%, what per cent is C? B? A?

7. If we call B 100%, what per cent is D + C? C + A?

8. Because $1.50 = 1\frac{50}{100} = \frac{150}{100}$, 1.50 = what per cent?

207. Reading per cents. We have found that 2.75 may be read "two and seventy-five hundredths," or "two hundred seventy-five per cent."

9. Read, using the words *per cent:* 4.35, 0.75, 0.05, 1.00.

WRITTEN EXERCISE

Express as decimal fractions the numbers in Exs. 1–10:

1. 25%. 2. $33\frac{1}{3}$%. 3. 1%. 4. $2\frac{1}{2}$%. 5. 7.5%.
6. 125%. 7. $\frac{1}{2}$%. 8. $\frac{3}{4}$%. 9. 200%. 10. 0.3%.

Express as common fractions reduced to lowest terms the numbers in Exs. 11–20:

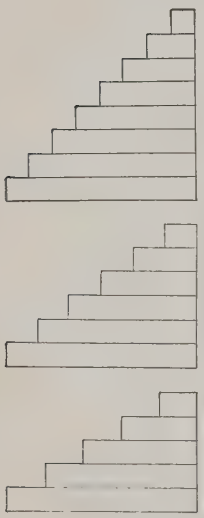
11. 32%. 12. 24%. 13. 36%. 14. 17%. 15. $82\frac{1}{2}$%.
16. 68%. 17. $6\frac{1}{4}$%. 18. 125%. 19. 375%. 20. $33.3\frac{1}{3}$%.

Express as per cents the following:

21. $\frac{1}{2}$. 22. $\frac{4}{5}$. 23. $\frac{7}{8}$. 24. $\frac{1}{3}$. 25. $1\frac{2}{5}$.

Important Per Cents to be remembered

ORAL EXERCISE



1. Give the per cent equivalents of the eighths.

2. $87\frac{1}{2}\%$ is how many times $12\frac{1}{2}\%$?

3. What is the quotient of $75\% \div 62\frac{1}{2}\%$? (Divide equivalent common fractions.)

4. What is the quotient of $62\frac{1}{2}\% \div 50\%$?

5. Give the per cent equivalents of the sixths.

6. Divide $83\frac{1}{3}\%$ by $33\frac{1}{3}\%$.

7. Give the per cent equivalents of the fifths.

8. Divide 1 by 80%.

WRITTEN EXERCISE

Draw figures as above and write a table of equivalents of the following:

1. The sevenths.
2. The ninths.
3. The twelfths.
4. The sixteenths.

5. Divide 20% by $16\frac{2}{3}\%$. (Divide the equivalent common fractions, and write the result both as a common fraction and as a decimal.)

In the same way perform these operations:

6. $50\% \div 37\frac{1}{2}\%$.
7. $33\frac{1}{3}\%$ of 75%.
8. $83\frac{1}{3}\%$ of 60%.
9. $80\% \div 50\%$.
10. $12\frac{1}{2}\%$ of 80%.
11. 60% of $33\frac{1}{3}\%$.

208. Since 90%, 0.90, and $\frac{90}{100}$ all mean the same, 90% of a number is the same as that number multiplied by 0.90.

Thus, 90% of 60 = .90 of 60 = 54,
85% of 70 = .85 of 70 = 59.5,
15% of 200 = .15 of 200 = 30,
12½% of 40 = .125 of 40 = ⅛ of 40 = 5.

ORAL EXERCISE

1. If you have 50 words in a spelling test, and are marked 98%, how many words did you spell correctly?

2. If you have 10 questions in arithmetic, and are marked 80%, how many did you answer correctly?

3. If you have 12 questions in geography, and are marked 75%, how many did you answer correctly?

4. If 60% of a class of 40 have not missed a day this year, how many have been present every day?

5. If a merchant pays 2 ct. apiece for pencils, and sells them at a profit of 50%, what does he receive for each?

6. If a milk dealer pays 5 ct. a quart for milk, and sells it at a profit of 20%, what does he receive per quart?

7. If a butcher buys meat at 18 ct. a pound, and sells it at a profit of 16⅔%, at what rate does he sell it per pound?

WRITTEN EXERCISE

1. 22% of 5475.
2. 65% of $9200.
3. 6% of $2496.
4. 87% of 4700 ft.
5. 95% of $625.
6. 66⅔% of $4131.
7. 72% of $345.
8. 17½% of $4000.
9. 35% of 5280 ft.
10. 31% of 4920 lb.
11. 62% of 3500 mi.
12. 87½% of $14.40.

ORAL EXERCISE

1. From these columns show that $20\% = \frac{1}{5}$, $40\% = \frac{2}{5}$, $60\% = \frac{3}{5}$, and $80\% = \frac{4}{5}$.

2. Also show that 20% is $\frac{1}{2}$ of 40%, $\frac{1}{3}$ of 60%, and $\frac{1}{4}$ of 80%.

3. Also show that 40% is $\frac{2}{3}$ of 60%, $\frac{1}{2}$ of 80%, and $\frac{2}{5}$ of 100%.

4. Also show that $60\% = 1\frac{1}{2}$ times 40%, $\frac{3}{4}$ of 80%, and $\frac{3}{5}$ of 100%.

				20%
			20%	20%
		20%	20%	20%
	20%	20%	20%	20%
20%	20%	20%	20%	20%
20%	40%	60%	80%	100%

These columns should be written on the board. Columns of other per cents, like $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, 25%, and $33\frac{1}{3}\%$ are also very useful for introducing rapid drill work.

WRITTEN EXERCISE

1. Find $12\frac{1}{2}\%$ of 984; of 6116; of \$177.76.

2. Find $33\frac{1}{3}\%$ of 729; of \$923.22; of 5280 ft.

3. Find $16\frac{2}{3}\%$ of 5280 ft.; of \$1234.56; of 1728 cu. in.

4. A man bought a bicycle for \$40, and sold it at a loss of 35%. How much did he lose?

5. A man bought 83 sheep at \$6 apiece, and sold them at a profit of 15%. How much did he gain?

6. A dealer buys 25 cows at \$32 each, and sells them at a profit of 8%. How much does he gain on them all?

7. A man bought an automobile for \$700, and after using it two seasons sold it for 15% less than it cost. What was the selling price?

8. A farmer bought 75 A. of land at \$65 an acre, and 12 cows at \$35 a head. He sold the land at a profit of 10% and the cows at a loss of 5%. Did he gain or lose on the whole, and how much?

209. Base. The number of which some per cent is to be found is called the *base*.

210. Rate. The number of hundredths to be taken is called the *rate*.

Thus, in 6% of $200, $200 is the *base* and 6% is the *rate*. Sometimes 6 is called the *rate per cent*, but the terms *rate* and *rate per cent* are generally used to mean the same thing.

211. Percentage. The result found by taking a certain per cent of the base is called the *percentage*.

212. *The percentage is the product of the base and rate.*

Excepting *rate*, these terms are of little value in business, but in school they are occasionally convenient for designating different numbers in a problem.

WRITTEN EXERCISE

1. Given the base 725, and the rate 7%, find the percentage.

2. A man has a salary of $1200, and saves 18% of it. How much does he save?

3. If you go to school $51\frac{1}{73}\%$ of the days of the year, how many days do you attend?

4. If a man owes $1470, and pays $33\frac{1}{3}\%$ of his debts this year, how much remains unpaid?

5. A certain fertilizer contains $37\frac{1}{2}\%$ of lime. How many pounds of lime in 3 tons of fertilizer?

6. In a certain ore the metal is 9.8% of the total weight. How many pounds of metal in 1950 lb. of ore?

7. A dealer bought 10 automobiles at $750 each, and sold them at a profit of 20%. How much did he gain?

8. If the value of our silk manufactures is $107,260,000 annually, while ten years ago it was 80% as much, how much was it then?

ORAL EXERCISE

1. 4 is what part of 32? what per cent of 32?
2. 15 is what part of 30? what per cent of 30?
3. 3 is what per cent of 36? 4 is what per cent of 48?
4. 7 is what per cent of 42? 9 is what per cent of 54?

213. \$17.50 is what per cent of \$250?

Because \$1 is $\frac{1}{250}$ of \$250, therefore \$17.50 is $\frac{17.5}{250}$ of \$250, and $\frac{17.5}{250} = 0.07$.

$$250\overline{)17.5}\quad .07$$
$$17.5$$

214. We therefore see that *the percentage divided by the base equals the rate.*

WRITTEN EXERCISE

1. \$130 is what per cent of \$2600? of \$3900?
2. \$11.25 is what per cent of \$375? of \$56.25?
3. \$10.12 is what per cent of \$253? of \$202.40?
4. \$29.10 is what per cent of \$485? of \$174.60?
5. A foot is what per cent of a yard? of a rod?
6. A quart is what per cent of a gallon? of 25 gallons?
7. $\frac{2}{3}$ is what per cent of $\frac{1}{4}$? $\frac{3}{8}$ is what per cent of $\frac{1}{4}$?
8. 25% is what per cent of 75%? of 125%? of 250%?
9. A man invested \$1750 and lost \$192.50. What per cent did he lose?
10. A school having 275 pupils graduated 33. What per cent were graduated?
11. A man had \$2500 at the beginning of a year, and \$2775 at the end. How much did he gain? What per cent did he gain?

ORAL EXERCISE

1. If $\frac{1}{5}$ of a number is 7, what are 5 fifths of it? What is the number?

2. If 10% of a number is 30, what is 1%? What is 100% of it, or once the number?

3. If 25% of it is 7, what is the number?

State rapidly the numbers of which

4. 25% is 9. 5. $33\frac{1}{3}$% is 12. 6. $12\frac{1}{2}$% is 7.

7. 6% is 54. 8. 9% is 27. 9. 13% is 26.

215. If 28% of a number is \$12.60, what is the number?

Since 28% of the number is \$12.60,

1% " " " " $\frac{\$12.60}{28}$,

100% " " " " $\frac{\$1260}{28} = \$45$.

Since $\frac{\$1260}{28} = \frac{\$12.60}{.28}$ (§ 86), we see that

216. *The percentage divided by the rate equals the base.*

WRITTEN EXERCISE

1. If 20% of the people in a town are voters, and there are 236 voters, what is the population?

2. If 20% of your class are over 12 years old, and there are 8 such pupils, how many are there in your class?

3. If a farmer sold a horse for 80% of its cost, and received \$152, how much did the horse cost?

4. If a poultry raiser set some eggs, and 15% fail to hatch, how many did he set, the number failing to hatch being 33?

ORAL EXERCISE

1. If 4 times a certain number is 36, how do we find the number? If $1\frac{1}{2}$ times a certain number is 90, how do we find the number?

2. If .50 of a certain number is 300, how do we find the number? If 25% of a certain number is 60, how do we find the number?

3. How much is $\$1 - \$\frac{1}{4}$? 1 ft. $-$ 0.25 ft.? 1 yd. $-$ 25% of 1 yd.? $1 - 75\%$? $1 - 80\%$? $1 - 30\%$?

4. What part of anything is left after taking away $\frac{1}{4}$ of it? 75% of it? 65% of it? 99% of it? 1% of it?

217. A man sold an automobile for \$490, which was 30% less than it cost. How much did it cost?

1. Cost $-$ .30 of cost = .70 of cost, just as $\$1 - \$0.30 = \$0.70$.
2. If .70 of cost = \$490,
3. The cost = $\$490 \div .70 = \$700$.

WRITTEN EXERCISE

Find the numbers of which

1. 35% is 7.	2. 4% is 8.	3. 75% is 75.
4. 90% is 198.	5. 10% is 100.	6. 20% is 440.
7. 13% is 22.36.	8. 17% is 42.5.	9. 33% is 10.89.

10. After losing 15% of its population a town had 5950 inhabitants. How many had it before losing the 15%?

11. A dealer sold a bicycle for \$24.50, which was 30% less than the marked price. What was the marked price?

12. A man bought a set of furniture for \$52, which was 20% less than the marked price. What was the marked price?

ORAL EXERCISE

1. If $1\frac{1}{4}$ times a number is 50, what is the number?
2. If 1.25 times a number is 25, what is the number?
3. If 125% of a number is 250, what is the number?

Find the numbers of which

4. 120% is 24.
5. 200% is 16.
6. 150% is 60.
7. 125% is 100.
8. $133\frac{1}{3}$% is 80.
9. 300% is 30.

218. A man sold a set of furniture for \$78, which was 30% more than it cost. How much did it cost?

1. Cost + .30 of cost = 1.30 times cost, just as 1 ft. + .3 ft. = 1.3 ft.
2. If 1.30 times cost = \$78,
3. The cost = \$78 ÷ 1.30 = \$60.

WRITTEN EXERCISE

1. A dealer sold pencils at 5 ct. each, which gave him a profit of 25%. How much did they cost him a dozen?

2. A farmer purchased 35 sheep, and sold them for \$252, which gave him a profit of 20%. What did they cost him?

3. A dealer sells some goods for \$31.05, thus making a profit of 15% on the cost. How much did the goods cost him?

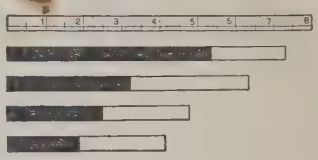
4. A manufacturer sells a suit of clothes to a merchant at a profit of $12\frac{1}{2}$%. The merchant sells it to you for \$12, and makes a profit of $33\frac{1}{3}$%. How much does it cost the merchant? the manufacturer?

5. An agent buys 100 doz. eggs, which he sells to a grocer at a profit of 20%. The grocer sells the eggs for \$22.44, and makes a profit of 10%. What did the eggs cost the grocer per dozen? the agent?

Where Our People live

219. The rectangles show population in millions, as numbered on the scale above.

State	*Population*	*Per cent in cities*
New York	7,300,000	73%
Pennsylvania	6,400,000	51%
Illinois	4,900,000	51%
Ohio	4,200,000	46%



The dark part shows the numbers living in cities or villages of 4000 or over. Thus in New York state 73% of 7,300,000 people, or 5,329,000 people, lived in such cities or villages at the opening of the century.

WRITTEN EXERCISE

1. Draw a diagram as above, on the same scale, for Missouri, the population being supposed to be 3,200,000, and the city population 35% of the whole.

2. Also for Iowa, 2,300,000, city population 21%.

3. Also for California, 1,500,000, city population 49%.

4. Of our total population of 5,308,000 in 1800, 207,012 lived in cities. What per cent lived in cities?

5. Of our total population of 76,303,000 in 1900, 28,079,504 lived in cities. What per cent lived in cities?

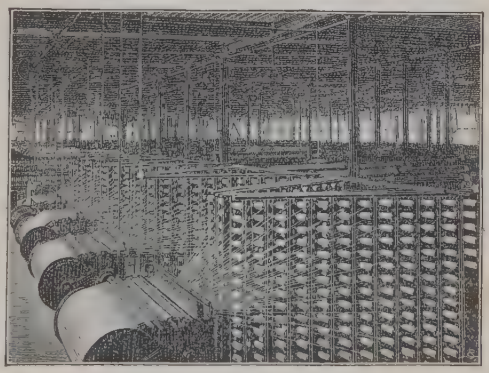
6. In 1820 our population was 9,600,000, and 124,800 lived in cities of over 100,000 inhabitants. What per cent was this?

7. In 1900 our population was 76,303,000, and 14,192,358 lived in cities of over 100,000 inhabitants. What per cent was this?

Manufacturing the Clothing for Our People

WRITTEN EXERCISE

1. If the United States manufactured $345,000,000 worth of cotton cloth in a certain year, and wool goods worth 90% as much, and hosiery worth $\frac{1}{3}$ as much as the wool goods, what was the value of each of these products?



2. If twenty years ago only 2% of the 18,000,000 spindles (for spinning cotton) were in the southern states, while now 42% of the 21,558,600 spindles are there, what was the number then? What is it now?

3. If in the cotton-producing states there are 500 mills, having a total of 5,001,500 spindles, how many spindles are there on an average to each mill?

4. If twenty years ago the northern mills used 1,165,000 bales a year, and the southern mills 18% as much, while now the northern mills use 2,262,000 bales and the southern mills use 85% as much, how many bales did the latter use then? How many do they use now?

5. If the value of our wool manufactures is now $316,800,000 annually, while ten years ago it was 82% as much, how much was it then?

6. In New York City 20,000 workers were employed in making men's clothes in a certain year, and the output was $103,000,000 worth a year. What was the average output per worker?

7. There were also 19,000 workers employed there in making women's clothes, and the output was $102,030,000 worth a year. What was the average output per worker?

8. In a certain coat factory 9 women make buttonholes at $2\frac{1}{2}$ ct. apiece. At 4 buttonholes to a coat, and 600 coats a week, what are the average weekly wages of each?

9. In this factory a woman receives $5\frac{1}{2}$ ct. a vest for basting, and she can baste 120 vests a week. What are her weekly wages?

10. A dressmaker uses in making a waist $2\frac{1}{2}$ yd. of white Persian lawn worth 24 ct. a yard, $4\frac{1}{4}$ yd. of embroidery worth 16 ct. a yard, 6 buttons worth 40 ct. a dozen, and 10 ct. worth of thread. How much does the material cost?

11. A working girl in a city shop pays $3 a week for board and room, 5 ct. a day 6 days in the week for car fare, and 15 ct. a day 6 days in the week for luncheon. Her incidental expenses are 60 ct. a week. If her wages are $6 a week, how much does she save a week? How much in a year of 50 working weeks?

12. Of the $39,271,900 worth of knit goods produced in this country in a certain year, New England produced $27\frac{1}{3}\%$, the middle states $66\frac{2}{3}\%$, the western states 6%, and the South practically none. Twenty years later New England produced 18% of the $101,337,000 worth manufactured, the middle states 64%, the West 12%, and the South 6%. Find the value of the product for each section for each year.

Our Fisheries

WRITTEN EXERCISE

1. This is part of the fleet that landed 176,774,300 lb. of fish at Boston and Gloucester in a certain year. Of this amount, 63% was landed at Boston. How many pounds were landed at Boston?

2. The value of the total catch in Ex. 1 was $4,193,600, and that of Gloucester was $2,725,840. The latter amount was what per cent of the former?

3. If $2,800,000 is invested in the Lake Erie fisheries, and the annual product is $1,176,000, this is what per cent of the amount invested?

4. If the herring fishery produces $435,120, this is what per cent of the total $1,176,000?

5. If Maryland has 42,800 persons engaged in fisheries, Virginia 65% as many, and New York 21% as many, how many have Virginia and New York together?

6. Whales are not really fish, but whaling is usually classed as a fishery. The United States recently had 48 vessels engaged in whaling. If they secured 11,904 bbl. of sperm oil in one year, valued at $49 a barrel, what was the average amount and value of oil produced by each?

7. If they also secured 3792 bbl. of whale oil, valued at $35 a barrel, what was the average for each vessel, in quantity and value?

8. If they also secured 321,600 lb. of whalebone, valued at $2.70 a pound, what was the average for each vessel, in quantity and value?

9. If Florida has 4472 men engaged in the fishing industry, and this is 40% of the total number engaged in this work in the Gulf states, what is the total number? The total number is what per cent of the number in Florida?

10. Sponge gathering, a Florida industry, is usually ranked as one of the fisheries. If it amounted in one year to 13½% of the $2,340,000 received from the Gulf-states fisheries, what was its value?

11. If the oyster industry of Louisiana and Mississippi amounted to 23½% of this same total, $2,340,000, what was the value?

12. The principal fishing industry in North Carolina is catching shad. What is the value of shad which are worth 28% of the whole annual income, in a year when the fisheries bring in $1,300,000?

13. In catching salmon in the rivers flowing into Puget Sound, a seine measuring about 100 fathoms long and 35 ft. wide is often used. A fathom being 6 ft., how many square feet in such a net?

14. The Indians there often use a seine that is only ⅓ as long and ⅓ as wide. What is its area? This area is what part of that of the large net?

15. If a boat brings back from a day's fishing 25 salmon averaging 7 lb. 8 oz. each, what is the weight of the day's catch? What is it worth at 2 ct. a pound?

DISCOUNT

ORAL EXERCISE

1. How much is \$200 less \$50? \$100 less 25% of itself?

2. If from 1 we take $\frac{1}{4}$, what is left? If from 1 we take 25%, what is left?

3. How much is 1 less $\frac{1}{5}$? $\frac{100}{100} - \frac{20}{100}$? 1 − .20? 1 − 20%?

4. If you buy a \$1 book for 10% less than the list price, how much does it cost you?

220. Discount. A deduction made on a price or amount is called a *discount*.

Discounts are usually reckoned as so many per cent.

221. Rate of Discount. The per cent of discount is called the *rate of discount*.

222. What is the discount and the selling price, when the list price is \$245.50 and the rate of discount is 12%?

12% of \$245.50 = \$29.46, discount.
\$245.50 − \$29.46 = \$216.04.

\$245.50 list price
.12 rate of discount
\$29.46 discount

\$245.50
29.46
\$216.04 selling price

WRITTEN EXERCISE

1. How much will a set of Scott's works, marked \$14, cost at 20% discount? (Also take 80% of \$14.)

2. If you can buy a \$7 suit of clothes at 15% discount, how much will you gain on the marked price?

3. If the list price of your books, pencils, and paper is \$2.50 a year, how much will they cost at 12% discount?

4. At a bargain sale an \$8 chair was sold at 30% discount. For how much did it sell?

Find the discounts on goods of which the list price and rate of discount are given in Exs. 5–13:

5. \$430, 15%. 6. \$281, 20%. 7. \$425.50, 10%.
8. \$1235, 17%. 9. \$2350, 14%. 10. \$43.26, $33\frac{1}{3}$%.
11. \$42.64, 25%. 12. \$6336, $6\frac{1}{4}$%. 13. \$234.54, $16\frac{2}{3}$%.

Find the selling price, given the list price and rate of discount as follows:

14. \$275, 20%. 15. \$575, 5%. 16. \$462, 3%.
17. \$650, $12\frac{1}{2}$%. 18. \$425, 12%. 19. \$320, 14%.
20. \$12.75, 20%. 21. \$6202, 14%. 22. \$52.84, $16\frac{2}{3}$%.
23. \$26.32, 50%. 24. \$4235, 22%. 25. \$14.43, $33\frac{1}{3}$%.

26. A \$17.50 set of china is marked down 15%. What is the selling price?

27. If you buy \$1.80 worth of fireworks at 15% discount, how much do they cost?

28. Your book dealer buys a 65-ct. arithmetic at 22% discount. How much does it cost him?

29. A \$31 overcoat is marked "25% off" at a bargain sale. What is the selling price?

30. If you buy a \$1.20 sled at 10% discount, how much does it cost? Suppose the discount is 20%? 25%?

31. How much will 2 dozen blackboard erasers cost at 45 ct. a dozen, discount 10%? discount 20%?

32. A man paid \$150 for a carriage, and after using it for a year sold it at a discount of $16\frac{2}{3}$%. What did he receive for it?

33. A man bought a \$900 automobile at 15% discount. After using it for a season he sold it for 15% less than it cost. What did he receive for it?

REVIEW OF DENOMINATE NUMBERS

ORAL EXERCISE

1. Repeat the table of length; of square measure; of cubic measure.

2. Repeat the table of weight.

3. Repeat the table of dry measure; of liquid measure.

Teachers should drill frequently upon the facts of these tables. The mere routine recitation of the tables is not so valuable as the frequent use of the facts in simple oral problems.

223. Table of dozens. In some kinds of business this table is common:

12 units = 1 dozen.
12 dozen, or 144 = 1 gross (gr.).
12 gross, or 1728 = 1 great gross.

The term *great gross* is going out of use.

224. Years ago the word *score* was often used for 20.

4. How many pens in 2 gross? in 6 gross? in 8 gross?

5. How many dozen in $\frac{2}{3}$ of a gross? in $3\frac{1}{2}$ gross?

6. What will a gross of padlocks cost at \$5 a dozen?

7. How many years old is a man who is 4 score? One who is 3 score? One who is 4 score and 10?

WRITTEN EXERCISE

1. How many are 7 great gross? $9\frac{2}{3}$ gross?

2. What will 9 gr. of tablets cost at 25 ct. a dozen?

3. At \$7.20 a gross what will a dozen lamp chimneys cost?

4. A school bought a gross of blackboard erasers, but $8\frac{1}{3}\%$ were damaged. How many were all right?

225. Counting sheets of paper. Formerly, in counting sheets of paper,

24 sheets = 1 quire.

20 quires = 480 sheets = 1 ream.

Now, for convenience in counting, 500 sheets are more often called a *ream*, and the word *quire* is used only for folded note paper, other paper being usually sold by the pound.

ORAL EXERCISE

1. How many sheets of fine writing paper in 2 quires? in 5 quires? in $3\frac{1}{2}$ quires? in 10 quires? in $12\frac{1}{2}\%$ of a quire?

2. If this paper costs 15 ct. a quire, how much will 4 quires cost? How many sheets will there be?

Notice that 4 times 24 is the same as 4 times $(25 - 1)$, or $100 - 4$.

3. At 20 ct. a quire, how much will 7 quires of paper, with \$1 worth of envelopes, cost?

4. If a dealer buys paper at 19 ct. a quire, and sells it at 25 ct. a quire, how much will he make on 10 quires?

WRITTEN EXERCISE

1. How many sheets in 17 quires? in $33\frac{1}{3}\%$ of a quire?

2. In 7 dozen reams of 500 sheets each, how many more sheets than in a dozen reams of 480 sheets each?

3. At $16\frac{2}{3}$ ct. a quire, how much will $7\frac{1}{2}$ quires of paper cost? (Remember aliquot parts.)

4. At $12\frac{1}{2}$ ct. a quire, how much will $9\frac{1}{2}$ quires of paper cost? $6\frac{1}{2}$ quires? a dozen quires?

5. At 24 ct. a quire for paper, and 1 ct. each for envelopes, how much will it cost for paper, envelopes, and postage for eight letters from where you live to some other place in this country? Allow 1 sheet to a letter.

MEASUREMENTS AND COMPARISONS

ORAL EXERCISE

1. Draw on the blackboard a picture of a parallelogram; of a rectangle; of a square.

Pupils are, of course, already familiar with certain of these figures. The formal definitions are not of so much importance as the ability to recognize the figures by name.

226. Quadrilateral. A figure bounded by four straight lines is called a *quadrilateral.*

227. Trapezoid. A quadrilateral which has one pair of opposite sides parallel is called a *trapezoid.*

228. Parallelogram. A quadrilateral which has two pairs of opposite sides parallel is called a *parallelogram.*

229. Rectangle. A parallelogram whose angles are right angles is called a *rectangle.*

230. Square. A rectangle whose sides are all equal is called a *square.*

231. Polygon. A figure bounded by straight lines is called a *polygon.*

The figures pictured above are all special kinds of polygons.

2. What is the perimeter of a square 12 ft. 6 in. on a side?

3. What is the perimeter of a parallelogram two of whose sides are 10 ft. 8 in. and 9 ft. 4 in.?

4. What is the perimeter of a quadrilateral whose sides are 16 ft., 14 ft., 13 ft., 12 ft.?

232. Equilateral triangle. A triangle whose three sides are equal is called an *equilateral* triangle.

Triangles classified with respect to sides

233. Isosceles triangle. A triangle two of whose sides are equal is called an *isosceles* triangle.

234. Scalene triangle. A triangle whose three sides are unequal is called a *scalene* triangle.

Triangles classified with respect to angles

235. Right-angled triangle. A triangle having one right angle is called a *right-angled* triangle.

236. Acute-angled triangle. A triangle having three acute angles is called an *acute-angled* triangle.

237. Obtuse-angled triangle. A triangle having one obtuse angle is called an *obtuse-angled* triangle.

WRITTEN EXERCISE

1. What is the perimeter of an equilateral triangle $275\frac{2}{3}$ ft. on a side? (Draw the pictures in all such cases.)

2. A triangle has a perimeter of 342 ft. 8 in. One side is 90 ft. and another is 50% more; how long is the third?

3. One side of an isosceles triangle is 35 ft., and one of the equal sides is 35% longer; what is the perimeter?

4. Cut off the three angles of a triangle and place them together. The sum equals how many right angles?

ORAL EXERCISE

1. How long is the side of an equilateral triangle whose perimeter is 45 in.? 9 ft. 6 in.? $9\frac{1}{2}$ yd.? 63 ft. 9 in.?

2. How long is the side of a square whose perimeter is 84 in.? 8 ft. 4 in.? 16 yd. 8 in.? $60\frac{1}{2}$ rd.? 120 in.?

3. How long is the fourth side of a quadrilateral whose perimeter is 85 in., and three of whose sides are 10 in., 20 in., 30 in.?

4. The straight-line distance from Chicago to St. Louis is 270 mi.; from St. Louis to Cincinnati, 310 mi.; from Cincinnati to Chicago, 250 mi. What is the perimeter of this triangle?

WRITTEN EXERCISE

How long are the sides of the equilateral triangles whose perimeters are as follows?

1. 113 ft. 3 in. 2. 127 yd. 3. 88 ft. 3 in.

How long are the sides of the squares whose perimeters are as follows?

4. 110 ft. 4 in. 5. 61 yd. 2 ft. 6. 118 ft. 8 in.

7. 42 ft. $3\frac{1}{2}$ in. 8. 3 yd. 2 ft. 4 in. 9. 227 mi. 40 rd.

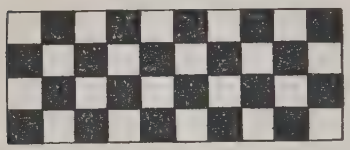
10. The perimeter of an isosceles triangle is 100 ft., and the unequal side is 33 ft. Find the length of one of the equal sides.

11. The perimeter of an isosceles triangle is 88 ft., and the unequal side is 20% of the perimeter. Required the length of each side.

12. The perimeter of a scalene triangle is 24 ft. One side is $33\frac{1}{3}\%$ of the perimeter; another is $37\frac{1}{2}\%$ of the perimeter. How many feet in each of the three sides?

ORAL EXERCISE

1. How many square inches in a rectangle 1 in. high and 10 in. long? 4 in. high and 10 in. long?

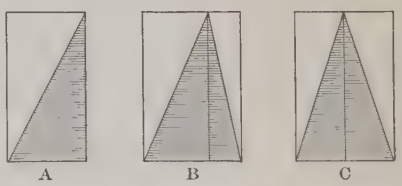


2. How do we find the area of a rectangle? Give reasons.

State the areas of the following rectangles:

3. 5 ft. by 9 ft.
4. 6 ft. by $4\frac{1}{2}$ ft.
5. 1 ft. by 8 in.
6. 8 ft. by $7\frac{1}{4}$ ft.
7. 9 in. by $8\frac{1}{3}$ in.
8. 9 in. by $7\frac{1}{3}$ in.
9. 3 yd. by $1\frac{1}{2}$ ft.
10. 1 rd. by 10 ft.
11. 10 yd. by 1 rd.

12. How do the areas of these triangles compare with the areas of the rectangles? Then a triangle is what part of a rectangle of the same base and same height?



238. Base. The line on which a figure appears to stand is called the *base*.

239. Height or altitude. The perpendicular distance from the highest point above the base to the base is called the *height* or *altitude* of a triangle or parallelogram.

240. Area of a rectangle. If a rectangle is 4 in. by 10 in., we find the area by taking 4 times 10 sq. in. We do not multiply 10 in. by 4 in., but 1 sq. in. by 4×10.

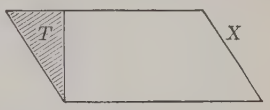
241. *The area of a rectangle equals the product of the base and height.*

242. Area of a triangle. From the above figures,

The area of a triangle equals half the product of the base and height.

ORAL EXERCISE

1. If we cut triangle T from this parallelogram and place it where X is, what kind of a figure have we? Try it. What does this tell us about the area of a parallelogram?



State the areas of the parallelograms whose bases and heights are given in Exs. 2–7:

2. 11 ft., 9 ft.
3. 6 ft., $9\frac{1}{2}$ ft.
4. 8 ft., $10\frac{1}{4}$ ft.
5. 2 yd., $1\frac{1}{2}$ ft.
6. 4 in., $6\frac{3}{4}$ in.
7. 10 ft., $5\frac{1}{2}$ yd.

State the areas of the triangles in Exs. 8–13:

8. 7 ft., 6 ft.
9. 8 ft., $4\frac{1}{2}$ ft.
10. 3 ft., 6 in.
11. $3\frac{1}{2}$ ft., 4 ft.
12. $2\frac{3}{4}$ in., 8 in.
13. 10 ft., $8\frac{1}{5}$ ft.

243. Area of a parallelogram. *The area of a parallelogram equals the product of the base and height.*

WRITTEN EXERCISE

Find the areas of the parallelograms in Exs. 1–6:

1. $19\frac{3}{4}$ in. by 16.8 in.
2. 14.7 ft. by 16.5 ft.
3. 171.4 ft. by 27.2 ft.
4. 3.75 in. by $2.33\frac{1}{3}$ in.
5. 6 ft. 4 in. by 4 ft. 6 in.
6. 16 ft. 6 in. by 8.25 ft.

244. What is the area of a triangle whose base is 10 rd. and height 8 rd.?

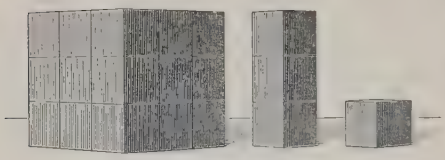
Because it is half of a rectangle 10 rd. by 8 rd., the area is $\frac{1}{2}$ of 10 times 8 sq. rd. = 40 sq. rd.

Find the areas of the triangles in Exs. 7–10:

7. 31.2 in. by 27.3 in.
8. 16.75 in. by 14.2 in.
9. 32 ft. 6 in. by 12 ft.
10. 17.3 ft. by 1 rd.

ORAL EXERCISE

1. If C = 1 cu. in., what is the volume of B? of A? Explain how you would find the volume of a box 4 in. by 3 in. by 2 in.



State the volumes of the solids whose dimensions are given in Exs. 2–7:

2. 4 in., 5 in., 6 in.
3. 2 in., 3 in., 10 in.
4. 3 in., 3 in., 7 in.
5. 6 in., 8 in., 10 in.
6. 2 in., 3 in., 12 in.
7. 9 in., 9 in., 10 in.

245. Volume of solid. *The volume of a rectangular solid equals the product of its three dimensions.*

This means a solid whose faces are rectangles. We do not say 4 in. times 3 in. times 2 in., but 4 times 3 times 2 cu. in. It is customary, however, to write 4 in. × 3 in. × 2 in., or 4″ × 3″ × 2″, reading it "4 inches by 3 inches by 2 inches."

WRITTEN EXERCISE

1. How many cubic inches in an aquarium 16 in. long, $8\frac{1}{2}$ in. wide, and $9\frac{1}{2}$ in. deep?

2. How many cubic feet of masonry in a wall 40 ft. long, $30\frac{1}{2}$ ft. high, and $1\frac{1}{2}$ ft. thick?

3. A cellar 24 ft. by 39 ft. by 6 ft. is to be excavated. How much will it cost at 40 ct. a load (cubic yard)?

4. The dimensions of a room are 14 ft., 12 ft., 9 ft. How many square feet of floor? of walls? of ceiling? How many cubic feet of volume?

WRITTEN EXERCISE

246. Measures occasionally used. In the following problems you may refer to these measurements:

31.5 gal. = 1 barrel. $7\frac{1}{2}$ gal. = 1 cu. ft., nearly.
231 cu. in. = 1 gallon. $4\frac{1}{7}$ cu. ft. = 1 barrel, nearly.
$24\frac{3}{4}$ cu. ft. = 1 perch of stonework.
2150.42 cu. in. = nearly $1\frac{1}{4}$ cu. ft. = 1 bushel.
35 cu. ft. = the volume of 1 ton of coal, nearly.
500 cu. ft. = the volume of 1 ton of hay, nearly.
22 bricks, including the mortar = 1 cu. ft., nearly.

All measures refer to inside dimensions, in examples like the following. 1 ton of hay varies from about 450 cu. ft. to 550 cu. ft., depending on the quality and how much it has settled.

1. How many bushels in a bin 7 ft. by 6 ft. by 4 ft.? (Use $1\frac{1}{4}$ cu. ft. = 1 bu. in all such cases.)

2. How many bushels in a bin 9 ft. by 5 ft. by 3 ft.?

3. How many bushels in a box 18 in. wide, 3 ft. long, and 2 ft. deep?

4. How many gallons in a tank 6 ft. by 5 ft. by 4 ft.?

5. How many gallons in an aquarium 14 in. long, 9 in. wide, and 11 in. deep?

6. How many gallons in a tank $6\frac{1}{4}$ ft. long, $3\frac{1}{2}$ ft. wide, and $2\frac{1}{3}$ ft. deep?

7. A cistern is 6 ft. square at the bottom and 5 ft. deep. How many barrels will it hold? (Use $4\frac{1}{7}$ cu. ft. = 1 bbl. in all such cases.)

8. An iron tank is 8 ft. long, $6\frac{1}{2}$ ft. wide, and $4\frac{3}{4}$ ft. deep. How many barrels does it contain?

9. A rectangular watering trough is 6 ft. long, 18 in. wide, and 14 in. deep. How many barrels does it contain?

10. A coal bin is 22 ft. long, 13 ft. wide, and 8 ft. deep. About how many tons of coal will it hold?

11. A coal bin is 16 ft. long, $8\frac{1}{2}$ ft. wide, and $6\frac{1}{4}$ ft. deep. About how many tons of coal will it hold?

12. A haymow is 22 ft. long, 14 ft. wide, and 9 ft. high. About how many tons of hay will it hold?

13. A barn has two haymows, each 24 ft. long, 15 ft. 6 in. wide, and 10 ft. high. About how many tons of hay will both hold?

14. How many cords of wood in a pile 4 ft. wide, 7 ft. high, $32\frac{1}{2}$ ft. long?

15. How many cords of wood in a pile 4 ft. wide, $6\frac{1}{2}$ ft. high, 44 ft. long?

16. If a schoolroom is 30 ft. long, 15 ft. wide, and 12 ft. high, and 180 cu. ft. should be allowed to each person, how many will the room accommodate?

17. A tank holds 82,500 gal. About how many cubic feet does it contain?

18. An aquarium contains $3\frac{4}{5}$ cu. ft. About how many gallons does it hold?

19. An aquarium is 18 in. long, $9\frac{1}{4}$ in. wide, and $10\frac{1}{8}$ in. deep. How many gallons does it hold?

20. A can containing 1732.5 cu. in. is full of milk. What is the milk worth at 6 ct. a quart?

21. A wall is $32\frac{1}{2}$ ft. long, 2 ft. thick, and $32\frac{1}{2}$ ft. high. How many perches of masonry does it contain?

22. A brick wall is to be built 42 ft. long, $1\frac{1}{2}$ ft. thick, and $6\frac{1}{4}$ ft. high. About how many bricks will be needed?

23. How much will a mason charge for laying a wall 20 ft. long, $1\frac{1}{2}$ ft. thick, and $5\frac{1}{2}$ ft. high, at $\$1.87\frac{1}{2}$ a perch? at $\$1.75$ a perch?

CHAPTER II

I. PERCENTAGE AND ITS APPLICATIONS. SIMPLE INTEREST. MEASUREMENTS

THE FORMAL SOLUTION OF PROBLEMS

247. Solving problems. We have already (page 115) learned something about stating the solutions of problems. We are now ready to consider this part of arithmetic more fully. There are three matters which are especially important.

1. *Work rapidly and accurately.* The business man demands this first of all. Rapidity not only means money in the time saved, but it usually means accuracy, for when a man works rapidly he keeps his mind on his work. To insure accuracy, *always check your work.* One of the best checks is that of estimating the answer in advance. If the answer is not about what was expected, there is probably a mistake. This has been explained on page 88.

2. *Work as the business man does.* He does not put his work in steps, with signs like $\div$ and $+$. He works in columns because it is quicker and he can more easily check his work. He also makes his work look neat, because he finds slovenly work leads to inaccuracy.

3. *Analyze the problems.* This means that we are to state the reasons. The business man does not write out his analyses, but he thinks them. We are asked to give analyses both orally and in writing, so we will learn to think accurately as the business man does.

248. Oral analysis. Many problems are so easy as to be solved orally, without using pencil and paper. We usually see merchants solving problems this way, and we should do so whenever possible.

249. Illustrative problems. 1. If a dealer pays \$10 for 8 books of the same kind, what is the price of each?

No one would think of solving such a problem on paper. If asked to analyze (explain) it, we would say: "Since 8 books cost \$10, 1 book costs $\frac{1}{8}$ of \$10, or \$1.25."

In analyzing, speak briefly and accurately. Use your own language, not feeling that you must use the exact language of the book or the teacher.

2. If a stationer buys 100 sheets of blotting paper for \$3.50, and retails it for 5 ct. a sheet, what does he gain?

Analysis. Since he sells 1 sheet for 5 ct., he sells 100 sheets for 100 times 5 ct., or \$5. (Don't stop to say "500 ct."; reduce this to \$5 mentally.) He therefore gains the difference between \$3.50 and \$5, or \$1.50.

3. A dealer pays \$24 for 120 doz. pencils. What does each one cost?

Analysis. Since 120 doz. cost \$24, 1 doz. costs $\frac{1}{120}$ of \$24, or 20 ct. Therefore 1 pencil costs $\frac{1}{12}$ of 20 ct., or $1\frac{2}{3}$ ct.

4. If a dealer pays \$3.60 for a dozen knives, and sells them at 50 ct. each, how much does he gain? This is what part of the cost?

Analysis. Since he sells 12 at 50 ct. each, he receives 12 times 50 ct., or \$6. He therefore gains \$6 − \$3.60 = \$2.40. \$2.40 is $\frac{2.40}{3.60} = \frac{24}{36} = \frac{2}{3}$ of the cost.

If the teacher begins a year's work with Chapter II, it will be found that these oral problems form an excellent review of the old work and an introduction to the new.

ORAL EXERCISE

When the teacher has given the number of the problem, give (1) the result and (2) the analysis.

1. At $12\frac{1}{2}$ ct. a pound, what will 8 lb. of cheese cost?

2. If 4 doz. oranges cost $1.80, what does 1 doz. cost?

3. At $492 a year, how much is the rent of a house for a month?

4. At $6 a ton (of 2000 lb.), how much will 3000 lb. of coal cost?

5. At $2 a ton, how many tons of soft coal can be bought for $96?

6. At 20 ct. a dozen, how many dozen eggs can be bought for $1.20?

7. At $5 a barrel, how many barrels of flour can be bought for $625?

8. At $6 a hundred, how many pounds of meat can be bought for $636?

9. At $20,000 a mile, how many miles of railroad can be built for $150,000?

10. At 15 ct. a dozen, how many dozen paper pads can be bought for $1.65? How many pads?

11. If 1 load of mortar contains 30 hods, how many loads are equal to 510 hods?

12. A jacket which cost a merchant $5.67 was sold for $7.25. How much did the merchant gain?

13. A dealer bought 12 doz. pairs of shoes for $360, and sold them at $3 a pair. How much did he gain?

14. If a bricklayer's hod contains 20 bricks, how many loads must he carry for a drain requiring 2920 bricks?

ORAL EXERCISE

1. At 40 ct. a yard, what do 9 yd. of silk cost?

2. At \$1.20 a yard, what do $5\frac{1}{2}$ yd. of silk cost?

3. At 90 ct. a yard, what do $2\frac{1}{2}$ yd. of cloth cost?

4. At 25 ct. a yard, what do $8\frac{1}{2}$ yd. of gingham cost?

5. At 35 ct. a yard, what do $10\frac{1}{2}$ yd. of madras cost?

6. At 96 ct. a yard, what do $10\frac{1}{4}$ yd. of pongee cost?

7. At $12\frac{1}{2}$ ct. a yard, what do 8 yd. of dimity cost? also 9 yd.? 10 yd.? 16 yd.? 24 yd.?

8. At 45 ct. a yard, what do 20 yd. of linen suiting cost? also $10\frac{1}{2}$ yd.? 5 yd.? 8 yd.? $2\frac{1}{3}$ yd.?

9. A lady buys a walking skirt for \$7.75, and 4 collars at $12\frac{1}{2}$ ct. each. What is her bill?

10. A man buys a pair of shoes for \$4.50, and a pair of slippers for \$2.75. What is his bill?

11. A lady buys a pair of gloves for \$1.50, and 3 handkerchiefs at 25 ct. each. What is her bill?

12. A lady bought 4 yd. of ribbon at 19 ct. a yard, a 25-ct. handkerchief, and a shirt waist for \$1.50. What was her bill?

13. A lady bought a hat for \$8.50, and 2 yd. of ribbon at 35 ct. a yard. How much change should she receive for a \$10 bill?

14. On the way home she ordered 4 lb. of meat at 23 ct. a pound, and paid for it with a \$1 bill. How much change did she receive?

15. She stopped at the grocer's and ordered 3 heads of lettuce at 9 ct. each, 2 bunches of asparagus at 6 ct. each, and \$1.75 worth of preserves. What was her bill?

ORAL EXERCISE

1. What will $2\frac{1}{2}$ gal. of milk cost at 7 ct. a quart?

2. How many yards in a coil of rope 2700 ft. long?

3. When lobsters sell at 20 ct. a pound, what will $1\frac{3}{4}$ lb. cost?

4. At 20 T. to the load, how many tons of coal in $5\frac{1}{2}$ car loads?

5. If a boy solves 3 out of 5 problems, what per cent does he solve?

6. If $2\frac{1}{2}$ lb. of porterhouse steak cost 55 ct., what is the price per pound?

7. If a man buys a horse for \$72 and pays $\frac{5}{6}$ down, how much does he still owe?

8. If it takes $3\frac{1}{2}$ yd. of gingham for a girl's dress, how much does the cloth cost at 30 ct. a yard?

9. At \$1.25 a yard, how much will three strips of carpet cost, the lengths being 13 yd., $4\frac{1}{2}$ yd., 7 ft. 6 in.?

10. A man buys 7 lb. of rice at 12 ct. a pound, and hands the grocer a dollar bill. How much change is due him?

11. I have 6 pictures to be hung, each requiring 7 ft. of picture wire. Find the cost of the wire at 1 ct. a yard.

12. What is the cost of a bunch of carrots at 13 ct., a box of strawberries at 18 ct., a quart of new potatoes at 12 ct., and a bunch of water cress at 5 ct.?

13. Sugar is 8 ct. a pound, and our recipe for candy calls for $1\frac{1}{4}$ lb. For our class we shall need 6 times the amount stated in the recipe. What will the sugar cost?

14. How much will a 9-word telegram from London to Chicago cost, at 24 ct. a word to New York, and 40 ct. for 10 words or less from New York to Chicago?

250. Written analysis. In the written work we wish to find out two things: (1) if we can solve problems quickly and accurately as they do in business; (2) if we clearly understand the solution. For this reason give both the analysis and the computation as in the following example, unless the teacher directs omitting one. In the simpler examples there is only one step in the analysis.

251. Illustrative problem. A cubic foot of water weighs $62\frac{1}{2}$ lb. What is the weight of the water in a cubical tank 6 in. on each edge?

Analysis:	*Work:*
1. 6 in. $= \frac{1}{2}$ ft.	$8)\overline{62\frac{1}{2}}$
2. $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of 1 cu. ft. $= \frac{1}{8}$ cu. ft. (Explain orally.)	$7\frac{6\frac{1}{2}}{8}$
3. $\frac{1}{8}$ of $62\frac{1}{2}$ lb. $= 7\frac{13}{16}$ lb. $= 7$ lb. 13 oz.	$7\frac{13}{16}$

WRITTEN EXERCISE

1. At $37\frac{1}{2}$ ct. a yard, how much will $15\frac{1}{2}$ yd. of cloth cost?

2. If 175% of a number is $194\frac{1}{4}$, what is the number?

3. At 680 bu. to a car, how many cars will it take to carry 11,560 bu.?

4. At 40,000 shingles to a car, how many cars will it take to carry 760,000 shingles?

5. The President of the United States receives $50,000 a year. How much does he receive a month?

6. A bushel of wheat weighs 60 lb., how many bushels of wheat in a car load weighing 21,240 lb.?

7. A bushel of oats weighs 32 lb., how many bushels of oats in a car load weighing 20,000 lb.?

8. How many chests of tea, weighing 25 lb. each, and costing 48 ct. a pound, can be bought for $1500?

9. If 32 bricks will pave 1 sq. yd., how many will be needed to pave a space 63 ft. by 36 ft.?

10. If a homing pigeon makes a flight of 1270 mi. in 13 da. $5\frac{1}{2}$ hr., what is the average speed per hour?

11. How many railway ties placed 2 ft. 6 in. from center to center would be required for a double line of road 13 mi. long?

12. One of the yachts in the international races made 30 mi. in 3 hr. 12 min. 30 sec. What was the average time per mile?

13. A cubic foot of pure cast gold weighs 1204 lb., and a cubic foot of granite weighs 172 lb. Gold is how many times as heavy as granite?

14. How many tons' pressure will a hurricane exert on the front of a city building 150 ft. high and 100 ft. wide, allowing 45 lb. pressure to the square foot?

15. How long would it take a line of soldiers, the line being $\frac{1}{2}$ mi. long, to pass a fixed point, if they took 75 steps a minute, and the steps averaged 2 ft. 8 in. in length?

16 In a certain arid region in the West they put in 250 mi. of main irrigating ditches at an average cost of $10,000 a mile, and 400 mi. of lateral ditches at $1250 a mile. Required the total cost.

17 By this irrigation (Ex. 16) they made 125,000 acres of desert land productive and worth $125 an acre. What was the value of the redeemed land? How much was the value above the cost of irrigating?

18 A fair breeze blows 880 ft. a minute, and a hurricane 7040 ft. a minute. In each case how many feet a second? In each case how many miles an hour? Answer to two decimal places only.

252. Illustrative problems. 1. If a grocer bought 200 bbl. of flour at \$3.67 a barrel, and sold it for \$4.13 a barrel, how much did he gain?

	\$4.13
	3.67
	.46
	200
	\$92.00

1. \$4.13 − \$3.67 = \$0.46, the gain on 1 bbl.
2. 200 times \$0.46 = \$92, " " " 200 bbl.

2. A merchant bought 500 yd. of India silk at 49 ct. a yard. He sold 350 yd. at 60 ct. a yard, and the rest at a bargain sale at 48 ct. a yard. How much did he gain?

1. 500 times \$0.49 = \$245, the cost.
2. 350 " \$0.60 = \$210, received from 350 yd.
3. 150 " \$0.48 = \$72, " " 150 "
4. \$210 + \$72 = \$282, total receipts.
5. \$282 − \$245 = \$37, the gain.

19. If a man earns \$30 a week, and spends \$21.50, how much can he save at this rate in 48 weeks?

20. If a dozen pairs of scissors cost \$1.84, and were sold at 28 ct. a pair, how much did the seller gain?

21. If 1 gal. = 231 cu. in., what is the value of the oil in a can containing 4158 cu. in., at 7 ct. a gallon?

22. The cost of a piece of cloth is \$63, and the cost per yard is \$1.12½. How many yards are there in the piece?

23. If 35 bales of cloth contain 182,700 yd., and each piece contains 58 yd., how many pieces are there in each bale?

24. If 42 bales of cloth contain 41,412 yd., and each bale contains 34 pieces, what is the average number of yards in a piece?

25. A grocer bought 10 bu. of new potatoes at \$1.12½ a bushel, and sold them at 5 ct. a quart. How much did he gain on the lot?

26. A grain elevator contained 16,250 bu. of wheat, worth 82 ct. a bushel. If $\frac{1}{5}$ of it is destroyed by fire, what is the loss?

27. A man paid \$300 for a horse and carriage, the carriage costing \$50 more than the horse. How much did he pay for each?

28. A merchant bought 1200 pieces of cloth, 48 yd. to the piece, for \$23,040, and sold it for 50 ct. a yard. How much did he gain?

29. A dealer buys 3000 men's collars at \$1.92 a dozen, and sells them at 25 ct. each. How much does he gain on the total number?

30. A meat dealer bought 3650 lb. of beef at the average price of $12\frac{1}{2}$ ct. a pound, and sold it at the average price of 17 ct. How much did he gain by the transaction?

31. A dealer buys 50 arithmetics at 39 ct. each, and 40 readers at $37\frac{1}{2}$ ct. each. At what uniform price must he sell them in order to make \$8.70 on the whole transaction?

32. A man sold a load of hay for \$13.32. The wagon and hay together weighed 4160 lb., and after unloading, the wagon weighed 1200 lb. At what price per ton was the hay sold?

33. A fruit buyer receives \$210 for 6 weeks' work, and spends \$14.50 a week for board, \$13.74 a week in traveling, and \$3 a week for other purposes. What is his average gain per week?

34. A merchant bought 2000 yd. of pongee silk at $87\frac{1}{2}$ ct. a yard. He marked it \$1.10 a yard and sold 950 yd. at that price. He then sold 750 yd. at 10% off, and the rest at 50% off. Did he gain or lose on the lot, and how much?

253. Unitary analysis. Some examples are easily solved by a particular kind of analysis called *unitary analysis.*

254. Illustrative problems. 1. If a man invested some money and gained 10%, and then had $275, how much did he invest?

1. The sum invested + 0.10 of itself = 1.10 times itself.
2. Since 110% of the sum invested = $275,
   1% of the sum invested = $275 ÷ 110 = $2.50.
3. Therefore 100% of the sum invested = 100 times $2.50 = $250.

Notice that the problem was solved by finding the *unit* per cent, 1% of the sum. Hence the name, *unitary analysis.*

2. If 5 tons of coal cost $27.50, what will 7 tons cost?

1. Since 5 T. cost $27.50, 1 T. costs $\frac{1}{5}$ of $27.50 = $5.50.
2. Therefore 7 T. cost 7 times $5.50 = $38.50.

Notice again the *unit,* 1 ton.

WRITTEN EXERCISE

1. If 3000 ft. of lumber cost $96.60, what will 16,100 ft. cost at the same rate?

2. If a man gained 16% on an investment and then had $3770, how much did he invest?

3. If a man invested some money and lost 11%, and then had $1869, how much did he invest?

4. At $460 per 100 bbl., how many barrels of flour can be bought for $73.60? for $782? for $8556?

5. At $5.60 a ton of 2000 lb., what will 2775 lb. of coal cost? 1575 lb.? 1850 lb.? 3225 lb.? 6450 lb.?

6. At $12.50 per hundred pounds, what will $4\frac{1}{2}$ lb. of cheese cost? $7\frac{1}{4}$ lb.? 9 lb. 4 oz.? 12 lb. 12 oz.?

7. At $5.20 a hundred pounds, how many pounds of meat can be bought for $639.60? for $587.60? for $4004?

255. Occasionally examples like this are met:

If 4 loads of hay will last 6 horses 8 weeks, 8 loads will last how many horses 3 weeks?

1. If 4 loads will keep 6 horses 8 wk.,
2. 4 loads " " 8×6 horses 1 wk. Do you see why?
3. 1 load " " $\frac{1}{4}$ of 8×6 horses 1 wk.
4. 1 load " " $\frac{1}{3}$ of $\frac{1}{4}$ of 8×6 horses 3 wk.
5. 8 loads " " $8 \times \frac{1}{3}$ of $\frac{1}{4}$ of 8×6 horses 3 wk.
6. $8 \times \frac{1}{3}$ of $\frac{1}{4}$ of 8×6 horses $= 32$ horses.

Notice the two *units* in step 3, 1 load and 1 wk.

Such problems were formerly solved by the unexplained rule of compound proportion. In actual business they are but little used.

WRITTEN EXERCISE

1. If it takes 4 boys 16 da. to build a playhouse, how long will it take 1 boy working at the same rate?

2. In Ex. 1 how long will it take 8 boys working at the same rate? also 2 boys? also 6 boys? also 10 boys?

3. If 3 reapers can cut 90 acres in 2 da., how long will it take 5 reapers to cut 3000 acres?

4. How long will it take 25 men to do a piece of work which 12 men can do in 15 days, working at the same rate?

5. If 42 bu. of oats last 16 horses a week, how many bushels will be needed for 24 horses for 3 wk., at the same rate? for 48 horses for $1\frac{1}{2}$ wk.?

6. If a man working 10 hr. a day can complete a piece of work in 12 da., in how many days can he complete it working 8 hr. a day?

7. If 702 bricks are required for a walk 39 ft. long and 4 ft. wide, how many bricks will be required for a walk half as wide again and 3 times as long?

8. If it costs \$702 to excavate a cellar $27' \times 39' \times 9'$, what will it cost to excavate a cellar $30' \times 36' \times 12'$ at the same rate?

9. A California hop picker picks 3 bags of hops a day, averaging 60 lb. to a bag. At 2700 lb. to the acre, how long will it take him to pick an acre of hops? At this rate, how many men are needed to pick an acre in 5 da.? in 3 da.? in $7\frac{1}{2}$ da.?

10. At the rate given in Ex. 9, how many men will it take to pick 75 acres in 15 days?

11. At the rate given in Ex. 9, how many days will it take 90 men to pick 60 acres?

12. At the rate given in Ex. 9, how many acres can 75 men pick in 6 days?

13. These hops are baled in bales 20 in. wide, 20 in. high, and 5 ft. long. If such a bale weighs 200 lb., how much does 1 cu. ft. of baled hops weigh?

14. A well at Madison, Wisconsin, furnishes enough water to irrigate 110 acres of land 2 in. deep every 10 days. At this rate, how many acres can it cover to a depth of 1 in. every day?

15. Another well at Chamberlain, South Dakota, furnishes enough water to irrigate 657 acres to a depth of 4 in. every 10 days. At this rate, how many acres can it cover to a depth of 2 in. every 3 days?

16. In Wisconsin experiments it was found that an 8-horse-power farm engine could draw from a lake to a height of 26 ft. sufficient water to cover $22\frac{1}{8}$ acres a day to a depth of 1 in. At these rates, how many acres could be covered half as deep by a 16-horse-power engine raising the water only 13 ft.?

ORAL EXERCISE

1. If 3 collars cost 45 ct., what will 2 cost?

2. If 2 yd. of cloth cost \$3, what will 3 yd. cost?

3. If 4 spools of silk cost 24 ct., what will 5 cost?

4. If 2 tons of hay cost \$18, what will 11 tons cost?

5. At 40 ct. a quart, what will $\frac{7}{8}$ of a quart of cream cost? $1\frac{3}{4}$ qt.? $2\frac{1}{2}$ qt.? 3 gal.?

6. If $\frac{3}{4}$ of a barrel of beans cost \$6, what will a barrel cost? 2 bbl.? 5 bbl.? $6\frac{1}{2}$ bbl.? 10 bbl.?

7. At the rate of 72 mi. in 2 hr., how far will a train go in 3 hr.? in 5 hr.? in $1\frac{1}{2}$ hr.? in 4 hr. 30 min.?

WRITTEN EXERCISE

1. At the rate of \$110 for 40 sheep, what will 24 cost?

2. After losing 31% of his capital a man has \$3726. What was his capital?

3. At the rate of \$216 for 300 bu. of wheat, how much will 700 bu. cost? 50 bu.? 275 bu.? 1000 bu.?

4. If 11 rd. of ditching cost \$16.50, how much will 20 rd. cost, at the same rate? 50 rd.? 75 rd.? $5\frac{1}{2}$ rd.?

5. At the rate of 980 lb. to 2 bales of cotton, how much will 3 bales weigh? 7 bales? 9 bales? 35 bales?

6. If $\frac{3}{4}$ of an acre of land costs \$90, how much will $\frac{7}{8}$ of an acre cost, at the same rate? $2\frac{1}{2}$ acres? 35 acres?

7. The Twentieth Century Express makes the 980 mi. from New York to Chicago in 20 hr. At this rate, how far does it travel in 6 hr.? in $7\frac{1}{2}$ hr.? in 9 hr. 45 min.?

8. Every 10 people in the United States consume on an average 650 lb. of sugar annually. At this rate, how much does a family of 3 consume? a family of 7? of 9?

256. Illustrative problems. 1. A man paid \$78.75 for the building of a stone wall $52\frac{1}{2}$ rd. long. How much would he have paid, at this rate, if it had been 75 rd. long?

$$\frac{75 \times 78.75}{52.5} = \frac{\overset{\overset{225}{\cancel{3}}}{\cancel{75}} \times \overset{\overset{\cancel{1575}}{\cancel{7875}}}{\cancel{5250}}}{\underset{\underset{\underset{2}{\cancel{14}}}{\cancel{210}}}{\cancel{5250}}} = 112.50$$

1. $\$78.75 \div 52\frac{1}{2} = \$1.50$, cost of 1 rd.
2. 75 times $\$1.50 = \$112.50$, cost of 75 rd.

2. Two men pasture cattle in the same field, one pasturing 10 head for 3 wk., and the other 8 head for 4 wk. They pay \$12.40 rent. What is the share of each?

Since 10 cattle in 3 wk. eat as much as 30 cattle in 1 wk., and 8 cattle in 4 wk. eat as much as 32 in 1 wk., they all eat as much as 62 in 1 wk. Hence the first man should pay $\frac{30}{62}$, and the second $\frac{32}{62}$ of the rent. Therefore we have:

1. 3 times 10 cattle = 30 cattle.
2. 4 " 8 " = 32 "
3. Total " = 62 " for 1 wk.
4. $\frac{30}{62}$ of \$12.40 = \$ 6.
5. $\frac{32}{62}$ " " = \$ 6.40
6. The sum = \$12.40, checking the work.

WRITTEN EXERCISE

1. If 8 tons of hay cost \$76, what will 5 tons cost?

2. If 35 A. of land cost \$3220, what will 125 A. cost?

3. If sound travels 3696 ft. in 3.3 sec., how far does it travel in 7 sec.? in $9\frac{1}{2}$ sec.? in 1 min. 45 sec.?

4. If 15 bales of cotton weigh 7305 lb., what will 4 bales weigh? 7 bales? 16 bales? 25 bales?

5. If a pole 6 ft. high casts a shadow 8.3 ft. long, how high is an adjacent tree that casts a shadow 74.7 ft. long at the same time?

6. If a stack of brick $2' \times 4' \times 7'$ contains 1000 bricks, how many bricks in a stack $6' \times 10' \times 14'$?

7. If a tree 25 ft. high casts a shadow 20 ft. long, how long will be the shadow of a tree 30 ft. high at the same time?

8. If a boy 5 ft. high casts a shadow 3.2 ft. long, what is the height of a school building that casts a shadow 35.2 ft. long at the same time?

9. If it takes a boat 20 hr. to go the length of the Albemarle and Chesapeake Canal, 44 mi., how long will it take it to go 9 mi. at the same rate?

10. Three men pasture cattle in the same field, the first pasturing 8 head for 6 wk., the second 9 head for 5 wk., and the third 12 head for 4 wk. They pay \$28.20 rent. What is the share of each?

11. If a roofer can lay 2 squares (a square meaning 100 sq. ft., lapped and laid) of slate in 10 hr., how many men will it take to cover, in 5 da. of $7\frac{1}{2}$ hr. each, an A-shaped roof 100 ft. long, each slope being 30 ft. wide?

12. If 3 squares (see Ex. 11) of roofing slate weigh a ton, what is the weight of the slate on a roof 72 ft. long, and 20 ft. wide on each slope?

13. The average width of shingles being 4 in., and the shingles being laid 4 in. to the weather, how many will be required to cover a square (100 sq. ft.) of roof? How many if they were laid $4\frac{1}{2}$ in. to the weather?

14. If the total product of butter in the United States is 1,430,000,000 lb. a year, and the number of cows used for this purpose is 11,000,000, what would be the total product if there were 4,000,000 more cows, the same rate being maintained?

257. Stating a problem in one step. It is often a good plan to indicate the solution of a problem all in a single step, as in the following example:

If 1 cu. ft. of water weighs $62\frac{1}{2}$ lb., and gold is 19.5 times as heavy as water, how much is 1 cu. ft. of gold worth at \$18.60 an ounce avoirdupois?

1 cu. ft. of gold weighs 19.5 times $62\frac{1}{2}$ lb., or 19.5 times $62\frac{1}{2}$ times 16 oz.

Therefore the value is

$$19\tfrac{1}{2} \times 62\tfrac{1}{2} \times 16 \text{ times } \$18.60,$$

or

$$\frac{39 \times 125 \times 16 \text{ times } \$18.60}{2 \times 2}.$$

We may now cancel and work this out more easily than by multiplying step by step.

WRITTEN EXERCISE

1. Finish the solution of the above problem.

2. When sound travels 1122 ft. per second, how many miles away is a lightning flash which is seen 13.2 sec. before the thunder is heard?

The velocity of sound varies slightly with the temperature.

3. Soldiers marching quickstep take 120 paces a minute, averaging 28 in. each. At this rate, how many miles can a company march in an hour?

4. If the circumference of a wheel is $3\frac{1}{7}$ times the diameter, and the drive wheel of a locomotive is 6 ft. in diameter, how many revolutions will it make in going from New Orleans to New York, 1344 mi.?

5. What is the expense of covering a floor with plain matting 1 yd. wide, the room being 18 by $21\frac{1}{2}$ ft., and the matting being worth 30 ct. a yard, allowing 6 in. extra on each strip for turning under? Draw a plan.

ORAL EXERCISE

1. At the rate of \$2.10 for 3 bu. of wheat, what will 11 bu. cost? 16 bu.? 25 bu.? 40 bu.?

2. At the rate of \$25 a dozen, what will 8 silver tablespoons cost? What will 16 cost? 18? 20?

3. At the rate of 38 tons of hay to 19 acres, how many tons will 49 acres produce? 57 acres? 90 acres?

4. At the rate of 75 bu. of wheat to 3 acres, how many bushels will 50 acres produce? 12 acres? 40 acres? 275 acres? 450 acres?

WRITTEN EXERCISE

1. If a piece of cast iron $10'' \times 8'' \times 2''$ weighs 41.6 lb., what is the weight of a piece $16'' \times 10'' \times 3''$?

2. If 475 horse power from some engines costs \$13,490 a year, what will 60 horse power cost at the same rate?

3. A dealer bought 25 bu. of grass seed at \$2.90 a bushel. He paid \$1.20 for paper bags, and sold the seed in bags at 10 ct. a quart. How much did he gain?

4. Two steamers started for Europe on the same route at the same time. After 4 days one had averaged 448.7 mi. a day, and the other 502.1 mi. How far apart were they?

5. A box of arithmetics cost a dealer \$53.94, including \$1.48 freight. He sold the arithmetics for 50 ct. each, thus gaining 7 ct. on the list price of each book. How many did he buy?

6. A man bought two adjacent farms containing the same number of acres. For one he paid \$95 per acre, and for the other \$130 per acre. The two farms cost him \$31,950. How many acres did he buy?

258. The equation form. There is a modern and a much clearer way of solving many kinds of examples, known as the *equation method.* This will now be studied.

ORAL EXERCISE

1. What number added to 232 makes 250? 300?
2. What number subtracted from 250 leaves 170?
3. What number multiplied by 12 equals 132? 144?
4. What number divided by 12 equals 9? 90? 9000?
5. If 180 be subtracted from a certain number, the remainder is 25. Required the number.
6. If 320 be multiplied by a certain number, the product is 960. Required the number.
7. If 440 be divided by a certain number, the quotient is 40. Required the number.
8. The square of a certain number (the product of the number multiplied by itself) is 121. Required the number.
9. What is the number whose half is 55? Whose fourth is 70? Whose tenth is 75.2? Whose third is 92?
10. Tell the missing number (x) in each statement: $30 + x = 95$; $125 - x = 95$; $9 \times x = 720$; $550 \div x = 11$.

WRITTEN EXERCISE

In these statements, x *stands for what missing numbers?*

1. $425 + x = 973$.
2. $x + 741.3 = 963.8$.
3. $173\frac{1}{2} + x = 342\frac{3}{4}$.
4. $687.25 + x = 963.5$.
5. $x + 57.4 = 99.23$.
6. $609.37 + x = 1243.48$.
7. $427 - x = 298$.
8. $521.2 - x = 137.5$.
9. $34 \times x = 799$.
10. $448 \div x = 28$.

259. Unknown quantity. In an expression like $5+x=11$, x is called an *unknown quantity.*

260. Equation. An equality that is true for some value of an unknown quantity is called an *equation.*

For example, $2+3=5$ is an equality; $2+x=5$ is an equation. The two terms are often used as meaning the same.

261. Indicated multiplication. Because x and $\times$ look so nearly alike when written, the $\times$ is usually omitted when x is used, $5\times x$ being written $5x$, and read "five x."

262. To solve an equation means to find the value of the unknown quantity.

For example, to solve the equation $x+17=26$ is to find that $x=9$. To solve the equation $6x=48$ is to find that $x=8$.

263. How to solve. If we have an equation like $x+5=11$, we may think of the two sides as just balancing. Any change made on one side must be made on the other, to keep the balance. If we take 5 from the left side, we must also take 5 from the right side, leaving $x=6$.

264. *To solve an equation, do to one side what is necessary to leave* x *by itself, and then perform the same operation on the other side.*

For example, if $3x=15$, divide both sides by 3, so as to leave x by itself, whence $x=5$.

Also, if $\frac{x}{3}=7$, multiply by 3, because $3\times\frac{x}{3}=x$, thus leaving x by itself, whence $x=21$.

265. Teachers often find it valuable to represent the required numbers by n, the number of dollars by d, the number of dollars of cost by c, and so on with other initial letters. But x and y must finally be used as the universal conventions of mathematics.

266. Illustrative problems. 1. I have thought of a number; I add 7 to it; then I subtract 3; the result is now 28. What is the number?

1. Let x = the number.

Then $x + 7$ is the number with 7 added, and $x + 4$ is the remainder after subtracting 3, and the problem tells us that this remainder equals 28.

2. Therefore $x + 4 = 28$.

3. Subtracting 4 from both sides, $x = 24$.

To *check* this, put 24 in the statement of the problem.

2. What number is that which, being doubled, and 7 being then added, equals 29?

1. Let x = the number.

Then $2x$ = double the number, and $2x + 7$ = double the number with 7 added, and the problem tells us that this equals 29.

2. Therefore $2x + 7 = 29$.

3. Subtracting 7 from both sides, $2x = 22$.

4. Dividing both sides by 2, $x = 11$.

WRITTEN EXERCISE

1. What number multiplied by $1\frac{1}{2}$ equals 96?

2. What number multiplied by 1.04 equals 7.28?

3. If 5 times a certain number, less 70, is 80, what is the number? (Solve also without using x.)

4. If 72 times a number, together with 72, is 792, what is the number? (Solve also without using x.)

5. If to twice a number I add 15, the result is 75. What is the number? (Solve also without using x.)

6. I have thought of a number; I add 7; I add this result to itself; I take away 12; the result is 42. Required the number.

267. Illustrative problems. 1. A horse and carriage together cost \$375, and the carriage cost twice what the horse did. What did the horse cost?

1. Let x = the *number of dollars* the horse cost.
Then $2x$ = " " " " " carriage cost,
and $x + 2x$ = " " " " both cost, which is 375.
2. Therefore $3x = 375$, because x (that is, $1x$) and $2x$ are $3x$.
3. " $x = 125$, by dividing both sides by 3.
Therefore the horse cost \$125.

2. After increasing 6%, a certain sum amounts to \$318. What is the sum?

1. Let x = the *number of dollars* in the sum.
Then $0.06x$ = " " " " increase,
and $1.06x$ = " " " " after the increase.
2. Therefore $1.06x = 318$.
3. " $x = 318 \div 1.06 = 300$.
Therefore the sum is \$300, as is easily proved.

WRITTEN EXERCISE

1. After gaining 12%, a certain sum amounts to \$392. What is the sum?

2. If to $\frac{1}{5}$ of a certain number I add 730, the result is 855. What is the number?

3. A boy lost $\frac{2}{5}$ of his marbles and had 18 left. How many had he at first?

4. A man sold $\frac{3}{7}$ of his cattle and had 68 left. How many had he at first?

5. A certain sum was increased 50% and then amounted to \$505.50. What was the sum?

6. If your father is 3 times as old as you, and if the sum of your ages is 52 yr., how old are you?

268. Illustrative problem. A man sold a carriage for \$120, which was 20% less than it cost. How much did it cost?

	Complete analysis:	*Actual work:*
1. Let	x = the *number of dollars* of cost.	$.80\,x = 120$
Then	$0.20\,x =$ " " " " lost,	$8\overline{)1200}$
and	$x - 0.20\,x =$ " " " " received.	150

2. Therefore $0.80\,x = 120$, because $1\,x - 0.20\,x = 0.80\,x$.

3. Therefore $x = 120 \div 0.80 = 150$.

Therefore the cost was \$150, as is easily proved.

WRITTEN EXERCISE

1. A man sold a house for \$2250, which was 20% less than it cost. How much did it cost?

2. With the \$2250 he bought 20 acres of land from Mr. A, who thereby gained 25% on what it cost him. How much did Mr. A pay per acre for the land?

3. A man sold a farm of 100 acres at the rate of \$56.25 an acre. He lost 10% on the cost of the farm. How much did the farm cost him?

4. A dealer bought two horses at the same price. He sold one, at a profit of 20%, for \$102. The other he sold at a loss of 10%. How much did he receive for the latter?

5. A certain village, after gaining 10% on its population in 1900, had 10% less population than another village of 4180 inhabitants. What was the population of the first village in 1900?

6. A furniture dealer bought 5 sets of furniture at \$65 a set, 3 at \$90, and 2 at \$40. The \$40 sets he sold at a profit of 30%, the \$65 sets at a profit of 20%, but on the \$90 sets he lost 10%. How much did he gain on the lot?

PERCENTAGE

ORAL EXERCISE

1. Express as per cents: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$.
2. Express as per cents: $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, 1.
3. Express as per cents: $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{10}$.
4. If $\frac{3}{8}$ of a number is 9, what is the number?
5. If $37\frac{1}{2}\%$ of a number is 6, what is the number?
6. If $62\frac{1}{2}\%$ of a number is 15, what is the number?
7. If 75% of a number is 66, what is the number?
8. Express as common fractions: 20%, $33\frac{1}{3}\%$, $87\frac{1}{2}\%$.
9. Express as common fractions: $6\frac{1}{4}\%$, $8\frac{1}{3}\%$, $12\frac{1}{2}\%$.
10. How much is 25% of \$8000? $33\frac{1}{3}\%$ of \$9000?
11. How much is $16\frac{2}{3}\%$ of \$120? 10% of \$225.50?
12. If $87\frac{1}{2}\%$ of a number is 1400, what is the number?
13. 30 is $16\frac{2}{3}\%$ of what number? $33\frac{1}{3}\%$ of what number?
14. 40 is $66\frac{2}{3}\%$ of what number? $83\frac{1}{3}\%$ of what number?
15. Tell the short way of finding $12\frac{1}{2}\%$ of a number; of finding $66\frac{2}{3}\%$ of a number.
16. Tell the short way of finding $6\frac{1}{4}\%$ of a number; of finding $16\frac{2}{3}\%$ of a number.

WRITTEN EXERCISE

1. $37\frac{1}{2}\%$ of \$164.
2. $6\frac{1}{4}\%$ of \$33.60.
3. $83\frac{1}{3}\%$ of \$1110.
4. $66\frac{2}{3}\%$ of \$72.30.
5. $8\frac{1}{3}\%$ of \$145.20.
6. $16\frac{2}{3}\%$ of \$417.30.
7. 125% of \$4020.
8. $183\frac{1}{3}\%$ of \$2166.
9. 150% of \$2178.
10. 160% of \$475.50.
11. 175% of \$5260.
12. $133\frac{1}{3}\%$ of \$286.41.

269. Percentage. That part of Arithmetic that treats of per cents is often called *percentage*.

270. To find some per cent of a number. The most important problem in percentage is the one already studied, of finding some per cent of a number.

271. Illustrative problems. 1. What is 4% of \$62.25?

Here we may multiply \$62.25 by .04, as in the first process below; or we may take 1%, by moving the decimal point two places to the left, and then multiply by 4. The second process is considered the simpler by some teachers.

First process:	*Second process:*
\$62.25	\$0.6225 = 1% of \$62.25
.04	4
\$2.4900	\$2.4900

Result = \$2.49.

2. How much is 0.50% of \$1000?

This means $\frac{1}{2}$% of \$1000, or $\frac{1}{2}$ of \$10, or \$5.

WRITTEN EXERCISE

1. 22% of \$4870.
2. 35% of \$64.80.
3. 620% of \$4964.
4. 750% of \$4780.
5. 0.75% of \$2000.
6. 500% of \$987.25.
7. $33\frac{1}{3}$% of \$1773.30.
8. $0.33\frac{1}{3}$% of \$6312.
9. A man whose income was \$1500 a year spent 48% of it. How much did he save?
10. In a city having 3210 school children, $16\frac{2}{3}$% never missed a day at school last year. How many had perfect attendance?
11. A man bought 240 barrels of apples, and 5% of them were spoiled during the winter. He sold 50% of the remainder. How many had he left?

272. To find what per cent one number is of another. 25 is what per cent of 225?

Since 1 is $\frac{1}{225}$ of 225, therefore 25 is $\frac{25}{225}$ of 225, or $\frac{1}{9}$ of 225, or $11\frac{1}{9}\%$ of 225.

$$9\overline{)1.00}$$
$$.11\frac{1}{9}$$

We might also say: "$x\%$ of 225 = 25.

Hence $x\% = 25 \div 225 = 11\frac{1}{9}\%$."

Hence, as found on page 138,

The percentage divided by the base equals the rate.

WRITTEN EXERCISE

Find what per cent the first number is of the second:

1. 25, 750.
2. $1\frac{1}{2}$, $7\frac{1}{2}$.
3. 37, 148.
4. \$1.80, \$3.
5. 5 ft., $37\frac{1}{2}$ ft.
6. 7 ft., 7 yd.
7. 528 ft., 2 mi.
8. \$4.90, \$29.40.
9. \$8.60, \$77.40.
10. 73 ft., 194 yd. 2 ft.

11. A man's income is \$1650 a year, and he spends \$693. What per cent of his income does he save?

12. In a certain village 576 out of the 1200 pupils in school are boys. What per cent are boys? girls?

13. The purity of gold is measured in carats, or 24ths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in an 18-carat ring?

14. What is the per cent of pure gold in a watch case that is 16 carats fine? in a chain that is 12 carats fine? in a ring that is 14 carats fine?

15. If $6\frac{1}{4}\%$ of a class of 32 have been neither tardy nor absent during the year, and $12\frac{1}{2}\%$ have not been absent, how many have been neither tardy nor absent? How many have not been absent?

273. To find a number when a certain per cent of it is given. \$8.40 is 12% of what amount?

$$0.12\,x = \$8.40$$
$$x = \$8.40 \div 0.12 = \$70.$$

$$.12\overline{)8.40}$$
$$12\overline{)840}$$
$$70$$

We therefore find, in a way different from that on page 139, that

The percentage divided by the rate equals the base.

WRITTEN EXERCISE

1. \$1.05 is 15% of what amount?
2. \$2.08 is 32% of what amount?
3. 6 ft. 2.8 in. is 22% of what length?
4. 9.9 in. is 22% of how many yards?
5. 63 lb. 7 oz. is 35% of what weight?
6. 112 ft. 6 in. is 45% of what length?
7. A man saves \$675.20 a year, which is 32% of his income. How much is his income?
8. A school has 20% of its pupils in the sixth grade, which numbers 29. How many are there in the school?
9. If a school is in session 50% of the days of a certain year, and is in session 183 days, is that a leap year or not?
10. A man buys some lumber, and sells it for \$60, thereby gaining 20% on the cost. The \$60 is what per cent of the cost? What is the cost?
11. A certain school has 119 boys, which is 85% of the number of girls. How many girls are there in the school? What is the total number of pupils?
12. A man sells 41 sheep, which is $33\frac{1}{3}$% of all he owned. He received \$5.50 a head. At this rate, what is the value of the sheep remaining in his flock?

ORAL EXERCISE

1. If $1\frac{1}{2}$ times a number is 6, what is the number?
2. If $1\frac{1}{3}$ times a number is 16, what is the number?
3. If 1.50 times a number is 12, what is the number?
4. If $1.33\frac{1}{3}$ times a number is 8, what is the number?
5. If 125% of a number is 250, what is the number?

274. To find a number when the number plus a certain per cent is given.

What amount increased by 50% equals \$900?

If x is the amount,

then $x + .50\,x = \$900$,

or $1.50\,x = \$900$,

and $x = \$900 \div 1.50 = \$600$.

Check. \$600 + 50% of \$600 = \$600 + \$300 = \$900.

Compare this with the method on page 141.

WRITTEN EXERCISE

1. A man's salary has been increased 22% this year. It is now \$1525. What was it last year?
2. If I have added 60% to the size of my farm, and now have 240 acres, how many did I have before the increase?
3. If you weigh 78.1 lb., and have increased 42% in weight in the last 5 years, how much did you weigh five years ago?
4. If our regular army has been increased 21%, and now numbers 73,810, how many did it number before the increase?
5. If a library has increased in size $16\frac{2}{3}$% in the past five years, and now has 8323 volumes, how many volumes had it before the increase?

ORAL EXERCISE

1. If $\frac{3}{4}$ of a number is 90, what is the number?

2. If 75% of a number is 60, what is the number?

3. If I spend 25% of my money, and have \$30 left, how much had I in the first place?

4. If a number is decreased by $\frac{1}{3}$ of itself, and then equals 80, what is the number? If it is decreased by $33\frac{1}{3}\%$ of itself, and then equals 40, what is the number?

275. To find a number when the number minus a certain per cent of itself is given. What number decreased by 42% of itself equals 406?

If x is the number,

then $x - .42\,x = 406,$

or $.58\,x = 406,$

and $x = 406 \div .58 = 700.$

Check. $700 - 42\%$ of $700 = 700 - 294 = 406.$

WRITTEN EXERCISE

1. What number decreased by $2\frac{1}{2}\%$ of itself equals 11.7?

2. If a man sells 12% of his hens, and has 66 left, how many had he at first?

3. If a man spends 5% of his salary for clothes, and then has \$855 left, what is his salary?

4. If you have read all but 40% of a book, and have read 234 pages, how many pages has the book?

5. If the width of a schoolhouse is 10% less than the length, and the width is 46.8 ft., what is the length?

6. If you lose 10% of a kite string, and have 585 ft. left, how long was the string? How many feet did you lose?

WRITTEN EXERCISE

1. If 45% of the 240 pupils in a school are boys, how many are girls?

2. A man purchased 530 head of cattle and sold 30% of them. How many had he left?

3. If a man pays 12% of his income for house rent, which is $15 a month, how much is his income?

4. If an automobile going 14 mi. an hour increases its speed 37%, at what rate will it then be traveling?

5. If the boys in a certain school are 45% of the total, and there are 66 girls, how many boys are there?

6. What sum increased by $33\frac{1}{3}\%$ of itself becomes $600? What sum decreased by $33\frac{1}{3}\%$ of itself becomes $600?

7. A stenographer can write 105 words a minute, which is 25% more than she could write three months ago. What was her rate then?

8. A stenographer can write 95 words a minute. Three months ago she wrote 20% less than now. What was her rate then?

9. A cubic foot of platinum weighs 1220 lb., and a cubic foot of pressed gold weighs 10 lb. less. Platinum is what per cent heavier than gold?

10. A bank averaged $1200 a day of new deposits in November, and 26% more a day in December. What were the average daily deposits in December?

11. A clerk's salary is $12.00 a week. If he spends 20% of it for rent, 10% for clothes, and 25% for other expenses, and saves the rest, how much does he save in a year?

12. The championship record in baseball in a certain year was 52 games won, 14 lost. What per cent of the games were won?

13. One of the best records of baseball in recent years was when Brooklyn won the pennant with a total of 82 games won, 54 lost. What was the per cent of games won?

14. Two of the best running averages in baseball were 597 times at the bat, 139 runs, and 543 times at the bat, 145 runs. What was the per cent of runs in each case?

15. The area of the Atlantic Ocean is 24,651,410 sq. mi., and this is 49% of the area of the Pacific Ocean. What is the area of the latter?

16. A boy last year weighed 63.9 lb., which was 10% less than he weighs now. The 63.9 lb. is what per cent of his present weight? How much does he weigh now?

17. The population of the earth at the beginning of the Christian era is estimated at 54,000,000. At the opening of this century it was estimated at 1,487,900,000. The latter population is what per cent of the former?

18. Of the 161,800,000 people who spoke European languages in 1800, 31,450,000 spoke French. Of the 401,700,000 in 1900, 51,200,000 spoke French. What per cent spoke French in each of these years?

19. Of the same (Ex. 18) totals, 20,520,000 spoke English in 1800, and 111,100,000 in 1900. What per cent spoke English in each of these years?

20. I own a half interest in a farm worth $6000. The annual incomes from my share for the last five years have been $120, $90, $178, $210, and $152. What has been the average annual rate of income?

21. A bar of iron 1 in. in diameter and 1 ft. long weighs 3.368 lb. What is the weight when the length is increased 25% ?

22. A man bought a house and lot in a village for $3480, and sold it at an advance of $12\frac{1}{2}\%$. What was the selling price?

23. If the weight of an iron plate 1 ft. square and 1 in. thick is 40.42 lb., what is the weight when the thickness is decreased $12\frac{1}{2}\%$?

24. A factory sells $133,833.48 worth of goods in a year, the cost of manufacturing being $101,389. The gain is what per cent of the cost?

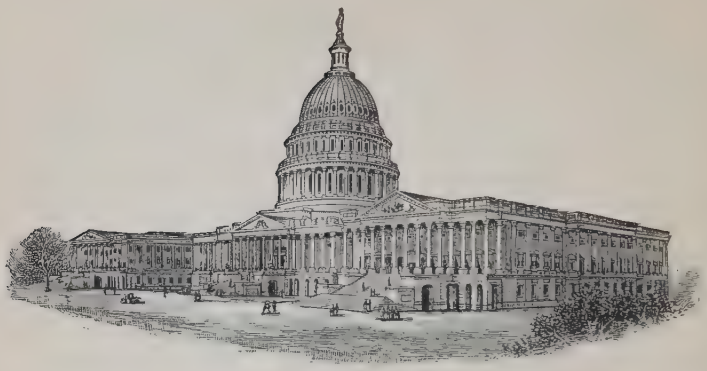
25. A factory sells $100,606.41 worth of goods in a year, the cost of manufacturing being $91,878. The gain is what per cent of the cost?

26. If 14 cu. ft. of chalk weighs 1 ton, and clay is 30% lighter than chalk, and sand is 20% lighter than clay, how many cubic feet of clay to a ton? How many cubic feet of sand?

27. A grain dealer bought 9000 bu. of wheat at 85 ct. a bushel. He sold a third of it at an advance of $33\frac{1}{3}\%$, a fourth of it at an advance of 25%, and the rest at 90 ct. a bushel. How much did he gain?

28. A dealer bought 2000 tons of coal for $9000. He sold 600 tons at $5.25 a ton. A strike then occurring, he was able to sell the rest at $6.75 a ton. What per cent did he make on his investment?

29. How much is 25% of $400? $400 is 25% of what number? $400 increased by 25% of itself is how much? $400 decreased by 25% of itself is how much? What sum increased by 25% of itself becomes $400?



SOME OF THE LARGE BUILDINGS OF THE WORLD

WRITTEN EXERCISE

1. The rotunda of the capitol at Washington is 95 ft. 6 in. in diameter, and the interior is 185 ft. 3 in. high. The diameter is what per cent of the height?

2. The tallest church spire in the world is that of the Cologne cathedral, 501 ft. The dome of the capitol at Washington, 287.5 ft., is what per cent as high?

3. The tallest column in the world is the Washington monument, 555 ft. high. This is what per cent of the height of the Alexander column at St. Petersburg, 175 ft.?

4. The greatest altitude above the sea, in the state of Delaware, is 280 ft. The height of a certain office building in New York is 322 ft. The office building is what per cent higher than the Delaware altitude?

5. The largest cathedral in the world is St. Peter's at Rome. It has a standing capacity of 54,000 persons. A school hall with a standing capacity of 405 persons has what per cent of floor space as compared with St. Peter's?

276. Per cents off. Sometimes merchants have bargain sales, to dispose of odd pieces or of goods that are going out of style. At such times they usually sell goods at a certain "per cent off."

For example, goods marked \$2.50, but sold 10% off, sell at \$2.50 − 10% of \$2.50, or \$2.25.

Merchants who sell at wholesale, that is, in large quantities to dealers, sell at a certain per cent off.

\$2.50
.25
\$2.25

ORAL EXERCISE

Find the price that must be paid for goods marked as stated in these examples, and discounted at the rates given:

1. \$2, 10% off.
2. \$6, 20% off.
3. \$5, 20% off.
4. \$4, 25% off.
5. \$3, $33\frac{1}{3}$% off.
6. \$10, 30% off.
7. \$25, 40% off.
8. \$30, 50% off.
9. \$50, 30% off.
10. \$60, $33\frac{1}{3}$% off.
11. \$1.60, 10% off.
12. \$100, 30% off.
13. \$3.50, 20% off.
14. \$3.50, 10% off.
15. \$4.50, 20% off.
16. \$4.50, $33\frac{1}{3}$% off.

WRITTEN EXERCISE

1. \$635, 6% off.
2. \$350, 15% off.
3. \$275, 15% off.
4. \$425, 35% off.
5. \$21.25, 8% off.
6. \$45.50, 20% off.
7. \$375.25, 4% off.
8. \$175.50, 20% off.
9. \$225.75, 20% off.
10. \$975.70, 10% off.
11. \$1527.50, 4% off.
12. \$310.00, 40% off.
13. \$33.33, $33\frac{1}{3}$% off.
14. \$2736.40, 3% off.

277. Several discounts. In some kinds of business two or more discounts are allowed. For example, a dealer may buy \$200 worth of hardware with discounts of 20%, 10% (20% and 10%). This means that 20% is to be deducted, and then 10% from the remainder.

The marked price is \$200.
This less 20% is \$160.
The \$160 less 10% is \$144, the cost, or *net price*.

ORAL EXERCISE

1. From \$50 take 10%, and 10% from the remainder.
2. From \$200 take 20%, and 1% from the remainder.
3. From \$100 take 25%, and 10% from the remainder.
4. From \$500 take 20%, and 25% from the remainder.
5. A man bought some paper at a marked price of \$80, with discounts of 25%, 10%. How much did he pay?

WRITTEN EXERCISE

How much must be paid for goods marked and discounted as stated in Exs. 1–10?

1. \$200, 30%, 20%.
2. \$700, 35%, 5%.
3. \$500, 25%, 15%.
4. \$500, 12%, 2%.
5. \$650, 10%, 10%.
6. \$400, 30%, 10%.
7. \$300, 15%, 10%.
8. \$275, 10%, 10%.
9. \$240, $12\frac{1}{2}$%, $33\frac{1}{3}$%.
10. \$800, 25%, 25%.

11. What is the difference between a discount of 50% on \$800, and the two discounts of 25%, 25%?

12. Is there any difference between a discount of 10%, 5% on \$900, and that of 5%, 10%? How is it on \$400? on \$200? on \$1200?

278. Bills discounted. Here is a bill for some goods purchased by a retail merchant from a wholesale dealer.

Chicago, December 17, 19.....

Mr. A. M. Course, Joliet, Ill.

Bought of Starr & Tiffany, Jewelers
378 Wabash Ave.

Terms cash in 60 da. *2%* and *1%*

Dec. 2.	*3 doz. Silver Forks @ $22*	*$66.*
"	*1/6 doz. Salad Forks @ $24*	*4.*
		$70.
	Less 2%, 1%	*2.09*
		$67.91

Received payment .
Starr & Tiffany

WRITTEN EXERCISE

Make out bills for the following, as on page 125:

1. Bought 40 arithmetics @ 70 ct., 65 geographies @ 90 ct., 25 readers @ 35 ct. Discount 12%, 5%.

2. Bought 3 doz. locks at $4.25, 4 doz. bolts @ $1.10, 2½ doz. No. 2478 hinges @ $2.35. Discount 30%, 5%.

3. Bought 4 doz. spoons @ $15, 8 doz. plated tablespoons @ $5, 6½ doz. teaspoons @ $10.50. Discount 8%, 5%.

4. Bought 3 doz. pencils @ at 25 ct., ½ doz. rulers @ $1.00, 1 pt. ink @ 25 ct., 4 lb. paper @ 30 ct. Discount 10%.

PROFIT AND LOSS ON PURCHASES

ORAL EXERCISE

1. What is \$2.50 with 10% added? with 10% subtracted?

2. What is \$1200 with 20% added? with 25% subtracted?

3. A dealer buys some books for \$100, and sells them at a profit of 20%. How much does he gain?

4. A dealer buys some cloth for \$50, and sells it at a profit of 20%. How much does he gain? What is the selling price?

5. A grocer bought \$15 worth of berries, but left them in the sun so long that he had to sell them at a loss of 10%. How much did he lose?

6. A man bought some goods for \$100, and sold them for \$125. He gained what per cent on the cost? He gained what per cent on the selling price?

7. If a man sells goods at a profit of 25%, the selling price is what per cent of the cost? If at a loss of 25%, the selling price is what per cent of the cost?

279. Computing profit or loss. The per cent of profit or loss is always reckoned on the *cost* of the goods.

280. Illustrative problems. 1. A man buys goods at \$40 and sells them at a profit of 20%. What is the selling price?

1. 20% of \$40 = \$8, profit.
2. \$40 + \$8 = \$48, selling price.

2. A man buys goods at \$60 and sells them at \$40. He loses what per cent?

1. \$60 − \$40 = \$20, loss.
2. Since x% of \$60 = \$20,
x% = \$20 ÷ \$60 = $33\frac{1}{3}$%.

ORAL EXERCISE

1. A hat dealer buys hats at $48 a dozen and sells them at $5 each. What is his per cent of profit?

2. A grocer buys starch at 50 ct. a box and sells it at 10% profit. What is the selling price?

3. A dealer buys stove polish at 3 ct. a cake and sells it at 4 ct. What is his per cent of profit?

4. A grocer buys matches at 48 ct. a dozen boxes and sells them at 54 ct. What is his per cent of profit?

5. A druggist buys some fine soap at the rate of $2.40 a dozen cakes, and sells it at 25 ct. a cake. What is his per cent of profit?

6. Some soap that a druggist bought at $1.50 a dozen cakes is damaged by dampness, so he sells it at 10 ct. a cake. What is his per cent of loss?

WRITTEN EXERCISE

1. A grocer pays $12 a gross for boxes of starch. He sells the starch at 10 ct. a box. What is his per cent of profit?

2. He buys washing powder in 1-lb. packages at $5.88 a gross, and sells it at 5 ct. a package. What is his per cent of profit?

3. Some of this washing powder that he bought at $5.88 a gross was damaged and had to be sold at 4 ct. a package. What was the per cent of loss?

4. A dealer bought 150 bags of salt, 56 lb. to the bag, for $63. He sold $\frac{2}{3}$ of the salt at the rate of 1 ct. a pound, and the rest at the rate of 2 lb. for a cent. Did he gain or lose, and what per cent?

Some Problems of the Grocer

ORAL EXERCISE

1. If I buy olives at $2.50 a dozen bottles, at what rate must I sell them to gain 20%? 30%?

2. If I buy mustard in bottles at $14.40 a gross, how much do I pay a bottle? If I sell it at 15 ct., what per cent do I gain? Suppose I sell it at 20 ct.?

3. If I buy pickles at $1.80 a dozen bottles, and wish to sell them so as to gain $33\frac{1}{3}\%$, at what price per bottle must I sell them? Suppose I wish to gain 20%?

4. Boneless codfish costs me 36 ct. a box. Some of it is damaged so that I have to sell it at a loss of 25%. At what price per box do I sell it? Suppose the loss were 50%?

WRITTEN EXERCISE

1. Macaroni costs me, in packages, $15 a gross. I sell it at 12 ct. a package. What per cent do I gain?

2. Gelatin costs me $11.52 a gross, in packages. At what rate per package must I sell it to gain 25%?

3. I buy allspice at the rate of $9 for 60 lb. I sell it at a profit of $33\frac{1}{3}\%$. At what price per pound do I sell it?

4. I pay $40.32 a gross for 1-lb. boxes of pepper, and I sell it at 35 ct. a box. What per cent do I gain?

5. Some packages of oatmeal crackers that cost me $10.80 a gross are damaged, and I have to sell them at 6 ct. a package. What per cent do I lose?

6. I buy 500 lb. of California prunes for $45, and sell 400 lb. at a profit of $33\frac{1}{3}\%$, and the rest at a loss of 50%. Do I gain or lose, and how much?

Some Problems of the Dry Goods Merchant

ORAL EXERCISE

1. Silk that costs me 90 ct. a yard must be sold at what price to allow me 30% profit?

2. If I sell at $1.75 a yard lace that cost me $1.50 a yard, what per cent do I gain?

3. Some flannel that cost me 48 ct. a yard I am selling at 60 ct. a yard. What per cent do I gain?

4. Gloves that cost me $160 per 100 pairs I am selling at $1.80 a pair. What per cent do I gain?

5. If I pay $500 for 100 pieces of ribbon, 10 yd. in a piece, and sell it at 65 ct. a yard, what per cent do I gain?

WRITTEN EXERCISE

1. Some buttons that cost me $1.08 a gross I am selling at 12 ct. a dozen. What per cent do I gain?

2. I buy 525 yd. of lawn for $94.50, and sell it at a profit of $33\frac{1}{3}\%$. At what price per yard do I sell it?

3. I buy hooks and eyes at $10 a gross of papers, and sell them at 9 ct. a paper. What per cent do I gain?

4. I pay $288 for 480 yd. of taffeta silk, and I sell it at 75 ct. a yard. How much do I gain, and what per cent?

5. In order to close out some gloves that cost me $9.60 a dozen pairs, I am selling them at 75 ct. a pair. What per cent do I lose?

6. I buy at an auction sale 252 yd. of embroidery for $20.16. I mark it 25% above cost, but because there is no sale for it I take off 10% from this price and thus sell it. Do I gain or lose, and how much?

281. Discounts with a profit. Dealers often allow a discount on goods, but usually so as still to allow a profit.

For example, if a hat cost a dealer \$1.50, and it is marked \$2, but is sold for \$1.75, the dealer makes 25 ct.

The 25 ct. profit is $16\frac{2}{3}\%$ of the \$1.50 of cost, and the 25 ct. discount is $12\frac{1}{2}\%$ of the \$2 marked price.

282. Illustrative problem. How much above cost must a man mark goods in order to take off 25% from the marked price and still make a profit of 20%?

He sells for marked price − 25% of it, or 75% of it.

He also sells for cost + 20% of cost, or 1.20 of cost.

Therefore 75% of marked price = 1.20 of cost,

and therefore " " = 1.20 of cost ÷ 75% = 1.60 of cost.

That is, the marked price is 160% of cost, or 60% above cost.

Some prefer to solve the problem like this:

Let \$1 = the cost.

Then \$1.20 = selling price, which is 25% below the marked price.

Therefore 75% of marked price = \$1.20,

and 1% " " " = \$1.20 ÷ 75 = \$0.016,

and 100% " " " = 100 × \$0.016 = \$1.60.

WRITTEN EXERCISE

How must a man mark goods in order to take off the rate named, and make the profit or loss indicated?

Discount	*Profit*	*Discount*	*Profit*
1. 20%,	20%.	2. 25%,	25%.
3. 30%,	30%.	4. 30%,	25%.
5. $33\frac{1}{3}\%$,	$33\frac{1}{3}\%$.	6. 50%,	10%.
7. 0%,	$16\frac{2}{3}\%$.	8. $22\frac{1}{2}\%$,	0%.

9. Discount 30% and *loss* 9%; 50% and *loss* 10%.

10. Discount 40% and *loss* 10%; $33\frac{1}{3}\%$ and *loss* $33\frac{1}{3}\%$.

COMMISSION

ORAL EXERCISE

1. How much is \$200 plus 10% of itself? plus 1%?

2. How much is \$3 plus 30% of itself? plus 6%?

3. How much is \$150 plus 1% of itself? plus $33\frac{1}{3}$%?

4. If you bought \$15 worth of eggs for a dealer, receiving 5% for your work, what would you make?

5. If you bought \$2 worth of eggs for a dealer, and he paid you 10% more than cost, what would you make?

283. Commission. The pay received by one person for buying or selling for another is often reckoned at a certain per cent of the price, and is called a *commission.*

For example, if a man buys \$1000 worth of potatoes for another man, at 5% commission, he would receive 5% of \$1000, or \$50.

WRITTEN EXERCISE

1. Required the commission on an \$850 sale, at $2\frac{1}{2}$%.

2. Required the commission on a \$1200 sale, at $5\frac{1}{2}$%.

3. A grocer buys \$300 worth of apples, but loses 10% of them by decay. The rest he sells for 23% more than they cost. How much does he gain?

4. A man sells at an auction \$292 worth of property for another man, and receives 10% commission. How much does he get? How much is left for the other man?

5. A dealer in fruit in a city sends a man to the fruit region to buy for him. He pays him 2% commission. The buyer purchases \$2500 worth in one place, \$3200 worth in another, and \$4100 worth in another. What is his total commission?

ORAL EXERCISE

1. How much is $2\frac{1}{2}\%$ commission on \$400? on \$8000?
2. How much is $3\frac{1}{3}\%$ commission on \$600? on \$9000?
3. How much is $12\frac{1}{2}\%$ commission on \$240? on \$6400?
4. How much is $16\frac{2}{3}\%$ commission on \$366? on \$6240?

284. Commission on sales. In large city stores salesmen often receive a salary, and also a commission on the amount of their sales, to induce them to sell all the goods possible.

WRITTEN EXERCISE

1. A clerk receives \$4 a week, and 2% commission on goods sold. If he sells \$372 worth of goods in a week, what is his income for the week?

2. A clerk receives \$10 a week, and $1\frac{1}{2}\%$ commission on goods sold. If he sells \$492 worth of goods in a week, how much is his income for the week?

3. A city grocer sends a man out to his customers' residences every morning to take orders. If the man receives 8% commission on all orders, and if he takes orders to the amount of \$9800 a year, how much does he earn?

4. A saleswoman is offered her choice of \$8.50 a week, \$5 a week with 1% on all sales, or 4% on all sales. She takes the last plan, and her sales for a year average \$265 a week. How much does she gain over each of the other two plans?

5. In a large dry goods store in San Francisco a woman is employed to "shop" for out-of-town customers. She receives from the dealers a commission of 5% on all purchases. Her purchases amount to \$422 a month, on the average. How much is her commission during the year?

ORAL EXERCISE

1. At 30%, what is the commission on a $120 sale?

2. At 10% commission, what does a collection agent receive for collecting a debt of $95.70?

3. An agent sells 100 school desks at $3.75 each, and receives a commission of 10%. What does he receive?

4. A commercial traveler, whose sales average $52,000 a year, receives a commission of 5%. How much is this?

WRITTEN EXERCISE

1. What is an architect's commission on a house costing $9472, at 5%?

2. What is an auctioneer's commission for selling $358 worth of furniture, at 6%?

3. A real estate dealer sells 168 acres of land at $40 an acre. How much is his commission at $2\frac{1}{2}$%?

4. A lawyer charged $40 for collecting a debt of $750. This was the same as what per cent commission?

5. An agent sells our school $65 worth of maps, and his firm pays him $7\frac{1}{2}$% commission. How much is this?

6. A farmer sends some butter and eggs to the city, and they are sold for $72.50. He is charged 8% by his agent. What is the net amount received by the farmer?

7. A real estate dealer rents 21 city apartments for the owner, at $1500 each, and charges a commission of $2\frac{1}{2}$%. How much is the commission?

8. A salesman's sales in a department store in a city amounted, during the six days of one week, to $257.70, $103.10, $175.02, $132.30, $302.25, $50.63. He receives $2\frac{1}{2}$% on his sales. How much did he receive that week?

INTEREST

ORAL EXERCISE

1. If one man borrows money of another, how does he pay him for the loan, besides returning the money?

2. If you put money in a savings bank and leave it 6 mo. or 1 yr., will they pay you back only what you put in?

3. If any one where you live wished to borrow $100 for a year, what interest would he have to pay?

285. Interest. Money paid for the use of money is called *interest.*

For example, if I borrow $50 for a year, and pay $3 for the use of it, the $3 is interest.

286. Reckoning interest. Interest is reckoned as a certain per cent of the sum borrowed.

If the interest is 6% of the sum borrowed, and the sum is $200, the interest for 1 yr. is 6% of $200, or $12.

287. Principal. The sum borrowed is called the *principal.*

288. Rate. The per cent of interest is called the *rate.*

That is, if the interest is 6% of the principal, the rate is 6%.

WRITTEN EXERCISE

1. What is the interest on $375 for 1 yr. at 6%? at 4%?

2. What is the interest on $950 for 1 yr. at 5%? at 3%?

3. What is the interest on $235 for 1 yr. at 4%? at 5%?

4. If you had $27 in a bank that paid 3% a year, how much interest would you receive for a year?

5. How much are the interest and principal together of $475 at the end of 1 yr., the rate of interest being 6%?

ORAL EXERCISE

Find the interest for 1 yr. on the sums given in Exs. 1–12, at the rates specified:

1. \$100 at 4%. 2. \$300 at 6%. 3. \$60 at 3%.
4. \$500 at 5%. 5. \$900 at 3%. 6. \$50 at 4%.
7. \$400 at 6%. 8. \$1200 at 3%. 9. \$70 at 5%.
10. \$1000 at 4%. 11. \$3000 at 4%. 12. \$90 at 6%.

Find the interest on the following for 6 mo., first finding the interest for 1 yr., and then dividing by 2:

13. \$100 at 6%. 14. \$200 at 5%. 15. \$40 at 6%.
16. \$400 at 3%. 17. \$250 at 4%. 18. \$50 at 4%.
19. \$600 at 5%. 20. \$500 at 5%. 21. \$70 at 3%.
22. \$800 at 4%. 23. \$1000 at 3%. 24. \$80 at 5%.

WRITTEN EXERCISE

Find the interest for 1 yr. on the sums given in Exs. 1–15, at the rates specified:

1. \$652 at 4%. 2. \$375 at 5%. 3. \$27 at $3\frac{1}{2}$%.
4. \$325 at 3%. 5. \$433$\frac{1}{3}$ at 3%. 6. \$42 at $4\frac{1}{2}$%.
7. \$275 at 4%. 8. \$450.25 at 4%. 9. \$75 at $2\frac{1}{2}$%.
10. \$725 at 6%. 11. \$275.20 at 5%. 12. \$84 at $4\frac{1}{2}$%.
13. \$535 at 5%. 14. \$625.20 at 5%. 15. \$72 at $3\frac{1}{3}$%.

Find the interest on the following for 6 mo.:

16. \$82 at 5%. 17. \$175 at 4%. 18. \$350 at $2\frac{1}{2}$%.
19. \$75 at 6%. 20. \$375 at 5%. 21. \$675 at $3\frac{1}{3}$%.
22. \$45 at 4%. 23. \$450 at 6%. 24. \$450 at $4\frac{1}{2}$%.
25. \$35 at 2%. 26. \$325 at 3%. 27. \$864 at $3\frac{1}{3}$%.

CHECKS AND RECEIPTS

289. Bank checks. When a man pays money he often does it by an order on the bank where he keeps his money.

An order on a bank calling for the payment of money is called a *check.*

No. *1476* San Francisco, *January 7,* 19.....

The Fifteenth National Bank of San Francisco

Pay to the order of ..... *John Roberts* .....$*27.50*

Twenty-seven and $\frac{50}{100}$ ~~~~~~~~~~~~~~~~ Dollars

Charles Frederieks

290. Indorsing. This check is payable to the *order* of John Roberts. He must therefore order it paid by writing his name across the back, *indorsing* the check.

WRITTEN EXERCISE

1. Make out three checks payable to the order of members of the class. Hand them the checks for indorsement.

2. John borrowed $4 six months ago at 6% a year. Make out a check to pay the note and interest.

3. A man deposited $125 in a bank. He drew out by checks the following amounts: $14.62, $3.75, $21.50, $35, $2.17. How much had he left? Write the checks.

4. A man borrowed $75 a year ago at 6% interest, and $130 six months ago at 5% interest. How large a check must he draw to-day to pay all the principal and interest?

291. Receipts. When I owe you any money and pay you, I ask for a *receipt*, so there may be no misunderstanding about it. A receipt for money paid is as follows:

New Orleans, *February 2*, 19.....

Received from ..... *Frank Franklin* .....

Fourteen and $\frac{75}{100}$ ~~~~~~~~~~~~~~~~~~~~ Dollars

for *rent to date*

$14.75 *Joseph Joslin*

WRITTEN EXERCISE

1. Robert rents a house of you at $18.50 a month. Write a receipt for $3\frac{1}{2}$ months' rent to date.

2. I borrowed $125 a year ago at 6%. I have paid you the principal and interest to-day. Write a receipt for me.

3. I borrowed $65 a year ago at 5%. I have paid you the principal and interest to-day. Write a receipt for me.

4. Write receipts for the principal and interest on the following, the money having been borrowed just a year ago: $140 at 4%; $420 at 6%; $375 at 5%.

5. Mary has bought these things at your store: $2\frac{1}{2}$ yd. ribbon @ 20 ct., 2 pr. gloves @ $1.25, 3 collars @ 15 ct. Make out a bill for the goods, and receipt it.

6. Write receipts for the principal and interest on the following, the money having been borrowed six months ago: $250, 6%; $300, 5%; $150, 4%; $375, 6%.

7. Write a receipt covering the following payments made to-day: $372 and interest for 6 mo. at 5%; $280 and interest for a year at 6%; $1000 and interest for $1\frac{1}{2}$ yr. at 5%.

DENOMINATE NUMBERS REVIEWED

ORAL EXERCISE

1. Repeat the tables of dry measure and liquid measure.

2. Repeat the tables of length, square measure, and cubic measure.

3. How many things make a dozen? a gross? a great gross? a score? a quire? When are these used?

4. Draw on the blackboard the different kinds of triangles studied by you and name them. (See page 152.)

5. If a rectangle is 60 ft. long and 40 ft. wide, what is the area? Explain your work. (See page 154.)

6. If a triangle has a base 40 ft. and a height 40 ft., what is the area? Explain your work. (See page 154.)

7. If a parallelogram has a base 20 ft. and a height 15 ft., what is the area? Explain your work. (See page 155.)

8. If a rectangular solid is 7 ft. long, 6 ft. wide, and 2 ft. high, what is the volume? Explain your work. (See page 156.)

WRITTEN EXERCISE

1. What is the volume of a room 16 ft. by 18 ft., and 10 ft. high? $22\frac{1}{2}$ ft. by $24\frac{1}{2}$ ft., and 9 ft. 10 in. high?

2. What is the area of the floors of the same rooms?

3. What is the area of the walls of the same rooms, no allowance being made for openings?

4. Some 4-ft. wood is piled 5 ft. high. The pile is 2 rd. long. How many cords are there? Suppose it is 7 rd. long?

5. How many gallons in a tank 12 ft. 3 in. long, 4 ft. 8 in. wide, and 4 ft. 2 in. deep, allowing 231 cu. in. to a gallon? Suppose it is $13\frac{1}{2}$ ft. $\times$ $6\frac{1}{3}$ ft. $\times$ $3\frac{1}{6}$ ft.?

ORAL EXERCISE

1. A bin holds 385 cu. ft. of coal. At 35 cu. ft. to the ton, how many tons does it hold? how many pounds?

2. A brick wall contains 300 cu. ft. At 22 bricks to the cubic foot, how many bricks are in the wall?

3. A bin is 4 ft. by 3 ft., and 1 ft. deep. How many cubic feet does it contain? At $1\frac{1}{4}$ cu. ft. to the bushel, how many bushels? How many if it were 5 ft. deep?

292. Certain equivalents. The following, already referred to, are often considered important enough to be memorized:

2150.42 cu. in. = 1 bushel.
231 cu. in. = 1 gallon. (Page 157.)
7000 grains = 1 avoirdupois pound.
5760 grains = 1 troy pound. (Page 92.)

293. Teachers are advised to require memorizing the above equivalents only when their course of study so directs. In certain parts of the country other measures may be demanded by local conditions, as in those states which still make use of the Mexican measures: 1 vara = $33\frac{1}{3}$ in., 1900.8 varas = 1 mi., 5645.376 square varas = 1 acre. Such local demands should always be met.

WRITTEN EXERCISE

1. How many cubic inches in 1645 bu.? in $2743\frac{1}{3}$ bu.?

2. A grain elevator has a capacity of 812,000 bu. At $1\frac{1}{4}$ cu. ft. per bushel, what is its capacity in cubic feet?

3. A grain elevator has a capacity of 1,500,000 cu. ft. It charges $1\frac{1}{4}$ ct. per bushel for storing grain for the first 10 da. and $\frac{1}{2}$ ct. per bushel for each succeeding period of 10 da. If it is $66\frac{2}{3}\%$ full for exactly 30 da., what is its income for that time? (Allow $1\frac{1}{4}$ cu. ft. to a bushel.)

4. The avoirdupois pound is what per cent heavier than the troy pound? The troy pound is what per cent lighter than the avoirdupois?

5. Bran weighs 20 lb. to the bushel, and oats are 60% heavier. Timothy seed is $40\frac{5}{8}\%$ heavier than oats. How much does timothy seed weigh per bushel?

6. A bale of cotton averages 477 lb. in the United States, 719 lb. in Egypt, and 220 lb. in Brazil. The Egyptian bale is what per cent heavier than ours? The Brazilian is what per cent lighter than ours?

7. A barrel of flour weighs 196 lb., and a barrel of beef or pork weighs 200 lb. A barrel of flour is what per cent lighter than a barrel of beef? A barrel of beef is what per cent heavier than a barrel of flour?

8. Corn meal, barley, and buckwheat weigh 48 lb. to the bushel, while wheat, clover seed, peas, and white beans weigh 60 lb. Wheat is what per cent heavier than barley? Barley is what per cent lighter than wheat?

9. A rectangular tank is $6\frac{1}{4}' \times 4\frac{1}{2}' \times 2\frac{1}{2}'$. How many gallons of water does it hold?

10. A wagon box is 6 ft. long and 3 ft. 6 in. wide. How deep must it be to hold 1 cu. yd.?

11. At $\$1.12\frac{1}{2}$ a load (cubic yard), how long a ditch $1\frac{1}{2}$ ft. wide and 3 ft. deep can be excavated for $\$13.50$?

12. A 240-gal. tank is to be placed in an attic 4 ft. high. What length and width may the tank have?

13. What will it cost to plaster a room $16' \times 14'$, $9'6''$ high, at 18 ct. a square yard, allowing 96 sq. ft. for doors, windows, and baseboard?

14. The sum of all the edges of a certain cube is 138 in. What is the volume of the cube? the area of each face?

Add in Exs. 1–6:

1. 127 ft. 4 in.
   286 3
   491 7
   302 5
   27 2

2. 329 ft. 7 in.
   206 8
   129 11
   43 5
   209 10

3. 635 ft. 7 in.
   82 6
   39 10
   241 11
   43 8

4. 278 yd. 10 in.
   63 13
   49 17
   326 5
   42

5. 27 gal. 1 qt.
   92 3
   167 1
   43 2
   286 3

6. 427 bu. 1 pk.
   63
   429 3
   28 2
   65 3

7. 480 ft. 2 in. — 197 ft. 10 in.
8. $9\frac{1}{2}$ lb. + 9 oz.
9. 293 lb. 5 oz. — 169 lb. 12 oz.
10. $3\frac{2}{3}$ ft. + 8 in.
11. 402 yd. 5 in. — 327 yd. 12 in.
12. $8\frac{2}{3}$ rd. + 5 yd.
13. 572 bu. 1 pk. — 296 bu. 3 pk.
14. $7\frac{1}{2}$ bu. + 3 pk.
15. 235 gal. 1 qt. — 186 gal. 3 qt.
16. $6\frac{2}{3}$ mi. + 90 rd.
17. 42 times 102 lb. 1 oz.
18. 27 times 63 lb. 2 oz.
19. 32 " 249 lb. 2 oz.
20. 16 " 78 lb. 9 oz.
21. 75 " 243 ft. 2 in.
22. 96 " 329 ft. 7 in.
23. 35 " 127 yd. 1 ft.
24. 82 " 306 yd. 2 ft.
25. 48 " 206 gal. 1 qt.
26. 32 " 142 bu. 1 pk.
27. 87 lb. 8 oz. ÷ 4.
28. 95 lb. 2 oz. ÷ 3.
29. 648 gal. ÷ 4 gal.
30. 981 gal. ÷ 9 gal.
31. 302 lb. 8 oz. ÷ 2.
32. 428 lb. 6 oz. ÷ 4.
33. 125 ft. 10 in. ÷ 5.
34. 425 ft. 5 in. ÷ 25.
35. 275 lb. 15 oz. ÷ 25.
36. 625 lb. 5 oz. ÷ 25.
37. 13,328 bu. ÷ 28 bu.
38. 38,208 bu. ÷ 36 bu.
39. 20,720 mi. ÷ 56 mi.
40. 41,440 yd. ÷ 56 yd.

Find the difference between the following dates:

41. April 6, 1905, May 1, 1907.

42. July 4, 1909, May 1, 1911.

43. May 24, 1905, April 2, 1910.

44. July 4, 1776, January 1, 1920.

45. September 6, 1906, April 4, 1908.

46. June 7, 1907, September 2, 1909.

47. September 4, 1906, April 2, 1908.

48. August 2, 1908, February 1, 1910.

49. If a watch gains 13 sec. a day, how much will it gain in January?

50. July 4, 1912, falls on Thursday. On what day does April 1 of that year fall?

51. How many hours from 7 P.M., February 28, to 9 A.M., March 1, in a leap year?

52. If a watch loses 17 sec. a day, and is set correctly at noon July 1, what time will it indicate at noon August 1?

53. A piece of oak $12' \times 9'' \times 6''$ weighs how many times as much as one $8'\ 3'' \times 8'' \times 4''$?

54. A man can row across a lake, $2\frac{3}{4}$ mi., in 28 min. At this rate, how long will it take him to row the length of the lake, $6\frac{1}{4}$ mi.?

55. How many days from April 17 to July 16? from January 23 to September 10? from August 27 to December 5? (Give the exact number.)

56. Make a diagram of the parts of a room 18 ft. long, 15 ft. wide, and 9 ft. 6 in. high, that are usually plastered. Not allowing for openings and baseboard, how many square yards of plaster are needed?

II. SIMPLE INTEREST. RATIO AND PROPORTION

SIMPLE INTEREST

294. Finding interest. We have found the interest on money, at various rates, for one year, and also for half of a year. We shall now find it for other lengths of time.

ORAL EXERCISE

1. If the interest for 1 yr. is \$16, how much is it for $\frac{1}{2}$ yr.? for $\frac{1}{4}$ yr.? for $\frac{1}{8}$ yr.?

2. If the interest for 1 yr. is \$24, how much is it for 1 mo.? for 3 mo.? for 11 mo.?

3. If the interest for 1 yr. is \$8, how much is it for 3 yr.? for 5 yr.? for 7 yr.? for 8 yr.?

4. At 6%, how much is the interest on \$500 for 1 yr.? for 4 mo.? for 2 yr.? for 6 mo.? for 3 yr.?

WRITTEN EXERCISE

1. What is the interest on \$436, at 6%, for 1 yr.? for 6 mo.? for 4 mo.? for 8 mo.? for 10 mo.?

2. What is the interest on \$360, at 5%, for 1 yr.? for 6 mo.? for 10 mo.? for 1 mo.? for 9 mo.?

3. If the interest on a certain sum is \$93.84 for 1 yr., how much is it for 3 mo.? for 6 mo.? for 9 mo.?

4. If the interest on a certain sum is \$36.48 for 1 yr., how much is it for 1 mo.? for 5 mo.? for 10 mo.?

5. What is the interest on \$540, at 4%, for 1 yr.? for 1 mo.? for $\frac{1}{2}$ mo.? for 5 da.? for 1 da.? for 1 wk.?

6. What is the interest on \$175.50, at 4%, for 1 yr.? for 6 mo.? for 4 mo.? for 3 mo.? for 1 mo.? for 10 da.?

ORAL EXERCISE

What is the interest on the following sums?

1. \$100, 5%, 1 yr.
2. \$300, 6%, 1 yr.
3. \$500, 4%, 1 yr.
4. \$800, 5%, 1 yr.
5. \$200, 6%, 2 yr.
6. \$500, 5%, 2 yr.
7. \$120, 5%, 1 mo.
8. \$300, 5%, 6 mo.
9. \$100, 6%, 6 mo.
10. \$600, 3%, 6 mo.
11. \$700, 4%, 6 mo.
12. \$500, 3%, 4 mo.
13. \$200, 6%, 4 mo.
14. \$400, 3%, 8 mo.
15. \$300, 4%, 5 mo.
16. \$200, $3\frac{1}{2}$%, 6 mo.
17. \$100, $4\frac{1}{2}$%, 6 mo.
18. \$240, 5%, 11 mo.
19. \$200, 4%, 2 yr. 6 mo.
20. \$300, 4%, 2 yr. 1 mo.

295. Interest for years and months. We have seen that to find the interest for 2 yr. 6 mo., we find the interest for 1 yr., and multiply by $2\frac{1}{2}$. That is,

Find the interest for 1 year, and multiply by the number of years.

Thus, to find the interest on \$125 for 2 yr. 3 mo. at 5%.

1. 5% of \$125 = \$6.25, int. for 1 yr.
2. $2\frac{1}{4}$ times \$6.25 = \$14.06, " " 2 yr. 3 mo.

\$125
.05
\$6.25
$2\frac{1}{4}$
1 56$\frac{1}{4}$
12 50
\$14.06

WRITTEN EXERCISE

1. \$250, 6%, 6 mo.
2. \$300, $5\frac{1}{2}$%, 8 mo.
3. \$8200, $5\frac{1}{2}$%, $2\frac{1}{2}$ yr.
4. \$625, 4%, 3 yr. 6 mo.
5. \$35, 6%, 1 yr. 6 mo.
6. \$45.50, 4%, 2 yr. 3 mo.
7. \$750, 5%, 2 yr. 4 mo.
8. \$75.50, 6%, 2 yr. 6 mo.
9. \$225, 6%, 3 yr. 6 mo.
10. \$4250, $4\frac{1}{2}$%, 1 yr. 6 mo.

296. Interest for years, months, and days. Suppose one man borrows from another $400 on September 10, 1904, at 6%. What will the interest amount to on August 7, 1907?

1. 6% of $400 = $24, int. for 1 yr.
2. 2 times $24 = $48, " " 2 yr.
3. $\frac{1}{12}$ of $24 = $20, " " 10 mo.
4. $\frac{27}{30}$ of $\frac{1}{12}$ of $24 = $1.80, " " 27 da., or $\frac{27}{30}$ of $\frac{1}{12}$ of a year.
5. Total int. = $69.80, " " 2 yr. 10 mo. 27 da.

1907	8 mo.	7 da.
1904	9	10
2	10	27

WRITTEN EXERCISE

Find the interest as directed in Exs. 1–12:

1. $325, 1 yr. 8 mo. 5 da., 5%.
2. $300, 1 yr. 4 mo. 15 da., 6%.
3. $200, 3 yr. 6 mo. 10 da., 6%.
4. $150, 1 yr. 3 mo. 20 da., 5%.
5. $250, 2 yr. 3 mo. 10 da., 4%.
6. $475, 3 yr. 6 mo. 15 da., 6%.
7. $850, 2 yr. 7 mo. 6 da., 4½%.
8. $2500, 3 yr. 3 mo. 3 da., 3%.
9. $775, 1 yr. 6 mo. 24 da., 3½%.
10. $75, 4%, June 9, 1906 to May 2, 1908.
11. $225, 5%, July 8, 1908 to April 3, 1910.
12. $2000, 3%, May 7, 1907 to March 4, 1909.
13. If I borrow $50, at 6%, on February 8 of this year, how much will be due on May 2 of next year?
14. If Mr. Roberts borrows $350 from Mr. Bugbee, at 4%, on September 25 of this year, how much principal and interest will be due Mr. Bugbee on April 10 of next year?

297. Promissory note. A paper signed by a borrower, agreeing to repay money, is called a *promissory note.*

298. Face of note. The sum borrowed is called the *principal,* or, if a note is given, the *face of the note.*

299. Amount of note. The sum of the principal (or face) and interest is called the *amount.*

Notes should state value received, the date, face, rate, person to whom payable, and time of payment.

$75.^{00}$ BUFFALO, N.Y., *February 7,* 1907

Six months after date, for value received, I promise to pay to ..... *John Johnson* ..... or order, *Seventy-five and* $\frac{00}{100}$ ..... Dollars, with interest at 5 %

Frank Francis

300. Rate of interest. The rate of interest varies in different parts of the country. The United States has to pay about 2%. Savings banks pay 3% or 4%. People borrowing money in cities, on good security, pay from 4% to 6%.

WRITTEN EXERCISE

1. Compute the amount of the above note.

2. Write a promissory note, signed by X, payable to Y, for $25, due in 1 yr., at 6%. Find the amount.

3. Write a promissory note as before, for $250, due in 10 mo., at 5%. Find the amount.

4. Write a promissory note as before, for $400, due in 1 yr. 6 mo., at 4%. Find the amount.

301. Bank discount. When a man borrows from a bank he pays the interest in advance. Interest is not mentioned in the note, because it has already been paid.

Interest paid in advance on a note is called *discount.*

Teachers should call attention to the fact that the same word is used for bank and commercial (trade) discount, explaining that the process is the same in both cases, — finding some per cent of a number.

The technicalities of interest-bearing notes are explained later. In all work at present call 30 days a month.

302. Proceeds. The face of a note less the discount is called the *proceeds.*

303. Illustrative problem. What is the discount on a \$200 note for 6 mo. at 5%? What are the proceeds?

1. The discount (interest) for 1 yr. is 5% of \$200, or \$10.
2. For $\frac{1}{2}$ yr. it is $\frac{1}{2}$ of \$10, or \$5, discount.
3. \$200 − \$5 = \$195, proceeds.

WRITTEN EXERCISE

Find the discount and proceeds:

1. \$75, 1 mo., 6%.
2. \$90, 30 da., 6%.
3. \$250, 3 mo., 5%.
4. \$275, 3 mo., 5%.
5. \$425, 4 mo., 5%.
6. \$300, 90 da., 6%.
7. \$375, 6 mo., 6%.
8. \$400, 90 da., 6%.
9. \$150, 45 da., 6%.
10. \$400, 1 mo., $5\frac{1}{2}$%.
11. \$200, 60 da., 6%.
12. \$175, 2 mo., $4\frac{1}{2}$%.
13. \$300, 4 mo., $5\frac{1}{2}$%.
14. \$500, 2 mo., $3\frac{1}{2}$%.
15. \$2975, 63 da., 5%.
16. \$4250, 93 da., 6%.
17. \$1250, 10 da., 6%.
18. \$2500, 15 da., 6%.
19. \$1500, 20 da., 6%.
20. \$1250, 45 da., 5%.
21. \$2750, 30 da., $5\frac{1}{2}$%.
22. \$3520, 90 da., $4\frac{1}{2}$%.

304. Payee of a note. The person named in the note as the one to whom it is payable is called the *payee*.

305. Maker of a note. The one who signs the note is called the *maker*.

306. Indorsing notes. If the payee wishes to sell the note, he must, when it is payable to himself or *order*, indorse it.

A note is indorsed by the payee writing his name across the back. The indorser must pay the note if the maker does not.

If he wishes it payable to the order of John Roberts, he writes across the back, "Pay to the order of John Roberts," and signs his name.

A note payable to John Johnson or *bearer* may be sold without indorsement. Such notes are not common.

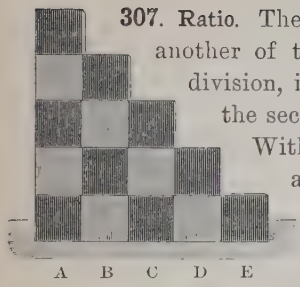
WRITTEN EXERCISE

1. F. H. Holmes borrows \$350, at 6%, for 1 yr., from A. B. Seel. He gives a note payable to Mr. Seel or order. Mr. Seel sells the note to B. C. Dean. Make out the note, indorse it, and find the amount due at the end of the year.

Make out and indorse, payable to the order of the buyer, the following notes, and find the amount due at the end of the time stated:

	Maker	*Payee*	*Buyer*	*Face*	*Rate*	*Time*
2.	J. R. Roberts	A. N. Cole	P. R. Jones	\$375	6%	6 mo.
3.	M. L. King	A. R. Owen	E. F. Pease	\$750	5%	2 yr.
4.	A. K. Jewett	S. M. Wood	E. L. Brown	\$75	6%	10 mo.
5.	B. S. Stevens	O. N. House	B. R. Lodge	\$250	5%	3 yr. 6 mo.
6.	O. M. Richards	D. E. Dodge	C. King	\$125	6%	1 yr. 2 mo.
7.	A. R. Castle	A. J. Jackson	G. F. Ray	\$250	$5\frac{1}{2}$%	2 yr. 4 mo.
8.	F. E. Taylor	A. B. Baker	G. F. Coe	\$350	$4\frac{1}{2}$%	1 yr. 3 mo.
9.	F. E. Glover	M. T. Thorn	F. Taylor	\$250	6%	4 yr. 1 mo.

RATIO



307. Ratio. The relation of one quantity to another of the same kind, as expressed by division, is called the *ratio* of the first to the second.

Without using the name, we studied about ratio in studying fractions. The ratio of \$2 to \$4 is $\frac{2}{4}$ or $\frac{1}{2}$; the ratio of 6 ft. to 5 ft. is $\frac{6}{5}$. In this picture the ratio of E to A is $\frac{1}{5}$; of D to C, $\frac{2}{3}$; of C to B, $\frac{3}{4}$; of D to B, $\frac{1}{2}$.

ORAL EXERCISE

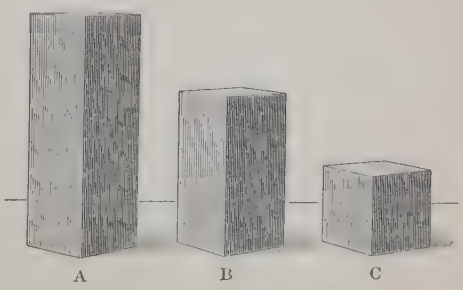
1. In the picture above, what is the ratio of B to A?

2. Also, what is the ratio of A to B? of E to C?

3. In the picture above, if A represents \$15, what values must be given to B, C, D, and E?

4. If each cube weighs 6 oz., give the ratios of A to B; B to C; C to D; E to D.

5. In the picture below, what is the ratio of C to B? of C to A? of B to A?



6. What is the ratio of A to C? of B to C? of A to B + C?

7. What is the ratio of any two equal numbers? of the lengths of $16\frac{1}{2}$ ft. and 1 rd.? of 1 mi. and 5280 ft.?

308. Writing ratios. A ratio may be written like a fraction, $\frac{2 \text{ ft.}}{3 \text{ ft.}}$, or with a sign of ratio (:), as 2 ft. : 3 ft.

We think of the ratio of 20 ft. to 4 ft. as $\frac{20 \text{ ft.}}{4 \text{ ft.}}$, 20 ft. : 4 ft., $\frac{20}{4}$, 20 : 4, or simply as 5. The word *ratio* is used for all.

309. Antecedent. The first term is called the *antecedent.*

310. Consequent. The second term is called the *consequent.*

$$\frac{2 \text{ ft.}}{3 \text{ ft.}} = \frac{2}{3} = \frac{\text{numerator}}{\text{denominator}} = \frac{\text{antecedent}}{\text{consequent}}.$$

311. Ratios always abstract. Because any number divided by a like number, as feet by feet, has an abstract quotient,

A ratio is always abstract, and its terms may be written as abstract numbers.

Teachers should use the fractional form at first, because the pupils are already familiar with it.

ORAL EXERCISE

Find the value of x *in Exs. 1–12:*

1. $\frac{x}{2} = 10.$
2. $\frac{x}{3} = 3.$
3. $\frac{x}{4} = 2.$
4. $\frac{x}{5} = 3.$
5. $\frac{x}{10} = 1.$
6. $\frac{x}{10} = \frac{1}{2}.$
7. $\frac{x}{7} = \frac{3}{7}.$
8. $\frac{x}{6} = \frac{2}{3}.$
9. $\frac{x}{5} = \frac{4}{5}.$
10. $\frac{2}{x} = \frac{1}{3}.$
11. $\frac{20}{x} = .4.$
12. $\frac{10}{x} = 1.$

13. What is the number whose ratio to 5 is 6?

14. What is the number to which 75 has the ratio 3?

15. The ratio of the height of a certain door to its width is $3\frac{1}{2}$. It is 7 ft. high. How wide is it?

16. The ratio of Fred's father's height to Fred's height is $\frac{3}{2}$. Fred's father is 6 ft. tall. How tall is Fred?

ORAL EXERCISE

1. Separate 9 in the ratio of 2 to 1.

2. What is the ratio of the number of boys to the number of girls in your class?

3. What is the ratio of the time you are in school to the whole 24 hours of the day?

4. What is the ratio of the length of summer, 3 months, to the whole year? to the rest of the year?

5. There were 6 cloudy days in September, the rest being clear. What was the ratio of cloudy to clear days?

6. In playing ringtoss, Will has a score of 15 and Mollie has one of 18. What is the ratio of Will's score to Mollie's?

312. Separating in a given ratio. Separate 25 ct. in the ratio of 2 to 3.

1. Since there are 2 parts in one group and 3 in the other, there are 5 parts in all.
2. Since one group has $\frac{2}{5}$, it has $\frac{2}{5}$ of 25 ct., or 10 ct.
3. The other has $\frac{3}{5}$ of 25 ct., or 15 ct.

Checks. 10 ct. + 15 ct. = 25 ct.; 10 : 15 = 2 : 3.

WRITTEN EXERCISE

1. Divide \$100 in the ratio of 4 : 1; of 7 : 3; of 1 : 9.

2. In a year having 165 cloudy and 200 clear days, what is the ratio of cloudy days to clear?

3. The ratio of clear to cloudy days in November was 3 : 2. How many were there of each?

4. Taking the area of the United States as 3,600,000 sq. mi., and the ratio of farm land to other land as 7 to 18, find the area of the farm land.

Miscellaneous Problems of the Farmer

WRITTEN EXERCISE

1. A farmer puts 5 acres into celery, setting out 20,000 plants to the acre. The yield being 1500 doz. heads to the acre, what is the ratio of the plants matured to the others?

2. He pays $95 an acre for seeds, fertilizers, labor, and other expenses, and sells the crop at 15 ct. a dozen heads. What is his profit on the 5 acres?

3. Another farmer tries setting out 30,000 plants to the acre, but only 80% mature, and these are so small that he has to put 16 in a bunch to sell for a dozen, and then gets only 14 ct. a bunch. His expenses are $100 an acre. At this rate, what is his profit on 5 acres?

4. A farmer has a 30-acre meadow yielding $1\frac{1}{2}$ tons of hay to the acre. If by spending $300 a year for fertilizers, he can bring the yield to 4 tons to the acre, how much more will he make a year, hay being worth $8 a ton?

5. A farmer reads that a good mixture of seed for his meadow is, by weight, as follows: timothy 40%, redtop 40%, red clover making up the rest. At 40 lb. of seed to the acre, how many pounds of each should he sow?

6. The following is, by weight, a good mixture of seed for a pasture: Kentucky blue grass 25%, white clover $12\frac{1}{2}$%, perennial rye $28\frac{1}{8}$%, red fescue $9\frac{3}{8}$%, redtop 25%. At 32 lb. to the acre, how many pounds of each are used?

7. A cow weighing 1000 lb. consumes the equivalent of $4\frac{1}{4}$ tons (2000 lb. to the ton) of dry fodder a year; a 100-lb. sheep, 770 lb.; every ton of live pork, 12 tons; and every ton of live horseflesh, 8.4 tons. Each class of animals consumes what per cent of its own weight of dry fodder a year?

8. For spraying potato vines 1 lb. of Paris green is used to 100 gal. of water. How much water should be used with an ounce of Paris green?

9. If hops lose 72% of their weight in drying, 100 lb. of dry hops represent how many pounds of green hops? How many pounds are lost in drying?

10. To make grafting wax a man takes 60% resin, 22% beeswax, and the rest linseed oil, measuring all by weight. How many pounds of each will he need for 15 lb. of wax?

11. He can also make grafting wax by taking equal parts of beeswax and linseed oil, and 3 times as much resin as of beeswax and oil together. Each is then what per cent of the total?

12. What per cent less rice is yielded by a field that produces 32 bu. to the acre than by one that produces 10 barrels to the acre, counting 45 lb. to the bushel and 162 lb. to the barrel?

13. How much will the material cost for spraying an orchard of 1000 trees twice during a season, allowing 5 gal. to a tree for each spraying, every 150 gal. containing 1 lb. of Paris green worth 20 ct. a pound?

14. A farmer wishes to spray his orchard. He can make lead poison by using white sugar of lead at $7\frac{1}{2}$ ct. a pound, and arsenate of soda at 5 ct. a pound, in the ratio of 7 : 3, or he can buy Paris green at 20 ct. a pound. Which is the cheaper by the 100 lb., and how much?

15. For protection against the cotton worm a grower uses $1\frac{1}{2}$ lb. of dry Paris green to the acre. A man can cover 15 acres a day, with the help of a horse. At 20 ct. a pound for Paris green, and $2.25 a day for a man and horse, how much will it cost to dust 45 acres?

16. A Tennessee farmer has 85 acres planted to peanuts. The average yield is 42 bu. to the acre, but by using a better fertilizer he increases the yield $33\frac{1}{3}\%$. How many bushels does the entire farm then yield?

17. In feeding 750 lb. of dry cornstalks to cattle it was found that 225 lb. were wasted, but that when the green stalks were put into a silo and then fed to the cattle, only 32 lb. were wasted out of 640 lb. What was the ratio of the per cent of waste in the latter case to that in the former?

18. A farmer paid out the following amounts for a 50-acre wheatfield: plowing, 9 da. @ \$2.75; harrowing and rolling, 6 da. @ \$2.50; 100 bu. seed @ \$1.15; drilling, 6 da. @ \$2.75; harvesting, \$40; threshing, \$75. The yield was 25 bu. to the acre, and he sold the crop at 73 ct. a bushel. How much did he make?

19. A man in Florida went into the business of raising pineapples. He paid \$250 for 5 acres of land, and up to the sale of his first crop had spent 50% of that amount for clearing, \$2000 for plants, 3 times as much for fertilizer as for clearing, \$200 for freight, express, and packages, \$1800 for a shed, and as much for labor as the land cost. His first crop averaged \$500 to the acre. His receipts were what per cent of his expenses?

20. A farmer asked his son to experiment on the cost of feeding chickens and ducks. The boy took 10 ducks and 10 chickens, and kept a record of their weight and the cost of their feed for 5 weeks. When he began, the 10 ducks together weighed $3\frac{3}{4}$ lb., and the chickens 1 lb. 14 oz. After 5 weeks the ducks weighed $30\frac{1}{4}$ lb., and the chickens $7\frac{5}{8}$ lb. The feed for the ducks cost 53 ct., and for the chickens 26 ct. What was the cost of each pound of increase in the ducks? in the chickens?

PROPORTION

313. Proportion. An expression of the equality of two ratios is called a *proportion.*

For example, the ratio \$2 : \$3 equals the ratio 10 ft. : 15 ft. Therefore \$2 : \$3 = 10 ft. : 15 ft. is a proportion. It is read, "\$2 is to \$3 as 10 ft. is to 15 ft." It may, of course, be written simply $2 : 3 = 10 : 15$, or $\frac{2}{3} = \frac{10}{15}$.

314. Illustrative problems. 1. If $x : 3 = 8 : 12$, we may find x as follows:

$\frac{x}{3} = \frac{8}{12}$. Multiplying by 3, as in § 264, $x = \frac{\cancel{3} \times \overset{2}{\cancel{8}}}{\cancel{12}_{\cancel{4}}} = 2$.

2. If $5 : x = 15 : 18$, we may find x as follows:

1. Because $5 : x = 15 : 18$, we must have $x : 5 = 18 : 15$.

2. That is, $\frac{x}{5} = \frac{18}{15}$, and $x = \frac{\cancel{5} \times \overset{6}{\cancel{18}}}{\cancel{15}_{\cancel{3}}} = 6$.

WRITTEN EXERCISE

Find the value of x *in Exs. 1–10:*

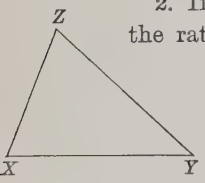
1. $x : 51 = 12 : 17$.
2. $x : 17 = 54 : 51$.
3. $x : 22 = 41 : 11$.
4. $x : 250 = 35 : 50$.
5. $x : 27 = 36 : 243$.
6. $x : 1001 = 8 : 143$.
7. $24 : x = 3 : 20$.
8. $35 : x = 7 : 91$.
9. $111 : x = 37 : 38$.
10. $19 : x = 209 : 143$.

11. I am thinking of a number which has to 17 the ratio 35 : 119. What is the number?

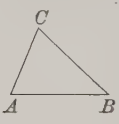
12. In a class in our school the ratio of boys to girls is 3 : 4, and there are 20 girls. How many boys are there?

ORAL EXERCISE

1. These two triangles have the same shape. If XY is twice AB, how does ZX compare with CA?

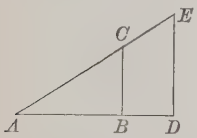


2. If the ratio of ZX to XY is $\frac{3}{4}$, what is the ratio of CA to AB?



3. What is the proportion existing between the sides AB, BC, XY, and YZ? State the proportion in more than one way.

4. If $XY = 1$ in., and if $ZX : XY = 3 : 4$, and if AB is half as long as XY, how long is CA?



5. In these triangles, what proportion exists between AB, AD, AC, and AE? If $AB = \frac{2}{3}\ AD$, what is the ratio of AC to AE?

6. If $AD = 1\frac{3}{16}$ in., $AB = \frac{9}{16}$ in., and $AC = \frac{10}{16}$, or $\frac{5}{8}$ in., how can we find the length of AE?

7. If DE represents the height of a man 6 ft. tall, BC the height of a boy, AD the shadow cast by the man, and AB the shadow cast by the boy, show how to find the height of the boy from measuring the shadows.

315. Proportional quantities. Quantities that are in proportion are said to be *proportional.*

316. Similar figures. Figures that are of exactly the same shape are said to be *similar.*

For example, the triangles in Ex. 1.

317. Proportional sides. *The corresponding sides of similar figures are proportional.*

For example, DE, BC, AD, AB, in the figures of Ex. 5.

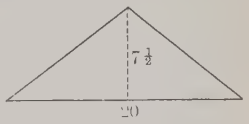
WRITTEN EXERCISE

1. The ratio of children in school to our total population is 1 : 5. If we have 85,000,000 population, how many children have we in school?

2. The ratio of the width of our schoolroom to the length is 7 : 9. The room is 21 ft. wide. How long is it?

3. If the United States has a standing army of 50,000 soldiers, and an army of 8,000,000 schoolboys, the ratio of soldiers to schoolboys is 1 to how many? State this fact as a proportion.

4. A carpenter wishes to construct a roof as shown here, the house being 20 ft. wide and the gable $7\frac{1}{2}$ ft. high. He wishes to know the length of slope. He draws a plan, representing 20 ft. by 8 in. How many inches will represent the $7\frac{1}{2}$ ft.?



5. He found this was not convenient, so he drew a right-angled triangle representing half the plan, and represented the 10 ft. by 10 in. Draw such a plan, representing the $7\frac{1}{2}$ ft. correctly, and measure the slope. How long did the carpenter find the slope of the roof to be?

6. Edward and Henry wished to find the height of the school building. Edward suggested standing a yardstick upright, measuring its shadow, and at the same time measuring the shadow of the school. The yardstick shadow was 4 ft., and the shadow of the building was 36 ft. How high was the school building? Draw a picture.

It is very easy to measure the height of trees and buildings in this way, or by sighting from the ground across the top of a pole to the top of a building, and pupils will find such work interesting and profitable.

Proportion used in Agriculture

WRITTEN EXERCISE

1. In a pound of grass there are 10.8 oz. of water. How many pounds of water in a ton of grass?

2. In 3 lb. of dried corn there are 6 oz. of liquid. How many pounds of liquid in 100 lb. of dried corn?

3. In 20 lb. of potatoes it is found that there are 4.2 lb. of solid matter. How much is there in 17.5 lb.?

4. In 25 lb. of oats it is found that there are 1.9 lb. of fat-producing matter. How much is there in 30 lb.?

5. In 15 lb. of corn it is found that there is 0.135 lb. of muscle-producing matter. How much is there in 20 lb.?

6. A farmer found that he needed 26 lb. of seed for $\frac{1}{2}$ acre. How many pounds does he need for 19.9 acres?

7. A Louisiana farmer obtained 9 hogsheads of molasses from 5 acres. At this rate, how many would he get from 93 acres?

8. In 35 lb. of cabbages there are $5\frac{1}{4}$ lb. of solid matter, the rest being liquids. How many pounds of solid matter in a ton of cabbages?

9. The insects eaten by a woodpecker one morning were found to number 3270, of which 1 out of 3 were harmless to man. How many harmful insects were destroyed?

10. To irrigate a farm at the rate of 2 in. in depth every 10 days requires for each acre a flow of 3.77 gal. a minute. At this rate, what flow is necessary for 3 in. in depth?

11. The water necessary to irrigate properly an acre of land can be stored in a circular reservoir 3.1 ft. deep and 55.5 ft. in diameter. Keeping the same diameter, how deep would the reservoir have to be for $3\frac{1}{2}$ acres?

WRITTEN EXERCISE

1. Blue jays' food consists of 1 lb. 3 oz. of noxious insects to 5 oz. of other animal food and 4 lb. 12 oz. of vegetable food. What per cent of this food is injurious to man?

This means, injurious to his agricultural interests.

2. Out of every 5 lb. of food taken by red-winged blackbirds, all but 10 oz. is made up of weed seed or of insects injurious to man. What per cent of this bird's food is injurious to man?

3. Out of every $7\frac{1}{2}$ lb. of food taken by phœbes, $8\frac{2}{5}$ oz. is vegetable. The rest consists chiefly of noxious insects, such as wasps and flies. What per cent of this bird's food consists of such insects?

4. In August, when grasshoppers and crickets are most harmful, 69% of a meadow lark's food is composed of these enemies of the farmer. Out of 10 lb. of food taken by these birds, how many ounces are of this kind?

5. Out of every 15 lb. of food taken by a kingbird, 1 lb. 8 oz. is vegetable, consisting almost entirely of wild fruits. The rest consists of insects injurious to man. What per cent of this bird's food is injurious to man?

6. Of 155 cuckoos observed for 3 hours, 46 black-billed cuckoos ate 1196 tree caterpillars and other pests, and 109 yellow-billed ones ate 2725. At this rate, the black-billed cuckoo destroys what per cent more than the yellow-billed?

7. A tree sparrow eats $\frac{1}{4}$ oz. of weed seed a day. Allowing 10 birds to 1 sq. mi., and supposing them to average 200 days a year in the state of Iowa, which has an area of 56,000 sq. mi., how many tons of weed seed are consumed a year by these birds alone?

Such reviews may be taken at the proper season of the year.

MEASURES

318. Capacity related to weight. The laws of the various states usually specify the weight of a bushel, requiring it to weigh at least the amount specified below. This table should not be committed to memory, but used for reference.

Corn (in the ear),	70 lb.	Oats,	32 lb.
Buckwheat, Barley,	48 "	Dried Apples,	26 "
Corn (shelled), Rye,	56 "	Timothy Seed,	45 "
Wheat, Clover Seed, Peas,		Dried Peaches,	33 "
Beans, White Potatoes,	60 "	Sweet Potatoes,	55 "

WRITTEN EXERCISE

1. How many tons to 1000 bu. of wheat? of oats?

2. How many bushels of white potatoes to the ton?

3. What number is to 12 as the bushel weight of sweet potatoes to that of white potatoes?

4. The weight of shelled corn is what per cent of the weight of the same amount of corn in the ear?

5. Barley is what per cent heavier than timothy seed? Timothy seed is what per cent lighter than barley?

6. What per cent more does 1 bu. of rye weigh than 1 bu. of barley? than 1 bu. of timothy seed? than 1 bu. of oats?

7. Clover seed is what per cent heavier than timothy seed? Timothy seed is what per cent lighter than clover?

8. In the above table it is possible to select four numbers so as to form a proportion. Do this, telling the products to which they refer.

9. State the ratios of the bushel weights of dried peaches to dried apples; of dried apples to dried peaches; of peas to beans; of sweet potatoes to white potatoes.

319. Weight of water. It is sometimes convenient to know that, practically, 1 cu. ft. of water weighs $62\frac{1}{2}$ lb.

ORAL EXERCISE

1. What is the weight of 2 cu. ft. of water? of 10 cu. ft.?

2. What is the weight of $\frac{1}{2}$ cu. ft. of water? of $\frac{1}{10}$ cu. ft.?

3. What part of 1 cu. ft. of water weighs $12\frac{1}{2}$ lb.? 25 lb.?

4. Silver is about ten times as heavy as water. About how much does 1 cu. ft. of silver weigh?

WRITTEN EXERCISE

1. What part of 1 oz. does 1 cu. in. of water weigh?

2. Water is half as heavy as sulphur. How much does 1 cu. ft. of sulphur weigh?

3. The ratio of the weight of cork to that of water is 6 : 25. How much does 1 cu. ft. of cork weigh?

4. A certain quality of gold is 19.3 times as heavy as water. How much does 1 cu. ft. weigh?

5. If silver is $10\frac{1}{2}$ times as heavy as water, how much is 1 cu. ft. of silver worth at 50 ct. an ounce (avoirdupois)?

6. If good milk is 3% heavier than water, what is the weight of a can of such milk containing 1 cu. ft., the can itself weighing 5 lb. 10 oz.?

7. Ice is 92% as heavy as water. I wish 100 lb. of ice to fill my refrigerator. The iceman sends me 2 cu. ft. He sends me how many pounds too much?

8. A man building a house wishes to put a tank for water in his attic. The tank is to be 5 ft. long, 4 ft. wide, and is to allow for $3\frac{1}{2}$ ft. depth of water. Allowing 600 lb. for the weight of the tank, how much will it weigh when full?

320. Weights compared with weight of water. We have found that 1 gal. contains 231 cu. in., and that 1 cu. ft. of water weighs nearly 62½ lb. Therefore

1 cu. in. of water weighs nearly $\frac{1}{1728}$ of 62½ lb.,
and 1 gal. " " " " $231 \times \frac{1}{1728}$ of 62½ lb.,
or nearly 8.3 lb., which is near enough for practical purposes.

ORAL EXERCISE

1. How much do 2 gal. of water weigh? 10 gal.?

2. How much does ½ gal. of water weigh? 100 gal.?

3. If a 3-gal. pail weighs 3 lb., how much will it weigh when full of water? when half full?

WRITTEN EXERCISE

1. How much does 1 bbl. (of 31½ gal.) of water weigh?

2. Milk being 3% heavier than water, how much does it weigh to the gallon? the quart? the pint?

3. Alcohol being 79% as heavy as water, how much does it weigh per gallon? per barrel of 31½ gal.?

4. Petroleum being 70% as heavy as water, how much does it weigh per gallon? per barrel of 31½ gal.?

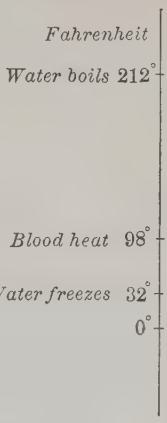
5. Copper being 8.95 times as heavy as water, how much does 1 cu. ft. of copper weigh? also 231 cu. in.?

6. Turpentine being 1% lighter than water, how much does it weigh to the gallon? the barrel of 31½ gal.?

7. A tank 7 ft. long by 5½ ft. wide is filled with water to the depth of 3 ft. How many gallons of water in it?

8. The water of the Dead Sea is so salty that the ratio of its weight to that of fresh water is 31 : 25. How much does 1 gal. of Dead-Sea water weigh?

321. Measuring temperature. We commonly use a thermometer known as Fahrenheit's. In scientific work the centigrade (a word meaning "100 degrees") is used. Because there are just 100° between the freezing and boiling points of water, on the centigrade thermometer, and the freezing point is at 0°, that would be easier than the Fahrenheit to work with if we were used to it.



The exercises refer only to the Fahrenheit.

ORAL EXERCISE

1. When the temperature is 5° below 0, how many degrees is it below the freezing point of water?

2. How many degrees between the freezing and boiling points of water? between blood heat and the freezing point?

WRITTEN EXERCISE

1. In shipping meat from Chicago and Kansas City, refrigerator cars reduce the temperature 64% below animal (blood) heat. What temperature is used?

2. In freight cars fruits can be shipped short distances when the outside temperature is 25°. If the inside temperature is 9½° higher, how much is it above freezing?

3. The lowest temperature at which bananas should be shipped is 13½° above freezing, lemons 3° above freezing, cheese 20° below freezing. Express these temperatures in the ordinary way.

Our Forests

322. Our timber industry. Our timber product is very great. Most states now have an Arbor Day to interest the people in preserving our forests.

WRITTEN EXERCISE

1. If there are 672,000,000 acres of woodland in the United States, how many square miles are there?

2. If there are $600,000,000 invested in lumbering, and the annual product amounts to $566,520,000, this is what per cent of the amount invested?

3. If 283,260 men are engaged in the business, and are paid $116,136,600 annually, what are the average wages of each?

4. If these 283,260 men cut and saw $566,520,000 worth of lumber annually, what is the average per man?

5. The loss from forest fires annually is about $3\frac{1}{2}\%$ as much as the value of the lumber produced. If the value of the lumber is $566,520,000, how much is the fire loss?

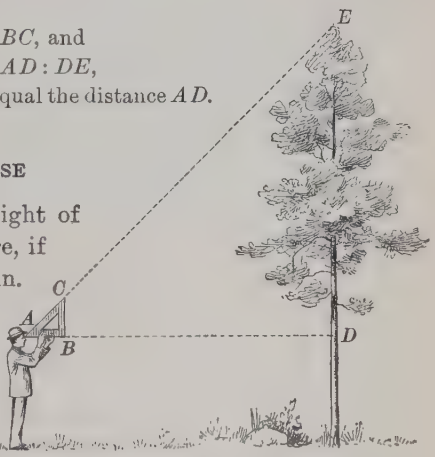
6. The average spruce tree supplies wood pulp for 500 lb. of paper. Taking the average weight of a book as 20 oz., how many trees had to be cut down to supply the paper for a million books?

323. Measuring trees. The height of a tree is often measured by proportion, as here shown.

This man holds a right-angled triangle in which $AB = BC$. He holds AB level and walks towards the tree until he just sees the top along AC. Then because

$$AB = BC, \text{ and}$$
$$AB : BC = AD : DE,$$

the height *above* D will equal the distance AD.



WRITTEN EXERCISE

1. What is the height of the tree in the picture, if the triangle is 5 ft. 4 in. from the ground, and AD is 23 ft. 8 in.?

2. Suppose a triangle were used which had $AB =$ twice BC. What would be the height of the tree if $AD = 75$ ft., and if D were 5 ft. 8 in. above the ground?

3. Suppose the triangle used had $AB = 2$ ft., $BC = 1$ ft. 6 in. What would be the height of the tree if $AD = 4$ rd., and if D were 5 ft. 6 in. above the ground? if $AD = 61$ ft., and if D were 5 ft. 3 in. above the ground?

4. What is the height of a tree that casts a shadow 88 ft. long at the same time that a man 5 ft. 9 in. high casts a shadow 7 ft. 8 in. long?

5. A boy whose eye is 5 ft. above the ground sights over the top of a 10-ft. pole and just sees the top of a tree. The boy is 6 ft. from the pole and 42 ft. from the tree. How high is the tree? Draw the picture.

324. Board measure. One foot of lumber, board measure (1 ft. B.M.), means a piece having an area of 1 sq. ft. on one surface, and being 1 in. or less in thickness.

A board 18′ long, 8″ wide, and 1″ or less thick contains $18 \times \frac{8}{12}$ ft. B.M., or 12′ B.M. A joist $12' \times 6'' \times 2\frac{1}{2}''$ (the sign × here meaning "by") contains $2\frac{1}{2} \times 12 \times \frac{1}{2}$ ft. B.M., or 15′ B.M.

ORAL EXERCISE

Find the number of feet B.M. in boards 1 in. or less in thickness, with dimensions as given:

1. $8' \times 3''$.	2. $16' \times 4''$.	3. $12' \times 4''$.	4. $8' \times 4''$.
5. $16' \times 6''$.	6. $10' \times 6''$.	7. $16' \times 3''$.	8. $12' \times 3''$.
9. $18' \times 12''$.	10. $16' \times 12''$.	11. $10' \times 12''$.	12. $18' \times 6''$.

WRITTEN EXERCISE

Find the number of feet B.M., the thickness being 1″:

1. $9' \times 18''$.	2. $10' \times 7''$.	3. $7' \times 16''$.	4. $22' \times 9''$.
5. $23' \times 10''$.	6. $17' \times 13''$.	7. $13' \times 9''$.	8. $8' \times 18''$.
9. $14' \times 10''$.	10. $28' \times 14''$.	11. $18' \times 14''$.	12. $16' \times 11''$.

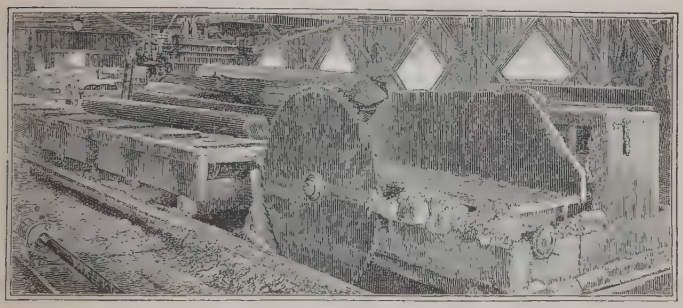
Also in the scantling in Exs. 13–18, to the nearest inch:

13. 6 ft. long, $1'' \times 2''$.	14. 8 ft. long, $2'' \times 2''$.
15. 8 ft. long, $2'' \times 3''$.	16. 8 ft. long, $2'' \times 4''$.
17. 10 ft. long, $2'' \times 5''$.	18. 10 ft. long, $2'' \times 6''$

Also in the planks in Exs. 19–24, to the nearest foot:

19. 10 ft. long, $6'' \times 1\frac{1}{4}''$.	20. 24 ft. long, $7'' \times 2''$.
21. 16 ft. long, $9'' \times 1\frac{1}{4}''$.	22. 12 ft. long, $7'' \times 1\frac{1}{4}''$.
23. 16 ft. long, $15'' \times 1\frac{1}{2}''$.	24. 14 ft. long, $10'' \times 1\frac{1}{4}''$.

325. Bills of lumber. Lumber is sold at so much "per M," meaning per 1000 ft. B.M.



WRITTEN EXERCISE

1. A man wishes to build a house $28' \times 32'$. He needs 4 sills, each $6'' \times 8''$, to put under the walls. How much will they cost at \$18 per M?

2. If he buys 34 sticks of timber, $2'' \times 6''$ and $28'$ long, how much will these cost at \$18 per M?

3. If he buys 60 joists, $2'' \times 4''$ and $18'$ long, how much will these cost at \$18 per M?

4. How much will the flooring for the house cost, taking the measurement of each floor as $28' \times 32'$, and allowing for two floors, the lumber costing \$40 per M?

5. How many feet of siding are necessary for this house, supposing it to be $28' \times 32'$ and $18'$ high, the siding being $5''$ wide and laid $4''$ to the weather, no allowance being made for gables or doors and windows?

6. If he puts on a sloping roof, each slope being $34' \times 18'$, how much will the shingles cost at \$4.75 per 1000, allowing 1000 shingles to 125 sq. ft.?

WRITTEN EXERCISE

1. How many feet (B.M.) of 2″ planking are needed for a barn floor 18′ × 25′?

2. Make out a bill for 15 planks 18 ft. long, 10″ × 2½″, and 12 joists 16 ft. long, 2″ × 4″, at $22 per M.

3. How many feet of lumber are needed for a floor 18′ × 14′, 1″ thick, and ⅛ of the floor area being added for matching?

4. An A-shaped roof is 25′ long, the rafters being 15′ long. Allowing 4½ shingles to the square foot, how many shingles are required to cover both slopes?

5. Not allowing for overlapping or for joists, how much lumber is required for a bin 8′ × 6′ and 5′ deep, open at the top, 1″ boards being used? What will it cost at $30 per M?

A man thinks of building this house. It is to be 34′ × 27½′. The walls are to be 14′ to the eaves, the ceilings in the first floor 9′ and in the second floor 8′. A builder estimates the cost of materials and labor. Part of the computation is given in Exs. 6–9; the rest is left for the class to work out.

6. Sills: 3 pieces, 6″ × 8″, 16′ long, 192′ B.M.
4 " 6″ × 8″, 14′ " (?)′ "
6 " 6″ × 8″, 12′ " (?)′ "
1 piece, 2″ × 8″, 14′ " 19′ "
1 " 2″ × 9″, 8′ " (?)′ "

7. Joists:	30	pieces, $2'' \times 9''$,	16′	long, (?)′	B.M.
	18	" $2'' \times 8''$,	12′	" (?)′	"
	13	" $2'' \times 8''$,	6′	" (?)′	"
	27	" $2'' \times 8''$,	16′	" (?)′	"
	27	" $2'' \times 8''$,	12′	" (?)′	"
	27	" $2'' \times 6''$,	16′	" (?)′	"
8. Studding, etc.:	5	" $4'' \times 6''$,	14′	" (?)′	"
	100	" $2'' \times 4''$,	14′	" 933′	"
	35	" $2'' \times 4''$,	10′	" 233′	"
	25	" $2'' \times 4''$,	18′	" (?)′	"
	45	" $2'' \times 5''$,	12′	" (?)′	"
	50	" $2'' \times 4''$,	8′	" 267′	"
	50	" $2'' \times 4''$,	14′	" 467′	"
	40	" $2'' \times 6''$,	18′	" (?)′	"
Bridging, etc.:				175′	"
Roof sheathing, etc.:				875′	"

9. The total in Exs. 6–8 should be 7847′ B.M. How much will this lumber cost at \$19 per M?

10. The man also needs 2000 ft. B.M. of siding @ \$45 per M; 1950 ft. of flooring @ \$22; 125 ft. of flooring for the porch @ \$40; 1500 shingles for the gables @ \$4.25 (that is, per 1000 shingles); 6200 shingles for the roof @ \$6.50. What will this material cost?

11. He also needs 28 foundation posts @ 30 ct. each; water tables, cornice, corner boards, etc., \$25; porch columns, \$5; 16 doors at an average cost of \$5.20 each; 16 windows at an average cost of \$5.50 each; hardware, \$40; chimney and plastering, \$200; painting, \$148; tin work, \$15.50; the work of 4 carpenters for 30 days @ \$3 a day each. What is the total cost of these items?

12. Allowing \$75 for extras, what is the total cost of the house?

Plastering

ORAL EXERCISE

1. What is the perimeter of a room 14 ft. by 16 ft.?

2. If the room is 9 ft. high, what is the area of the four walls, not allowing for openings?

3. What is the area of the ceiling of a room 18 ft. by 20 ft.? How many square yards?

4. What will it cost to plaster 60 sq. yd. at 30 ct. a square yard? at 33 ct. a square yard? at 35 ct. a square yard?

5. A corridor is 30 ft. long, 4 ft. wide, and 9 ft. high (above the baseboard). How many square feet of plaster are needed for the two sides and top, not allowing for openings?

WRITTEN EXERCISE

1. How many square yards of plastering in the walls and ceiling of a room 16 ft. by 20 ft., and 9 ft. high, deducting 14 sq. yd. for openings and baseboard?

2. How many square yards of plastering in the walls and ceiling of a hall 60 ft. long, 40 ft. wide, and 28 ft. high, deducting 36 sq. yd. for openings and baseboard?

3. What will it cost to plaster a schoolroom 32 ft. long, 18 ft. wide, and 12 ft. high, at 32 ct. a square yard, deducting 50% of the area of the walls on account of blackboards, baseboard, and openings?

4. On account of the danger of falling plaster, some school trustees decided to use pressed steel ceilings. There were 742 sq. yd. of ceiling in the building, and they let a contract for putting up the steel ceilings at 10 ct. a square foot. How much did the ceilings cost?

Carpeting

ORAL EXERCISE

1. How many strips of $\frac{3}{4}$ yd. carpet are needed for a room 18 ft. wide, not allowing for matching the pattern?

2. A carpet has an 18-in. pattern? Will there be any loss in matching if the strips are 15 ft. long? 14 ft. long?

WRITTEN EXERCISE

1. How many yards of carpet, 9-in. pattern, are needed for a room 11 ft. 3 in. wide and 12 ft. long, the carpet being 27 in. wide? Draw a plan showing how the strips lie.

2. How many yards of carpet, $\frac{1}{2}$-yd. pattern, are needed for a room 13 ft. 6 in. wide and 16 ft. long, the carpet being 27 in. wide? How much is wasted on each strip (except the first, with which the others must match)? Draw a plan.

Show that no allowance is ever necessary for the first strip.

Find how many yards of carpet are needed for the following, allowing for matching each strip (except the first), as indicated:

3. 18 ft. by 20 ft., 27-in. carpet, allowing 9 in.

4. 16 ft. by 18 ft., 27-in. carpet, allowing $\frac{1}{2}$ yd.

5. 14 ft. by 15 ft., 36-in. carpet, allowing $\frac{1}{4}$ yd.

Find the cost for carpeting these rooms with 27-in. carpet, allowing 4 inches on each strip (except the first), @ $1.20 a yard:

6. 18 ft. by 15 ft.
7. 18 ft. by 17 ft. 6 in.
8. 14 ft. 3 in. by 18 ft.
9. $13\frac{1}{2}$ ft. by 16 ft. 3 in.
10. 12 ft. by 13 ft. 6 in.
11. 4 ft. 6 in. by 23 ft. 8 in.

326. Papering. The standard width for estimating wall paper is 18 in., although it varies greatly. A single roll is considered to contain 8 yd. of paper. In estimating, paper hangers usually count 3 rolls to 100 sq. ft. Fractional parts of rolls cannot be bought.

ORAL EXERCISE

1. What will the border for a room 16 ft. by 20 ft. cost, at 25 ct. a yard?

2. Estimate the cost of the paper for a ceiling 15 ft. by 20 ft., at 40 ct. a roll.

3. A room has nearly 500 sq. ft. of wall to be papered. Estimate the cost of the paper at 50 ct. a roll.

4. A plain wall 14 ft. long and 9 ft. high above the baseboard is to be papered. How many rolls should you allow?

WRITTEN EXERCISE

1. The walls of a room are 8 ft. 6 in. above the baseboard, and the total length of the four walls, allowing for openings, is 72 ft. Allowing 6 in. on each strip for matching, what will the paper cost at 40 ct. a roll?

2. How many rolls of plain paper will it take to cover the four sides of a room 12 ft. high, 25 ft. long, and 20 ft. wide, the paper being 24 in. wide, allowing $1\frac{1}{2}$ rolls for windows and doors? At 35 ct. a roll, what will it cost?

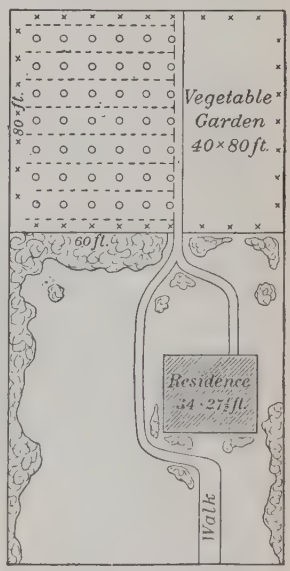
3. The walls of a room are to be papered with plain paper worth 30 ct. a roll. They are 9 ft. high above the baseboard, and the total length of the four walls, allowing for windows and doors, is 60 ft. At the usual estimate, what will the paper cost?

The cost of papering rooms at home should be considered.

Laying out a Garden and Grounds

WRITTEN EXERCISE

1. The house described on page 236 was built on a lot 100 ft. wide and 200 ft. deep. The owner took a piece 40 ft. by 80 ft. for vegetables, and one 60 ft. by 80 ft. for fruit. How many square feet in each?



2. He planted a row of grapevines around the garden and four corners, 10 ft. apart. How many did he plant?

3. Across the fruit garden he set out 3 rows, 60 ft. long, of dwarf pears, 1 row of dwarf cherries, and 1 row of dwarf plums, putting the trees 10 ft. apart and 10 ft. from his neighbor's line. How many trees did he set out?

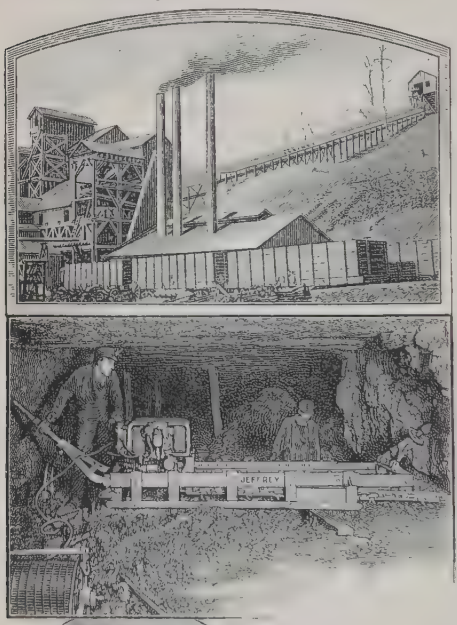
4. He set out a 60-ft. row of blackberry bushes, 3 ft. apart and 3 ft. from the fence; also 2 such rows of blackcaps and 2 of red raspberries. How many bushes did he set out?

5. He planted a bed of asparagus 4 ft. wide and 36 ft. long; 2 beds of parsnips and 2 of beets, each $1\frac{1}{2}$ ft. wide and 36 ft. long; 2 beds of tomatoes, and 2 of squash, each 3 ft. wide and 36 ft. long. What area did he give to each?

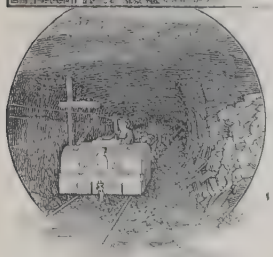
6. He planted shrubs about the lawn. These, with the walks, occupied 2200 sq. ft. How many square feet were left for the lawn? See if you can improve this design.

GENERAL BUSINESS APPLICATIONS

327. Our mining interests. We have frequently studied the greatest source of our national wealth, our farms.



Another source we have seen to be our forests. Still another important one is our mines. The first (raw) product of our mines was, at the beginning of the century, worth about $800,000,000 a year, the metals produced were worth about $500,000,000, and the petroleum products alone were worth over $70,000,000. We think of our gold output as valuable, but our copper is more so, our iron is three times as valuable, and our coal is worth four times as much.



The most valuable product of our mines is coal, from which coke and gas are made, and which furnishes the chief power for our manufacturing, for heating, and for general electricity.

WRITTEN EXERCISE

1. A short ton of bituminous (soft) coal contains 76.05% of carbon and 2.32% of ash. How many pounds are there of each?

2. If 2000 lb. of anthracite (hard) coal contain 1828 lb. of carbon, how much will 100 lb. contain? What is the per cent of carbon?

3. We mined 268 million tons of coal in a certain year when Great Britain mined 252 million tons. Our output was what per cent greater than theirs?

4. If we manufacture $75,700,000 worth of gas a year, the coal and other material used costing 27% of this amount, how much does the material cost?

5. If Maryland produces 5.1 million tons of soft coal, worth at the mines $5,000,000, what is the value of Pennsylvania's output of 81.6 million tons, at the same average price per ton?

6. If we produce 226 million tons of bituminous coal, worth $1.05 a ton at the mines, and 30% as much anthracite, worth 60% more per ton, what is the total value of our annual coal product?

7. A Wilkesbarre (Pennsylvania) coal shaft is 1042 ft. deep, and the opening is 12 ft. by 51 ft. Supposing it to be rectangular, it represents the excavating of how many loads of earth, 1 cu. yd. to a load?

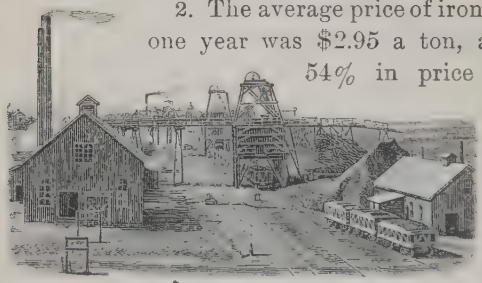
8. Near the bituminous mines are coke ovens where the coal is converted into coke by driving off the gas and moisture. We use 30 million tons of coal a year to make 19 million tons of coke. What is the loss per cent?

9. Of the 19 million tons of coke produced, Pennsylvania produced 67.4%. How many tons did she produce?

Our Iron Mines

WRITTEN EXERCISE

1. Iron ore is sold by the long ton. This is what per cent more than the short ton?



2. The average price of iron ore at the mines one year was \$2.95 a ton, and it decreased 54% in price the next year. What was then the average price?

3. In one year we made 2,106,500 kegs of cut nails, 100 lb. to the keg, and 300% more of wire nails. How many pounds of nails did we make in that year?

4. If we should make 2,100,000 short tons of steel rails a year, averaging 70 lb. to the yard, how many miles of rails would we make?

5. Iron melted from the ore is molded into short bars called pigs. Pig iron was worth \$16 a ton in a certain year. The next year it increased 10% in value, but the year following it decreased 10%. What was its value then?

6. At the opening of this century 39,758 wage earners in this country produced 14,432,154 tons of pig iron, while 10 years before that 33,415 men produced 8,854,975 tons. What was the average number of tons per man in each year, and the per cent of increase per man?

7. If we produced 17,160,632 tons of pig iron in one year, an increase of 13% on the year before, how many tons did we produce the year before?

328. Some of our metals. From the ores brought from our mines metals are extracted.

WRITTEN EXERCISE

1. When 700 lb. of copper was worth \$73.50, what was 250 lb. of the same quality worth?

2. A dealer sold 800 lb. of nickel at 55 ct. a pound, thus gaining 10% on the cost. How much did it cost him?

3. If a dealer buys 900 lb. of zinc at \$88 a short ton, at what rate per pound must he sell it to gain 50%?

4. In one year silver averaged 54.98 ct. an ounce, which was 102.8% of the average for the preceding year. What was the average then?

5. Silver is sold by the troy ounce. This is what per cent of the avoirdupois ounce (7000 gr. = 16 av. oz., 5760 gr. = 12 troy oz.)?

6. Quicksilver (mercury) is sold by the flask of 76½ lb. If a dealer buys it at \$38.25 a flask, at how much a pound must he sell it to gain 20%?

7. If a short ton of lead is worth \$96, how much is 750 lb. worth? If a dealer bought it at this rate and sold it at 8 ct. a pound, what was his per cent of profit?

Manufactured Products

WRITTEN EXERCISE

1. We made 50,168,000 gross of pins in a certain year, and 2% as many needles. How many needles and pins together did we make?

2. The value of the sewing machines manufactured in the first year of this century was \$21,603,200, or $143\frac{1}{3}\%$ of the value ten years before. What was the value then?

3. If we manufactured 1,825,769 watch movements in a certain year, and sold them for \$2,756,911.19, and the dealers sold them at an average price of \$3.02 each, what per cent profit did the dealers make?

4. We made 144,875 typewriters in a certain year, worth on an average \$38.50 each. We sold 40% of these to other countries. How much did we receive from the foreign sales?

5. If we made 2775 locomotives in a certain year, averaging \$10,000 each, and sold 20% of them abroad, how much did we receive from our foreign trade and how many locomotives did we sell abroad?

6. If, when our population was 82,155,000, we used \$2,738,500 worth of pins and needles a year, what will they cost for a population of 100,000,000, at the same rate? How much does this average a person?

7. It was recently estimated that the farmers in this country owned \$800,000,000 worth of agricultural implements, and that if it were possible to get help to do the harvesting without machinery it would cost \$684,000,000 a year more than it did. This gain is what per cent of the cost of the implements?

Some Problems of a Manufacturer

329. Capital. The money that a company has invested in such an enterprise as manufacturing is called its *capital.*

WRITTEN EXERCISE

1. The capital of a certain company is \$300,000, and its annual business is $3\frac{1}{2}$ times its capital. What is its annual business?

2. Its profits are 6% of its annual business. How much is this? This is what per cent of its capital?

3. The company carries \$325,000 worth of insurance, paying \$6 on each \$1000 per year. What is the cost?

4. One of the articles manufactured is sold at retail for \$3.50, the dealer making a profit of 40%. How much does the dealer pay for the article?

5. The manufacturer of the article mentioned in Ex. 4 pays for material \$1.30, labor 45¢, expenses and advertising 45¢, selling 15¢. How much does he make on each article? What per cent?

6. On which will the manufacturer make the greater rate of profit, — an article which costs \$2.35 to make, and which he sells at \$2.50 less 2%, or one costing \$1.84 and selling at \$2 less $3\frac{1}{2}$%?

7. The company sells on 30 da. credit, or at 3% discount if paid within 10 da. About 60% of the amount of the sales are discounted. What is the rate of discount on the total sales?

8. The company pays one of its traveling salesmen \$1500 a year, and allows him \$1500 a year additional for expenses. He sells \$75,000 worth of goods. His salary and expenses together are what per cent of his sales?

9. The buildings of the company are worth \$62,000, and the company allows 9% a year for repairs and depreciation in value. This means what annual expense?

10. In a year when the business amounted to \$954,000 the company invested $\frac{3}{4}$% of this sum in advertising in trade journals. How much did it pay for advertising?

11. The next year it invested \$51,350 in "national advertising" (in the popular magazines), which was 5% of the amount of its business for that year. What was the amount of its business? Taking the increase over the preceding year (see Ex. 10), did the advertising pay?

12. This company employs 200 girls at \$7.50 a week, 30 men at \$14.50 a week, 30 boys at \$5.75 a week. What is the weekly pay roll for this labor? How much does an increase of 10% in all the wages add to the weekly salary of one of the girls? to the total weekly pay roll?

13. The company employs great care in its sales, but its loss by poor debts last year, when its sales were \$1,125,000, was $1\frac{1}{3}$% of the sales. How many dollars did the company lose through poor debts?

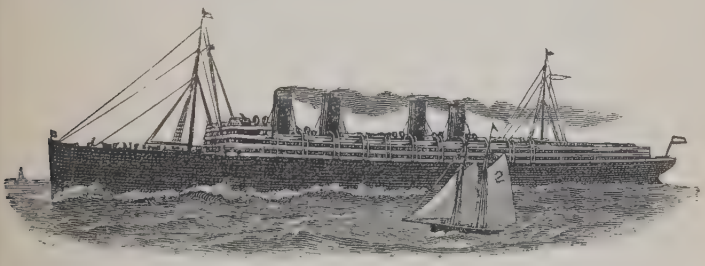
14. The company employs four men on a certain kind of work, at 28 ct. an hour. The pay roll shows the number of hours each man works. Fill in this pay roll. How will you check the work?

	Mon.	Tu.	Wed.	Th.	Fri.	Sat.	Wages	
E. Glover	8	8	8	8	8	8		
J. Thornton . . .	8	7	8	$7\frac{3}{4}$	8	8		
O. M. Gooley . . .	$7\frac{1}{2}$	$7\frac{1}{2}$	0	8	8	0		
Wm. Thompson . .	4	8	8	8	8	4		

Some of Our Shipping Interests

WRITTEN EXERCISE

1. The tonnage (weight) of the *Cedric* is 21,000 tons, which is $\frac{1}{17\frac{1}{4}}$ more than that of the *Celtic*. What is the tonnage of the *Celtic?*



2. If a steamer's hull is 40 ft. 4 in. deep and 700 ft. long, the depth is what per cent of the length? (Think of these dimensions compared with those of your school building.)

3. One of the steamship lines landed in New York on an average 1206 passengers per ship in a certain year, which was 90% of the average the year before. What was the average then?

4. One of the largest steamers will carry 80 barge loads of grain, at 8000 bu. to a barge. A freight car 40 ft. long will carry 1280 bu. What is the total length of a train carrying enough grain to load this steamer?

5. One of our American steamers pays $4760 in monthly wages to the 136 stokers who keep up the fires for its great engines. A German steamer pays $2864 to its 179 men who do this work. Each American stoker receives what per cent more monthly wages than the German?

330. The produce buyer, the shipper, and the freight bills. The word *produce* is commonly applied to the vegetable product of farms, as grain, hops, cabbages, and beans. If a man sends produce to some distant place, he is then a *shipper*. He may ship by cars or by boat. The buyer often puts produce in a large storage house, where it is kept so as not to be damaged, until he is ready to sell it. If he sells it delivered on the cars where he lives, he is said to sell *f.o.b.* (free on board).

WRITTEN EXERCISE

1. A produce buyer bought 250 bbl. of apples at $\$2.12\frac{1}{2}$ per barrel and sold them at a profit of $12\frac{1}{2}\%$. What did he gain?

2. A produce buyer bought 75 tons of cabbage at \$5.25 per ton. He paid 75 ct. per ton for storage and sold them at a profit of 15% on the total cost. What did he gain?

3. A produce buyer bought 50 tons of cabbage at \$5 per ton and put them in storage. He paid \$2 per ton for storage when he put them in. When he took them out he found that they had shrunk 25% while in storage. He then paid \$1 per ton for trimming and loading into cars, and sold them for \$10 per ton f.o.b. What was his gain or loss on the deal?

4. A buyer bought 1000 bbl. of apples in the fall, at \$2 per barrel. He decided to put them in storage for 3 months, which cost 40 ct. per barrel. At the end of that time he found that 6% of the apples had decayed. He then repacked them, paying 10 ct. per barrel of sound apples for repacking, and sold them at \$2.80 per barrel. What was his gain or loss on the deal?

5. A shipper sent 200 bbl. of fruit to a city, paying $2.30 per barrel f.o.b., and 20 ct. per cwt. (hundredweight) for freight. Allowing 160 lb. to the bbl., what did the shipment cost delivered at the city freight house?

6. A shipper's freight bills for 3 days were as follows:

Freight to	Totals	May 3	May 4	May 5
Columbus	$17.08	$3.40	$7.90	$5.78
Saint Joseph . . .	x	9.62	23.50	16.92
Cortland	x	8.27	16.75	31.75
Little River . . .	x	4.25	9.62	4.27
	y	z	z	z

Find the totals marked x, y, and z. The totals of the z's should equal what other total?

7. A shipper in Wilson, New York, sends 27,642 lb. of cabbages to Memphis. The rate is 47¢ per 100. What freight does he pay? This freight goes over three roads, the Rome and Watertown, the Wabash, and the Illinois Central, and these roads share according to the distances they carry the freight. The Rome and Watertown gets 21.99%, the Wabash 58.16%, and the Illinois Central the rest. How much does each road get?

8. A shipper sends 150 bbl. of apples to Kansas City. He pays 44¢ per cwt. At 160 lb. to the barrel, what freight does he pay? The freight agent allows 40¢ per cwt. to the following roads: Lackawanna Railroad, 22.62%; Michigan Central, 49.18%; Rock Island, 28.20%; and allows Mississippi bridge tolls 3¢ per cwt. and terminal switch charges 20¢ per ton. How much of the total freight charge did the roads, the bridge, and the terminal station receive?

Elementary Problems about Weather

WRITTEN EXERCISE

1. What is the difference in temperature between 4° below zero and the freezing point of water?

2. The average January temperature in Washington is 33°, in New York it is 10% less, and in Norfolk, Virginia, it is 20% more. What is it in each of these places?

3. The coldest temperature recorded by our weather bureau is in Poplar River, Montana, 63° below zero, while the lowest recorded at Key West, Florida, is 41° above zero. What is the average of these two?

4. The highest temperature recorded at any station of our national weather bureau is 118°, at Yuma. Red Bluff, California, has come within 2.95% of it, and Boisé City, Idaho, within 7%. What is the highest temperature of these places?

5. The average velocity of the wind in Buffalo is 11 mi. per hour throughout the year. It is $18\frac{2}{11}\%$ less in Chicago, and is $22\frac{2}{3}\%$ less in Denver than in Chicago. What is the average velocity in each of these places?

6. The highest wind velocity ever known in San Francisco was 60 mi. per hour. Such a wind blows with a force of 17.712 lb. per square foot. What is its pressure on the side of a school building 100 ft. long and 40 ft. high?

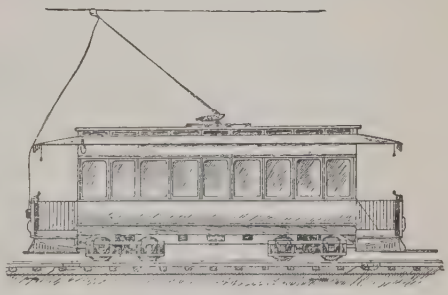
7. The average annual precipitation (rainfall) in Vicksburg is 55.7 in., in Nashville it is 5.7 in. less, in Yuma, Arizona, it is only 6% as much as in Nashville, and in Duluth it is 1000% as much as in Yuma. What is the precipitation in these various places?

It is often found to be helpful to keep records of the weather, illustrating the temperature by a curve as shown on page 121.

Our Electricity Interests

WRITTEN EXERCISE

1. If the number of miles of electric street railways increased in 12 years recently from 1250 to 22,500, what was the per cent of increase?



2. If the money invested in manufacturing electrical apparatus increased in the ratio of 2 to 9 in a certain ten years from \$19,000,000, how much was it at the end of the ten years?

3. If the value of the electrical apparatus produced in a certain year was \$96,000,000, an increase in the ratio of 5 to 24 in ten years, what was it ten years before?

4. If the cost of running street cars in a city is 19¢ per mile by horse power, $17\frac{3}{4}$¢ by cable, and 13¢ by electricity, what per cent cheaper is it to run them by electricity than by horse power? than by cable?

5. If the average number of telephone calls per day in this country was 9,387,600 in a certain year, an increase of 20% over the preceding year, what was the total number of calls for all of the preceding year?

6. If it is 25,000 mi. around the earth, what was the ratio of the distance around the earth to the length of wire of the Bell Telephone Company when this company had 3,250,000 miles of wire in its system?

MISCELLANEOUS PROBLEMS

ORAL EXERCISE

1. 15 is $\frac{1}{2}$ of what number? $\frac{1}{4}$ of what number?

2. 25 is $\frac{1}{3}$ of what number? $\frac{5}{7}$ of what number?

3. Express as decimals: $\frac{2}{5}$, $\frac{3}{4}$, $\frac{1}{25}$, $\frac{3}{50}$, $\frac{7}{25}$, $\frac{9}{50}$, $\frac{21}{50}$.

4. Express as common fractions: 0.5, 0.6, 0.25, 0.40.

5. Express as an integer and a common fraction: $1.33\frac{1}{3}$, $3.66\frac{2}{3}$, 5.75, 17.400, 32.125.

6. State the sums: $0.7 + 0.8$, $0.6 + 0.4$, $0.9 + 0.7$, $1.6 + 2.4$, $3.8 + 0.2$, $5.9 + 4.1$, $3.25 + 6.75$.

7. Multiply: 0.5×0.5, 4×0.5, 0.3×6, 0.25×4, 0.01×50, 7×0.7, 0.01×0.01, 0.05×0.09.

8. Divide: $2.5 \div 5$, $3.5 \div 7$, $0.49 \div 7$, $6.4 \div 8$, $0.72 \div 9$, $1.44 \div 12$, $14.4 \div 12$, $12.5 \div 5$, $1.25 \div 5$.

9. A foot is what decimal fraction of a yard? A quart is what decimal part of a gallon? of a peck?

10. Reduce 2 ft. 8 in. to inches; 2 lb. 8 oz. to ounces; 2 dollars and 8 cents to cents; 2 min. 8 sec. to seconds.

11. Reduce 36 in. to feet; 19 in. to feet and inches; 42 in. to yards and inches; 40 oz. to pounds and ounces.

12. State the sums: 3 ft. 8 in. + 4 ft. 4 in.; 2 ft. 9 in. + 3 ft. 9 in.; 3 yd. 20 in. + 5 yd. 20 in.; 7 lb. 10 oz. + 2 lb. 6 oz.; 4 bu. 2 pk. + 8 bu. 3 pk.

13. Subtract: 8 yd. 4 in. − 1 yd. 30 in.; 2 lb. 10 oz. − 15 oz.; 4 bu. 2 pk. − 3 pk.; 5 rd. 2 ft. − $3\frac{1}{2}$ ft.; 2 yd. 1 ft. − 1 yd. 2 ft.; 5 rd. $6\frac{1}{2}$ ft. − 2 rd. 13 ft.

14. Multiply: 2 ft. 6 in. by 2; 3 ft. 4 in. by 3; 4 ft. 3 in. by 4; 1 ft. 8 in. by 2; 2 lb. 2 oz. by 8; 1 lb. 4 oz. by 5.

15. Divide: 8 lb. 4 oz. by 4; 10 lb. 10 oz. by 5; 9 ft. 6 in. by 3; 1 lb. 10 oz. by 2; 1 lb. 8 oz. by 8; 3 yd. 4 in. by 10.

WRITTEN EXERCISE

First estimate the answer and write it down; then solve the problem and compare the answers (*see page 88*):

1. $\frac{5}{7} + \frac{16}{23}$.
2. $\frac{7}{8} + \frac{12}{13}$.
3. $\frac{6}{11} + \frac{17}{21}$.
4. $\frac{12}{17} + \frac{13}{20}$.
5. $\frac{11}{21} + \frac{13}{23}$.
6. $\frac{19}{23} - \frac{11}{20}$.
7. $\frac{16}{33} - \frac{5}{21}$.
8. $\frac{32}{35} - \frac{7}{15}$.
9. $\frac{27}{28} - \frac{13}{30}$.
10. At 23¢ a yard, what will $8\frac{3}{4}$ yd. of ribbon cost?
11. At 18¢ a pound, what will $7\frac{3}{4}$ lb. of meat cost?
12. At $12\frac{1}{2}$¢ a yard, what will 18 yd. of cotton cost?
13. At \$1.30 a yard, what will 27 yd. of cheviot cost?
14. At \$2.30 a yard, what will $15\frac{1}{2}$ yd. of velvet cost?
15. At 24¢ a yard, what will $14\frac{3}{4}$ yd. of gingham cost?
16. If $7\frac{3}{4}$ yd. of velvet cost \$12.40, what is the price per yard?
17. If a dealer pays \$18.75 for 150 lb. of figs, what does he pay per pound?
18. A woman buys $3\frac{1}{4}$ lb. of coffee for \$1.04. What does she pay per pound?
19. A train of cars goes 63.9 mi. in an hour and a half. What is its rate per hour?
20. A man pays \$216 a year for the rent of his house. What is the rate per month?
21. A laundryman pays \$5.40 for 160 cakes of soap. What does he pay per cake?
22. A dealer pays \$33 a dozen for hats. At this rate, what does he pay for 50 hats?
23. If a dealer pays 28¢ a pound for a certain grade of cinnamon, at what price per ounce must he sell it to gain 0.75¢ an ounce?

ORAL EXERCISE

Express as per cents:

1. $\frac{1}{2}$. 2. $\frac{1}{4}$. 3. $\frac{3}{4}$. 4. $\frac{1}{8}$. 5. $\frac{3}{8}$. 6. $\frac{5}{8}$.

7. $\frac{1}{16}$. 8. $\frac{3}{16}$. 9. $\frac{5}{16}$. 10. $\frac{4}{5}$. 11. $\frac{7}{25}$. 12. $\frac{9}{50}$.

Express as common fractions:

13. 25%. 14. $33\frac{1}{3}$%. 15. 20%.

16. $12\frac{1}{2}$%. 17. $16\frac{2}{3}$%. 18. $8\frac{1}{3}$%.

19. How much is $16\frac{2}{3}$% of 6? of 54? of 48? of 60?

20. \$15 is what per cent of \$45? of \$75? of \$150?

21. A quart is what per cent of a peck? of a gallon?

22. How much is $66\frac{2}{3}$% of 3? of 60? of 90? of 120?

23. How much is $6\frac{1}{4}$% of 16? of 32? of 64? of 160?

24. How much is $33\frac{1}{3}$% of 9? of 3? of 675? of 1233?

25. How much is 50% of 96? of 120? of 480? of 15?

26. How much is 25% of 60? of 88? of 500? of 1200?

27. How much is $12\frac{1}{2}$% of 40? of 64? of 888? of 16?

28. A man pays \$2.50 a year interest on \$50. What is the rate?

29. A school having 240 pupils graduated 30. What per cent were graduated?

30. A merchant gained \$360 on \$3000 worth of goods. What was his rate of gain?

31. A man pays \$24 interest for 6 mo. on a loan of \$1200. What is the rate per year?

32. How much is $8\frac{1}{3}$% of 120? of 240? of 600? of 2400? of 4800? of 6000? of 12,000?

33. 25 is 1% of what number? 10% of what number? 50% of what number? $33\frac{1}{3}$% of what number?

WRITTEN EXERCISE

1. $\frac{22}{25} \times \frac{161}{154}$.
2. $\frac{33}{35} \times \frac{63}{55}$.
3. $\frac{18}{17} \times \frac{102}{126}$.
4. $\frac{28}{25} \times \frac{261}{196}$.
5. $\frac{42}{43} \times \frac{344}{189}$.
6. $\frac{13}{44} \times \frac{121}{117}$.
7. $\frac{52}{53} \div \frac{364}{371}$.
8. $\frac{23}{40} \div \frac{161}{104}$.
9. $\frac{15}{37} \div \frac{105}{185}$.
10. $\frac{27}{32} \div \frac{135}{128}$.
11. $\frac{31}{41} \div \frac{62}{123}$.
12. $\frac{51}{64} \div \frac{17}{128}$.
13. $\frac{21}{22} + \frac{5}{11} + \frac{3}{44}$.
14. $\frac{32}{35} + \frac{1}{5} + \frac{5}{7}$.
15. $\frac{8}{9} + \frac{15}{16} + \frac{51}{144}$.

16. On a shipment of 525 bbl. of apples 4% were frozen. How many were not frozen?

17. A bookseller pays 50¢ less $\frac{1}{6}$ for a book, and sells it for 50¢. What is his per cent of profit?

18. A farmer sold 3 cows for $110.40, which was 8% less than they cost. What was the cost of each?

19. A boy was sick and missed 5% of the 180 school days of a year. How many days was he in school?

20. A man worked 320 days last year, at $3, and put 12% of his wages in the bank. How much did he put in the bank?

21. A dealer sold a suit of clothes for $4.50 more than it cost, thus making a profit of 25%. At what price did he sell the clothes?

22. A dealer sold an automobile for $90 less than the marked price, thereby throwing off 12%. What is the price at which he sold the automobile?

23. An agent charges a commission of $2\frac{1}{2}$% for selling some property, and thereby receives $36.25. At what price did he sell the property? What did he remit to his employer after deducting his commission?

24. A dealer bought a suit of clothes for $20 and sold it for $25. The suit cost the manufacturer $15. What was the manufacturer's rate of profit? What was the dealer's rate of gain on what he paid? on what the customer paid?

ORAL EXERCISE

1. How many cubic feet in a room 20 ft. by 15 ft. by 10 ft.?

2. A rectangle is 6 ft. wide and $33\frac{1}{3}\%$ longer. What is the perimeter?

3. A dealer buys \$126 worth of books at $\frac{1}{6}$ discount. What does he pay?

4. A school buys \$12 worth of maps at 20% discount. What do the maps cost?

5. After spending 10% of his money a man has \$45. How much had he at first?

6. After gaining 5% on his money a man had \$210. How much had he at first?

7. After selling 25% of his papers a newsboy had 27. How many had he at first?

8. At a bargain sale a woman buys \$18 worth of dry goods at $\frac{1}{3}$ off. What does she pay?

9. The width of a parallelogram is $33\frac{1}{3}\%$ of the length. The length is 21 ft. What is the area?

10. John has gained 10% in height this year, and is now 4 ft. 7 in. tall. How tall was he a year ago?

11. The base of a triangle is 50% greater than the altitude. The altitude is 20 in. What is the area?

12. One of the two equal sides of an isosceles triangle is 20 in., and the base is 25% of the sum of the equal sides. What is the perimeter?

13. A square and an equilateral triangle have equal perimeters. 120 in. The side of the square is what per cent of the side of the triangle?

WRITTEN EXERCISE

1. $3\frac{2}{3} \times 2\frac{7}{8}$.
2. $4\frac{1}{9} \times 3\frac{5}{7}$.
3. $2\frac{4}{5} \times 3\frac{7}{8}$.
4. $6\frac{3}{4} \times 5\frac{3}{8}$.
5. $4\frac{7}{9} \times 3\frac{3}{5}$.
6. $5\frac{3}{16} \times 2\frac{3}{8}$.
7. $16\frac{5}{8} \div 2\frac{3}{8}$.
8. $20\frac{4}{5} \div 3\frac{4}{5}$.
9. $16\frac{5}{7} \div 5\frac{4}{7}$.
10. $13\frac{11}{16} \div 4\frac{9}{16}$.
11. $22\frac{3}{5} \div 5\frac{13}{20}$.
12. $17\frac{1}{4} \div 2\frac{7}{8}$.

13. A bale of worsted weighing 75 lb. loses 8 oz. in reeling off. What is the per cent of loss?

14. It gains 0.45 lb. to the 75-lb. bale in dyeing. What is the per cent of gain?

15. It loses 12 oz. on the machines in being made into Astrakhan cloth. This is what per cent of the original weight of the bale?

16. This 75-lb. bale cost \$50.25, and it lost 4 oz. in the fulling mill. What was the value of the part lost?

17. The net loss (Exs. 13–16) is what per cent of the original weight? What is its value at 67¢ a pound?

18. A yard of this cloth contains 2 lb. of worsted at 67¢ a pound and $1\frac{1}{4}$ lb. of cotton at 18¢ a pound. Each is what per cent of the total cost of materials?

19. To the cost of materials in Exs. 16, 18, add 3¢ a yard for knitting the cloth, $2\frac{1}{2}$¢ a yard for dyes, $3\frac{1}{2}$¢ a yard for dyeing, and 20% of the cost of raw materials for factory expense, and find the total cost per yard.

20. A certain grade of this cloth costs \$1.48 a yard. It sells for \$2 less 10%. The salesman is allowed 5% on the list price. What profit will the factory make on 5000 yd.?

21. 75 lb. of worsted is made into Astrakhan cloth, at 2 lb. to the yard. In fulling, this cloth stretches 5 yd., and shrinks from 110 in. in width to 55 in. Does the piece gain or lose in area, and how much? What per cent?

ORAL EXERCISE

1. At $31\frac{1}{2}$ gal. to the barrel, how many barrels in 126 gal.? in 315 gal.? in 630 gal.?

2. At $4\frac{1}{7}$ cu. ft. to the barrel, how many barrels in 29 cu. ft.? in $8\frac{2}{7}$ cu. ft.? in $20\frac{5}{7}$ cu. ft.?

3. At $24\frac{3}{4}$ cu. ft. to the perch, how many perches in 99 cu. ft.? in $49\frac{1}{2}$ cu. ft.? in 2375 cu. ft.?

4. At 231 cu. in. to the gallon, how many gallons in 924 cu. in.? in 462 cu. in.? in 693 cu. in.?

5. At \$5.60 a ton, how much will 10 T. of coal cost? $\frac{1}{8}$ T.? $\frac{3}{8}$ T.? $10\frac{3}{8}$ T.? 10 T. 750 lb.? 2 T.? 20 T.?

6. At $31\frac{1}{2}$ gal. to the barrel, how many gallons in a 50-barrel water tank? in a 100-barrel water tank?

7. A vase contains 264 cu. in. of water. What fraction more than a gallon does it contain?

8. Allowing 35 cu. ft. to a ton of coal, how many tons will fill a bin containing 315 cu. ft.?

9. Allowing 22 bricks to the cubic foot, how many bricks in a wall containing 5000 cu. ft.?

10. An oat bin contains 50 cu. ft. How many bushels does it contain at $1\frac{1}{4}$ cu. ft. to the bushel?

11. A pile of wood is 8 ft. high, 4 ft. wide, and 16 ft. long. How many cords does it contain?

12. Allowing 500 cu. ft. of timothy hay to a ton, how many tons will fill a haymow containing 10,500 cu. ft.?

13. A cellar 21 ft. by 15 ft. is to be excavated to a depth of 6 ft. What will it cost at 50¢ a load (cubic yard)?

14. A wall is 60 ft. long, 5 ft. high, and 1 ft. thick. How many bricks does it contain at 22 to the cubic foot?

WRITTEN EXERCISE

1. 42% of $\frac{3}{14}$. 2. 24% of $\frac{25}{36}$. 3. 36% of $\frac{75}{81}$.
4. 56% of $\frac{25}{46}$. 5. 64% of $\frac{25}{32}$. 6. 84% of $\frac{25}{49}$.

7. If $\frac{7}{8}$ yd. of velvet costs \$2.80, what do $6\frac{1}{4}$ yd. cost?

8. A dealer paid \$341.25 for 10,500 ft. of lumber. What was the price per M?

9. How many bushels in a bin 7 ft. 6 in. long, 6 ft. 6 in. wide, and 5 ft. deep? (Take 1 bu. = 2150 cu. in.)

10. What is the number which must be subtracted from $7\frac{3}{16}$ in order to leave a remainder of $4\frac{25}{48}$?

11. How much will $7\frac{1}{8}$ mi. of telegraph wire cost at $33\frac{1}{3}¢$ a pound, allowing 1 lb. of wire to 22 ft.?

12. A dealer sold 72 barrels of flour at \$8.50 a barrel, thereby gaining \$43.20. What did each barrel cost him?

13. A farmer sold $\frac{3}{8}$ of his 152-acre farm, and afterwards sold 19 A. The part that remained was what per cent of the farm?

14. A man spends $\frac{3}{5}$ of his income for board and clothes, $\frac{1}{8}$ of it for other expenses, and saves \$330 a year. What is his income?

15. A farmer weighed the hay from $4\frac{1}{2}$ A. of a 32-acre meadow and found he had 11 T. 500 lb. At this rate, how many tons did he get from the meadow?

16. In taking an inventory a dry goods merchant finds that he has $6\frac{1}{4}$ pieces of pongee, $48\frac{1}{2}$ yd. in each piece, which cost him $37\frac{1}{2}$ ct. a yard. What is the total value?

17. How much will it cost to dig a ditch for a water main extending 15 city blocks, the ditch being 6 ft. deep and 2 ft. wide, at 50¢ a cubic yard, the blocks running 20 to the mile?

ORAL EXERCISE

1. If 4 collars cost 90¢, what will 6 cost?

The pupil should think, "Half as much more." Many such examples admit of these short methods.

2. If 7 pencils cost 21¢, what will 8 cost?
3. If 4 arithmetics cost $2.40, what will 5 cost?
4. If 3 lamp chimneys cost 39¢, what will 2 cost?
5. If 4 lb. of butter cost 88¢, what will 9 lb. cost?
6. If 2 yd. of cloth cost 62¢, what will 10 yd. cost?
7. If 3 pairs of cuffs cost 90¢, what will 5 pairs cost?
8. If 5 reams of paper cost 90¢, what will 3 reams cost?
9. If chairs cost $18 per half dozen, what will 7 cost?
10. If 3 yd. of carpet cost $1.56, what will 20 yd. cost?
11. If 3 tablets cost 24¢, how many can be bought for 40¢?
12. If a 6-lb. roast of beef cost $1.08, what will a 9-lb. roast cost?
13. If the length of 2 meters is 2.2 yd., how many yards in 7 meters?
14. A dealer pays $45 for 3 suits of clothes. What is the price per dozen?
15. At $15 per half dozen, what will a dozen and a half of fountain pens cost?
16. If an express train makes 90 mi. in 2 hours, how long will it take it to make 135 mi.?
17. If 4 men can lay some stone pavement in 5 days, how long will it take 10 men to lay it?
18. If 8 men can do a piece of work in 10 days, how many men will it take to do it in 16 days?

WRITTEN EXERCISE

1. A dealer sells for $238 some goods that cost him $59.50 less than this. What was his rate of gain on the cost?

$59.50 is the required per cent of what number?

2. A dealer sells for $2451 some goods that were marked $129 more than this. What rate of discount did he allow on the marked price?

$129 is the required per cent of what number?

3. A grocer sold some canned goods at a profit of 20%, making $70. What was the selling price?

4. A merchant paid $232.50 for some goods after a discount of 7% had been allowed. What was the marked price?

$232.50 is 93% of what number?

5. The total cost of a house, including the architect's commission of 5%, was $7644. How much was the commission?

6. After deducting his commission of 6% for selling some property, an auctioneer remitted $258.50. How much was the commission?

7. The sum of the principal and the interest at 5% on a note for a year is $246.75. What is the interest?

$246.75 is 105% of what number?

8. A man's salary this year is $935, an increase of 10%. What was it last year?

9. A number is increased 10%, and the result is decreased 20%. There remains 220. What is the number?

10. A man saved $427.50 this year, which is 10% less than he saved last year. What did he save last year?

11. If a man pays \$196 for 28 weeks' board, what will he pay for 37 weeks?

12. If the interest on \$255 is \$7.65 a year, what is the interest on \$385 at the same rate?

13. If it costs \$140 to insure some property for \$10,000, what will it cost to insure it for \$7500?

14. If the area of a certain field is 90 A., what is the area of one of exactly the same shape, but half as long?

If it is half as long, what about the width? the area?

15. If the interest on a certain sum for 2 yr. is \$38, what is the interest on twice that sum for 6 mo. at the same rate?

16. If the freight on some boxes is \$2.80 for a distance of 150 mi., what is it on the same boxes for 250 mi. at the same rate?

17. If \$2.50 pays the freight on 750 lb. for 450 mi., for how many miles will it pay the freight on half a ton at the same rate?

18. An agent has \$3248 to invest after deducting his commission of $1\frac{1}{2}\%$ on the amount invested. What does he invest?

19. If a 3-ft. post casts a shadow 4 ft. long at the same time that a church spire casts a shadow 116 ft. 8 in., how high is the spire?

20. A building known to be 90 ft. high casts a shadow that just reaches across the street. At the same time a boy 4 ft. 6 in. high finds that his shadow is 6 ft. long. How wide is the street?

21. A dealer has a balance of \$2815.20 due him from his agent. Instead of taking the money he instructs the agent to invest it in certain goods, deducting his commission of 2% on the amount invested. What is the commission?

CHAPTER III

I. A GENERAL REVIEW OF ARITHMETIC

WRITING NUMBERS

331. Reason for this review. This class has now studied most of the important subjects of Arithmetic. Soon we shall take the more advanced business applications. As an aid to this advanced work we should review the foundations of all work in Arithmetic, the common operations.

Where such a review is not thought to be advisable, it will of course be omitted.

332. Origin of our numerals. Our numerals, 1, 2, and so on to 9, originated in India about 2000 years ago. The zero (0) was added more than 1200 years ago, thus making the system the excellent one that we now know. Without the 0 it was but little better than the Roman system. The numerals were learned by the Arabs more than 1100 years ago, and by A.D. 1200 became somewhat known in Europe. About 400 years ago they became well known in schools and in business, and they are now used in most of the civilized world.

333. The great feature of this system. It is the *place value* that makes the Arabic or Hindoo system better than all others. The Roman VI means five + one, while 51 means five *tens* + one, the 5 having not only the value *five*, but the place value *tens*.

334. Decimal system. Because the places or orders increase tenfold to the left, and decrease by tenths to the right, we call our system a *decimal system*, from the Latin *decem*, meaning ten.

ORAL EXERCISE

1. How many different figures are used in the Arabic system? How many in the Roman system for numbers below a hundred?

2. If I write *ten five* in figures in the two systems, 105 and XV, what values are indicated? Why do the two numbers have different values? What is the use of the 0?

3. In the number 1,904,526,738, give the place value of each figure, as 8 units, 3 tens, and so on. Give the name of each period, as 738 units, 526 thousands, and so on.

4. What do you mean by the words *separatrix, order, period, decimal, naught, place value, billion?* Do not try to repeat definitions, but answer in your own way.

5. Why is our common system, the Arabic, better than the old Roman system? You might illustrate by trying to multiply one number by another.

6. Read the following numbers taken from book chapters: XXVIII, XLI, LXI, LXXXIX, XCIV, CLXVI.

WRITTEN EXERCISE

1. Write in the Arabic system: MDCLXIX, MCCCXLIV.

2. Write in the Roman system: 49, 79, 94, 96, 99, 146.

3. Write in the Roman system the number of the present year; of last year; of fifty years ago.

4. Write in the Arabic system the number twenty-one million, four hundred seventy-five.

5. Write in common figures the number one billion, one million, one thousand one, and one ten-thousandth.

6. Write in common figures the number of Ex. 5 decreased by 42,675; also by 12.245; also by 295,001.127.

335. Uniform scale. Because in our system of writing numbers each place has uniformly ten times the value of the place at its right, we are said to use a *uniform scale.* Because this is a scale of ten it is called a *decimal scale.*

336. Varying scale. A scale that is not uniform is called a *varying scale.*

Uniform Scale	*Varying Scale*
10 units = 1 ten.	2 pints = 1 quart.
10 tens = 1 hundred.	8 quarts = 1 peck.
10 hundreds = 1 thousand.	4 pecks = 1 bushel.

ORAL EXERCISE

1. What is an even number? Illustrate.

2. What name is given to numbers that are not even?

3. How can you tell whether or not a number is even?

4. What is meant by the word *integer?* the word *fraction?* the expression *decimal system?*

5. Give three illustrations of a varying scale. What kind of scale is that of United States money?

6. Tell some reasons why the decimal fraction is better for practical use than the common fraction, except in a few cases.

WRITTEN EXERCISE

1. Write in common figures the number one hundred forty-two thousand, eight hundred fifty-seven.

2. Multiply the number in Ex. 1 by 2, by 3, by 4, by 5, and by 6, and write the results in words.

The results in Ex. 2 are peculiar, all having the same figures as the number multiplied, but differently arranged.

3. Count by 3's from 3 to 27 and write the results. Multiply 37 by each of these numbers and write the nine products.

In Ex. 2 one of the interesting numbers of Arithmetic was considered, and another of the curiosities of number is seen in this example. With a view to further interest, several other curious properties are given in the following examples.

4. Count by 9's from 9 to 81 and write the results. Multiply 12,345,679 by each of these numbers and write the nine products.

5. Multiply 142,857 by 326,451 and tell the peculiarities of the partial products.

6. Multiply the following numbers by 7: 15,873, 31,746, 47,619, 63,492, 79,365, 95,238, 111,111, 126,984, 142,857, each number being 15,873 more than its predecessor.

7. Perform the multiplications indicated: 9×9, 99×99, 999×999, 9999×9999. Write a rule for such products, and apply it to writing the product of 99,999 by 99,999.

8. Perform the multiplications indicated: 11×11, 111×111, 1111×1111. Write a rule for such products, and apply it to writing the product of 111,111 by 111,111.

9. Multiply 143 by 7; by 11×7; by 135×7; by 234×7. From these results tell what the result would be of multiplying by 345×7; by 789×7.

10. Write the values of the following:

$0 \times 9 + 1$	$0 \times 9 + 8$
$1 \times 9 + 2$	$9 \times 9 + 7$
$12 \times 9 + 3$	$98 \times 9 + 6$
$123 \times 9 + 4$	$987 \times 9 + 5$
$1234 \times 9 + 5$	$9876 \times 9 + 4$

337. Units. We usually speak of things counted as *units*. If we are counting dollars, \$1 is the unit; if cents, 1 ct. is the unit; if dozens, 1 doz. is the unit; if thousands, 1000 is the unit. If we are counting fifths, $\frac{1}{5}$ is the unit, and we count $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and so on. A unit may, therefore, be one object, or a group of objects, or a part of some object.

ORAL EXERCISE

1. Count from $\frac{1}{8}$ to 1, using $\frac{1}{8}$ as the unit.

2. Count from 0.1 to 1, using 0.1 as the unit.

3. State the most common units of length and tell where we might use each.

4. State the most common units of area and tell where we might use each.

5. State the most common units of capacity (and volume), including liquid, dry, and cubic measures.

6. State some units suggested by these numbers:

$\frac{2}{3}\frac{2}{3}$ 0.125 325 ft. 263 bu. 5000 300

7. In the fraction $\frac{3}{2}\frac{1}{5}$, what is the fractional unit? What term of the fraction tells you this? What term tells how many such units are taken?

WRITTEN EXERCISE

1. Express 233 units, each unit being $\frac{1}{4}\frac{1}{2}\frac{1}{7}$; 1 ft.; 0.001.

2. Express in terms of the next smaller unit in the table: 37.5 ft., 48 gal., 23 mi., 48 yd., 75 lb., 6 T., 81 sq. ft.

3. Express in terms of the next larger unit in the table: 37 in., 42 ft., 36 oz., 960 rd., 4000 lb., 100 qt., 1728 in.

4. Express in terms of the unit 1 in. the following: 9 rd., 7 yd., 6 mi., 4 ft., 3 yd., 7 ft., $3\frac{1}{2}$ ft., $7\frac{1}{2}$ yd., $6\frac{1}{2}$ rd., $2\frac{1}{2}$ mi.

ORAL EXERCISE

1. In the common fraction $\frac{3}{4}$, name the terms and tell what each expresses about the fraction.

2. The fraction $\frac{3}{4}$ may be thought of as meaning 3 of what parts of a unit? Or it may be thought of as $\frac{1}{4}$ of how many units?

3. If a yard is called *one*, a foot will be called what fraction? An inch will be called what fraction? A rod will be called what mixed number?

4. How is a fraction reduced to lower terms? When is it said to be in its lowest terms? Why do you ever need to reduce fractions to lowest terms?

5. How do you proceed to add one fraction to another? to subtract one fraction from another? Why is this?

6. How do you know that $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$? Illustrate at the board by taking $\frac{1}{2}$ of $\frac{1}{3}$ of a foot. Show also that $\frac{1}{3}$ of $\frac{1}{2}$ has the same value as $\frac{1}{2}$ of $\frac{1}{3}$.

WRITTEN EXERCISE

1. Draw a rectangle 1 in. long. Find $\frac{1}{2}$ of $\frac{1}{4}$ of it; also $\frac{1}{4}$ of $\frac{1}{2}$ of it. Show that $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{4}$ of $\frac{1}{2}$.

2. In the same way show by a line that 2 times $\frac{1}{3}$ in. = $\frac{2}{3}$ in., and that $\frac{1}{3}$ of 2 in. = $\frac{2}{3}$ in.

3. Find the value of $\frac{1}{5}$ of $\frac{2}{3}$ in., representing the work by lines. Also find the value of $\frac{2}{3}$ of $\frac{1}{5}$ in.

4. In the same way show that $\frac{2}{5}$ of $\frac{2}{3}$ in. = $\frac{4}{15}$ in.; also that $\frac{2}{3}$ of $\frac{2}{5}$ in. = $\frac{4}{15}$ in.; also that $\frac{2}{5}$ of $\frac{3}{4} = \frac{3}{4}$ of $\frac{2}{5}$.

5. Mark an inch line into halves, and also into thirds. Show from it that the ratio of $\frac{1}{2}$ to $\frac{1}{3}$ is 3 : 2, or that $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$. Show also that $\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$.

ORAL EXERCISE

1. Read: $2\frac{3}{5}$, 2 ft. 3 in. Explain the difference in scales.

2. Read: 5.8, 5 lb. 8 oz., 5 hr. 8 min. State the scales.

3. Recite the tables of length, square measure, cubic measure, dry and liquid measure, weight, United States money, and time; also the table of dozens and gross.

4. Express as inches: 2 ft. 3 in., 5 ft. 8 in., 100 ft. 9 in., 1 yd., 1 yd. 2 in., 1 yd. 1 ft., 1 yd. 10 in.

5. Express as feet: 3 yd., 3 yd. 2 ft., 7 yd., 7 yd. 1 ft., 10 yd., 60 in., 2 rd., 4 rd., 2 rd. 3 ft., 2 rd. 2 yd.

6. Express as pounds or fractions of a pound: 32 oz., 8 oz., 4 oz., 1 T., 2 T., $3\frac{1}{2}$ T., 4 cwt., 1 T. 2 cwt.

7. Express as quarts, liquid or dry as the case may be: $20\frac{1}{2}$ gal., 2 gal., 2 bu., 3 pk., 2 pt., 5 gal. 2 qt., 2 pk. 1 qt.

WRITTEN EXERCISE

1. Write 3456 as dozens; as gross; as great gross.

2. Write as inches: 17 ft., 21 ft. 6 in., 3 yd., 4 rd.

3. Write as feet: 36 in., 144 in., 1728 in., 54 yd., $\frac{3}{5}$ rd.

4. Write as pounds: $2\frac{1}{2}$ T., 3.25 T., 144 oz., 3 T. 1520 lb., 4 cwt. 27 lb., 1584 oz.

5. Write as quarts: 53 gal., 26 bu., 17 gal. 3 qt., 98 pt., 31 bu. 2 pk., $31\frac{1}{2}$ gal., $75\frac{3}{4}$ gal., $17\frac{1}{2}$ bu.

6. Express as units: 6 gross 3 doz., $7\frac{1}{2}$ gross, $15\frac{1}{3}$ doz., 6 great gross, $15\frac{3}{4}$ doz., $\frac{1}{6}$ doz., $27\frac{1}{2}$ gross.

7. Explain what is meant by a uniform scale; a varying scale. Give an illustration of each.

8. Tell why a varying scale is not, in general, so good as a uniform scale. Illustrate by examples.

ADDITION REVIEWED

338. The principle of addition. The principle of addition is the same whether we add integers, fractions, denominate numbers, or expressions containing letters.

(1)	(2)	(3)
6 8	6 . 8	$6\frac{8}{11}$
9 7	9 . 7	$9\frac{7}{11}$
(15)(15) =	(15)·(15) =	$15\frac{15}{11}$ =
165, because 15 units = 1 ten + 5 units.	16.5, because 15 tenths = 1 + 5 tenths.	$16\frac{4}{11}$, because $\frac{15}{11} = 1\frac{4}{11}$.

In each case the number of units is the same. In each the addition 7 + 8 requires us to add 1 to the next higher order.

In oral addition, as of 68 and 97, it is usually better to begin at the left. In this case say: "97, 157, 165"; that is, 97 + 60 = 157, 157 + 8 = 165.

ORAL EXERCISE

Add, stating only the answers:

1. 23	2. 61	3. 37	4. 29	5. 34
49	28	48	63	48
6. 34	7. 26	8. 82	9. 68	10. 47
85	96	75	68	53
11. 121	12. 132	13. 126	14. 235	15. 128
69	48	57	75	89

16. Add 2 in., 3 in., 3 in., 7 in., 11 in. Express the result also as feet and a fraction; as feet and inches.

17. What do we mean by addition? by addends? by the sum? How do we check (or prove) our additions? Give examples.

WRITTEN EXERCISE

Add, checking (proving) the work by adding in reverse order. Time yourself.

1.	2.	3.	4.
472	47 ft. 2 in.	4 mi. 7 yd. 2 ft.	47 bu. 2 pk.
691	69 1	6 9 1	69 1
580	58	5 8	58
432	43 2	4 3 2	43 2
841	84 1	8 4 1	84 1
570	57	5 7	57

5. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{6}$ to twelfths and add. Also reduce to twenty-fourths and add.

6. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$ to eighths and add. Also reduce to decimal fractions and add, checking by reducing the result to an integer plus a common fraction.

7. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, by reducing to sixths; to twelfths; to eighteenths. Show that the results are equal.

8. Add $\frac{1}{8}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{2}{3}$, $\frac{7}{12}$. (Which is the better plan in this case, to reduce to the denominator 120, or to decimal fractions?)

9. Add $\frac{3}{5}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{7}{10}$. (To what common denominator may these fractions be reduced? Is it better to reduce to decimal fractions? Why?)

10. Add the following:

$5x + 1y$	501	5 ft. 1 in.	5 lb. 1 oz.
$6x + 5y$	605	6 5	6 5
$4x + 3y$	403	4 3	4 3
$7x + 6y$	706	7 6	7 6
$9x + 8y$	908	9 8	9 8

Notice that the first of these results reduces to the second if $x = 100$ and $y = 1$; to the third if $x = 1$ ft., $y = \frac{1}{12}$ ft., or 1 in.; to the fourth if $x = 1$ lb., $y = \frac{1}{16}$ lb., or 1 oz.

Add, checking the work by adding in reverse order. Time yourself.

11.	12.	13.	14.
$31.50	$62.35	$20.00	$35.75
26.75	49.82	35.40	87.66
15.42	81.46	20.76	92.33
51.13	73.98	83.92	83.47
73.06	20.05	99.81	62.54
52.70	30.00	78.56	81.85

15.	16.	17.	18.
$312.75	$402.75	$209.81	$175.54
63.40	75.75	801.19	285.46
281.00	163.87	634.77	87.92
127.05	91.83	75.23	93.41
31.63	182.48	62.97	62.78
428.70	328.62	47.03	88.39

19.	20.	21.	22.
$486.83	$128.92	$634.81	$189.98
642.75	871.08	528.92	235.42
208.05	346 52	365.19	628.75
100.00	853.50	471.08	832.63
378.92	671.18	275.50	798.49
604.83	832.00	325.50	629.36

23.	24.	25.	26.
$1276.35	$3246.81	$8234.61	$2983.41
397.63	2981.26	8327.92	3872.62
428.42	583.42	2842.68	2987.48
608.70	629.87	178.53	293.29
2304.69	3473.92	296.48	981.50
572.96	682.58	378.75	6287.49
3270.05	2987.62	2876.92	832.08

27. The following are some recent statistics of the great industries of the United States.

Industry	Number of Establishments	Money Invested	Number of Wage Earners	Annual Wages	Cost of Materials	Value of Products
Wool Manuf. .	2,636	$415,075,713	264,021	$92,499,262	$250,805,214	$427,905,020
Cotton Manuf. .	1,051	467,240,157	302,861	86,689,752	176,551,527	339,198,619
Iron and Steel .	661	573,119,275	222,264	120,723,092	522,071,772	803,344,591
Meat Industry .	921	189,198,264	68,534	33,457,013	683,583,577	786,603,670
Lumber . . .	33,035	611,611,524	283,260	104,640,591	317,923,548	566,832,984
Flour	25,258	218,714,104	37,073	17,703,418	475,826,345	560,719,063
Boots and Shoes	1,600	101,795,233	142,922	59,175,883	169,604,054	261,028,589
Publishing . .	15,305	192,443,708	94,604	50,214,051	50,214,904	222,983,569

Find the sums of the various columns.

Pupils should be timed in all such work, and accuracy should be insisted upon by requiring checks.

28. The following are some of the wealthiest countries in the world according to recent statistics, with the approximate money which each has.

Country	Population	Stock of Gold	Stock of Silver	Stock of Paper Money	Total Stock of Money
United States .	85,000,000	$1,174,600,000	$660,000,000	$437,800,000	
France	40,000,000	903,500,000	419,800,000	134,500,000	
Great Britain .	42,000,000	528,000,000	116,800,000	116,200,000	
Germany . . .	57,000,000	762,800,000	207,500,000	153,400,000	
Russia	131,000,000	714,600,000	103,200,000	——	
Austria-Hungary	47,100,000	257,000,000	80,000,000	39,900,000	
Australasia . .	5,500,000	128,600,000	6,100,000	——	

Add the columns. Fill the last column by adding the three preceding columns crossways to the right. How will you check the work?

SUBTRACTION REVIEWED

339. The principle of subtraction. The principle of subtraction is the same whether we subtract integers, fractions, denominate numbers, or expressions containing letters. The general principle is seen in the following:

(1)	(2)	(3)	(4)
61	6.1	$6\frac{1}{4}$	6 ft. 1 in.
23	2.3	$2\frac{3}{4}$	2 3
38	3.8	$3\frac{2}{4} = 3\frac{1}{2}$	3 ft. 10 in.

In each case the number of units is the same. In each case the subtraction $1 - 3$ requires us to increase the 1 by a unit of the next higher order before subtracting.

In oral subtraction, as of 23 from 61, it is usually better to begin at the left. In this case say: "61, 41, 38"; that is, $61 - 20 = 41$, $41 - 3 = 38$.

ORAL EXERCISE

Subtract, stating only the answers:

1. 82	2. 63	3. 72	4. 81	5. 73
27	19	35	24	49
6. 67	7. 42	8. 95	9. 90	10. 86
38	18	58	32	39
11. 102	12. 105	13. 204	14. 125	15. 143
43	36	95	67	85

16. What is meant by saying that "only units of the same kind can be added"? Can you not add \$2 and 3 ct.?

17. What is meant by saying that "only units of the same kind can be subtracted"? Can you not subtract 3 ct. from \$2?

WRITTEN EXERCISE

Subtract, checking each result by adding the subtrahend and remainder. Time yourself.

1. \$287.50 − 132.95
2. \$351.75 − 268.19
3. \$429.35 − 228.92
4. \$526.41 − 229.37
5. \$391.11 − 287.52
6. \$629.48 − 297.63
7. \$802.05 − 120.70
8. \$329.90 − 100.95
9. \$1235.00 − 926.75
10. \$2634.05 − 827.50
11. \$1963.00 − 298.50
12. \$5283.75 − 1296.55
13. \$285.42 − 196.55
14. 285 mi. 42 ft. − 196 mi. 55 ft.
15. 285 T. 42 lb. − 196 T. 55 lb.

Notice the similarity of the figures used in Exs. 13–15.

16. \$5432 − 2354
17. 54 mi. 32 ft. − 23 mi. 54 ft.
18. 54 T. 32 lb. − 23 T. 54 lb.

Notice the similarity of the figures used in Exs. 16–18.

19. \$1,725,342,628 − \$632,487,129.
20. \$27,142,628,341 − \$16,923,437,286.
21. \$2,421,002,625.25 − \$1,278,101,123.50.
22. 372 − 153
23. 3.72 − 1.53
24. 37 ft. 2 in. − 15 ft. 3 in.
25. 37 lb. 2 oz. − 15 lb. 3 oz.
26. 2 yd. 1 ft. − 1 yd. 2 ft.
27. 2 T. 1 lb. − 1 T. 2 lb.
28. 2 yr. 1 mo. − 1 yr. 2 mo.
29. 1915 yr. 2 mo. 3 da. − 1906 yr. 8 mo. 15 da.
30. 1920 yr. 0 mo. 6 da. − 1909 yr. 2 mo. 17 da.

31. Write out in your own way a statement of your method of adding numbers.

32. Write out in the same way a statement of your method of subtracting numbers.

33. $2738\frac{3}{7} - 1299\frac{5}{7}$.

34. $8721\frac{1}{7} - 1796\frac{3}{5}$.

35. $6592\frac{1}{3} - 4289\frac{2}{7}$.

36. $2872\frac{4}{5} - 1628\frac{1}{3}$.

37. $7291\frac{1}{2} - 2986\frac{3}{5}$.

38. $9289\frac{3}{8} - 2691\frac{5}{12}$.

39. 40 ft. $4\frac{1}{2}$ in. — 27 ft. $9\frac{1}{3}$ in.

40. 172 lb. $5\frac{1}{2}$ oz. — 69 lb. $10\frac{7}{8}$ oz.

41. 426 gal. $1\frac{3}{8}$ qt. — 197 gal. $3\frac{1}{3}$ qt.

42. $1345.25 + 175\frac{2}{3} + 375 - (\frac{3}{5} + \frac{3}{5} + \frac{3}{5})$.

43. 234 sq. ft. $9\frac{1}{2}$ sq. in. — 175 sq. ft. $98\frac{2}{3}$ sq. in.

44. 148 cu. ft. $521\frac{1}{8}$ cu. in. — 69 cu. ft. $827\frac{3}{5}$ cu. in.

45. 321 da. 6 hr. 2 min. 15 sec. — 125 da. 7 hr. 21.75 sec.

46. 273 da. 5 hr. 3 min. 17 sec. — 169 da. 23 hr. 43.25 sec.

47. The savings bank deposits in this country for the years 1900–1904 are indicated in this table. Find the increase for each year. Also find the increase of 1904 over 1900.

1900 . .	2,449,547,885
1901 . .	2,597,094,580
1902 . .	2,750,177,290
1903 . .	2,935,204,845
1904 . .	3,060,178,611

48. The five states having the largest population are here given, with the population according to a recent census. The population of New York was how much more than that of each other state?

New York . .	7,268,894
Pennsylvania	6,302,115
Illinois . . .	4,821,550
Ohio	4,157,545
Missouri . .	3,106,665

49. By the table of Ex. 48, the population of Pennsylvania was how much more than that of Illinois? of Ohio? of Missouri?

50. By the same table, the population of Illinois was how much more than that of Ohio? of Missouri?

MULTIPLICATION REVIEWED

340. The principle of multiplication. The principle of multiplication is the same for all numbers.

$7.50		7 ft. 5 in.	
9		9	
4.50	= 9 times $0.50	45 in.	= 9 times 5 in.
63	= " " $7	63 ft.	= " " 7 ft.
$67.50	= " " $7.50	66 ft. 9 in.	= " " 7 ft. 5 in.

In each case the number of units is the same. The results differ because the first number is on the scale of 10, the second on the scale of 12.

The principle is the same:

Multiply the denominations or orders separately. Then add the partial products, simplifying when possible.

In oral multiplication, as of $7.50 by 9, it is usually better to begin at the left. In this case we say: "$63, $4.50, $67.50."

ORAL EXERCISE

Multiply, stating only the answers:

1. 26	2. 37	3. 33	4. 43	5. 53
2	3	4	5	6
6. 25	7. 32	8. 54	9. 66	10. 70
7	8	9	11	12
11. $2.50	12. $7.50	13. $8.20	14. $3.30	15. $5.50
5	6	7	8	9

16. What is meant by saying that the multiplier must always be an abstract number? Illustrate.

17. What is meant by saying that the product must always be of the same denomination as the multiplicand?

WRITTEN EXERCISE

Multiply as indicated, noticing the similarity in the figures of the multiplicands:

1. By 7: 175, 17 ft. 5 in., 1 mi. 75 ft.
2. By 37: 427, 42 lb. 7 oz., 4 mi. 27 rd.
3. By 13: 936, $9.36, 93 ft. 6 in., 9036.
4. By 25: 428, 4 sq. ft. 28 sq. in., 4028.
5. By 1325: $63.25, 63 mi. 25 rd., 63,025.
6. By 59: $19.20, 19 gal. 2 qt., 19 yd. 2 ft.
7. By 2007: $1271.50, 127 da. 1 hr. 50 min.
8. By 625: 1728, 17 sq. ft. 28 sq. in., 17,028.

Multiply, writing the number of minutes required for Exs. 9–33:

9. 2765 × 127	10. 3402 × 348	11. 4246 × 293	12. 1563 × 892
13. 4762 × 987	14. 2983 × 896	15. 5786 × 469	16. 5999 × 987
17. $175.27 × 325	18. $287.62 × 625	19. $482.92 × 328	20. $576.75 × 476
21. $247.60 × 908	22. $409.75 × 607	23. $507.63 × 806	24. $707.07 × 999
25. 27 ft. 8 in. × 17	26. 29 ft. 9 in. × 32	27. 36 ft. 4 in. × 43	
28. 82 ft. 7 in. × 68	29. 12 lb. 8 oz. × 2	30. 16 lb. 9 oz. × 23	
31. 17 lb. 15 oz. × 27	32. 23 lb. 4 oz. × 24	33. 24 ft. 9 in. × 48	

341. Multiplying by fractions. In multiplying by fractions we may use the following forms to show the similarity, although we have shorter ones for actual work.

69	69	6.9	$\frac{4}{5}$
$\frac{2}{3}$	.2	.2	$\frac{2}{3}$
$23 = \frac{1}{3}$ of 69	$6.9 = .1$ of 69	$.69 = .1$ of 6.9	$\frac{4}{15} = \frac{1}{3}$ of $\frac{4}{5}$
2	2	2	2
$46 = \frac{2}{3}$ of 69	$13.8 = .2$ of 69	$1.38 = .2$ of 6.9	$\frac{8}{15} = \frac{2}{3}$ of $\frac{4}{5}$

In multiplying 69 by $\frac{2}{3}$ we simply *think* that $\frac{1}{3}$ of 69 is 23, and we multiply (mentally) 23 by 2.

In multiplying 6.9 by .2 we simply *think* that tenths multiplied by tenths is hundredths, and we multiply 69 by 2 and express the result as hundredths.

To multiply $3\frac{1}{3}$ by $2\frac{1}{2}$ we may say:

$$2\frac{1}{2} = \frac{5}{2}, \quad 3\frac{1}{3} = \frac{10}{3}, \quad \frac{5}{2} \times \frac{\overset{5}{\cancel{10}}}{3} = \frac{25}{3} = 8\frac{1}{3}.$$

ORAL EXERCISE

1. $\frac{3}{4}$ of $\frac{7}{8}$.	2. $\frac{2}{3}$ of $\frac{4}{7}$.	3. $\frac{2}{3}$ of $\frac{3}{4}$.
4. $\frac{5}{6}$ of $\frac{3}{4}$.	5. $\frac{1}{2}$ of $\frac{4}{5}$.	6. $\frac{1}{3}$ of $\frac{6}{7}$.
7. $\frac{1}{4}$ of $\frac{8}{9}$.	8. $\frac{4}{5}$ of $\frac{5}{8}$.	9. $\frac{7}{8}$ of $\frac{3}{10}$.
10. $\frac{3}{5}$ of $\frac{5}{7}$.	11. $\frac{1}{5}$ of $\frac{10}{11}$.	12. $\frac{5}{7}$ of $\frac{14}{15}$.
13. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$.	14. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.	15. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.
16. $\frac{3}{5}$ of $\frac{5}{8}$ of 8.	17. $\frac{2}{3}$ of $\frac{3}{4}$ of 10.	18. $\frac{1}{3}$ of $\frac{3}{7}$ of 7.
19. $\frac{5}{6}$ of $\frac{1}{2}$ of $1\frac{2}{3}$.	20. $\frac{2}{7}$ of $\frac{1}{2}$ of 14.	21. $\frac{1}{4}$ of $\frac{4}{5}$ of 50.
22. $\frac{1}{3}$ of $2\frac{1}{2}$.	23. $\frac{1}{4}$ of $1\frac{1}{3}$.	24. $\frac{1}{2}$ of $3\frac{1}{2}$.
25. $\frac{3}{4}$ of 44.	26. $\frac{2}{3}$ of 33.	27. $\frac{2}{3}$ of $1\frac{1}{2}$.
28. $\frac{5}{7}$ of 49.	29. $\frac{5}{7}$ of 77.	30. $\frac{4}{5}$ of 25.

31. How much will $\frac{5}{8}$ yd. of silk cost at 64¢ a yard?

32. How much will $\frac{7}{8}$ yd. of velvet cost at $1.60 a yard?

WRITTEN EXERCISE

1. 1.5×324.
2. $3\frac{4}{5} \times 65$.
3. $9\frac{2}{3} \times 72$.
4. $\frac{1}{2}$ of $73\frac{1}{2}$.
5. $\frac{2}{3}$ of $16\frac{3}{4}$.
6. $\frac{4}{5}$ of $19\frac{1}{2}$.
7. $2\frac{1}{2} \times 3\frac{2}{3}$.
8. $4\frac{2}{5} \times 6\frac{7}{8}$.
9. $5\frac{3}{8} \times 6\frac{1}{8}$.

10. State a brief rule for multiplying by a decimal fraction. Illustrate by 0.3 of 0.22.

11. State a brief rule for multiplying one common fraction by another. Illustrate by $\frac{2}{3}$ of $\frac{5}{7}$.

Multiply as indicated, timing yourself:

12. 23.4 × 4.5	13. 17.6 × 8.4	14. 35.7 × 6.9	15. 82.7 × 9.8
16. 99.8 × 7.9	17. 2.75 × .35	18. 7.48 × 1.25	19. 8.09 × 8.09
20. .632 × 8.25	21. .473 × 2.48	22. 6.42 × 3.27	23. 89.2 × 62.8
24. 34.9 × 30.5	25. 1.472 × 3.25	26. 6.043 × 1.37	27. 2.481 × 6.25
28. 0.735 × 6.25	29. 1.326 × 4.32	30. 2.307 × 53.6	31. 298.7 × .634
32. 812.8 × .437	33. 296.4 × .0124	34. 82.32 × .063	35. 487.6 × 2.083
36. \$14.92 × 2.35	37. \$15.75 × 3.42	38. \$14.87 × 2.91	39. \$65.04 × 9.09
40. 1484 × $3.2\frac{1}{2}$	41. 2636 × $4.1\frac{1}{4}$	42. 3792 × $6.23\frac{1}{3}$	43. 178.32 × $1.24\frac{2}{3}$

44. 49.74 × .0426

45. 82.49 × .0487

46. 693.4 × 83.7

47. $\frac{11}{36}$ of $\frac{18}{55}$.

48. $\frac{23}{24}$ of $\frac{12}{115}$.

49. $\frac{17}{32}$ of $\frac{64}{85}$.

50. $\frac{21}{25}$ of $\frac{75}{84}$.

51. $\frac{25}{32}$ of $\frac{16}{75}$.

52. $\frac{11}{12}$ of $\frac{13}{16}$.

53. \$327.76 × $13\frac{1}{2}$

54. \$139.83 × $37\frac{2}{3}$

55. \$164.24 × $34\frac{3}{8}$

56. \$325.84 × $39\frac{3}{4}$

57. \$841.40 × $63\frac{5}{7}$

58. \$935.44 × $87\frac{3}{4}$

59. 21 ft. 8 in. × $15\frac{1}{2}$

60. 38 lb. 2 oz. × $23\frac{3}{4}$

61. 62 yd. 27 in. × $25\frac{2}{3}$

62. $127\frac{2}{3} \times 135\frac{1}{4}$.

63. $229\frac{5}{8} \times 213\frac{2}{3}$.

64. $723\frac{3}{5} \times 734.8$.

65. $209\frac{5}{8} \times 235.5$.

66. $327\frac{2}{3} \times 333\frac{1}{3}$.

67. $187\frac{3}{16} \times 175\frac{1}{8}$.

68. At \$1.56 a yard, what will $14\frac{3}{4}$ yd. of silk cost?

69. At \$1.28 a yard, what will $37\frac{3}{8}$ yd. of carpet cost?

70. At \$5.60 a ton, what will 17 T. 250 lb. of coal cost?

71. What is the cost of $37\frac{1}{2}$ rd. of fence at $\$1.12\frac{1}{2}$ a rod?

72. When hay is selling at $\$9\frac{3}{4}$ a ton, what will $23\frac{1}{8}$ T. cost?

73. What is the number resulting from taking 32 ft. 9 in. as an addend 37 times?

74. At $78\frac{3}{4}$ lb. to the yard, what is the weight of a mile of double rails for a track?

75. A hexagon of equal sides is measured, and each side is found to be $19\frac{7}{32}$ in. What is the perimeter?

76. What is the number resulting from taking $17\frac{3}{7}$ as an addend 23 times, and then adding $\frac{1}{3}$ of $17\frac{3}{7}$ to the sum?

DIVISION REVIEWED

342. Nature of division. We have seen that division is the inverse of multiplication. That is,

because 2.5 times \$40 = \$100,
therefore \$100 ÷ \$40 = 2.5,
and \$100 ÷ 2.5 = \$40.

343. Two cases. There are, therefore, two cases of division:

1. *If the dividend and divisor are like numbers, the quotient is abstract.*

2. *If the dividend is concrete and the divisor abstract, the quotient is like the dividend.*

344. Similar classes in the first case. The similarity for various classes of numbers appears from the following:

572 ft. ÷ 24 in. may be reduced to 572 ft. ÷ 2 ft. = 286, or it may be reduced to 6864 in. ÷ 24 in. = 286;

55.2 ÷ 0.24 may be reduced to 55.20 ÷ 0.24, and treated as if it were 5520 ÷ 24 = 230;

$\frac{2}{3} \div \frac{4}{5}$ may be reduced to $\frac{10}{15} \div \frac{12}{15}$, and treated as if it were $10 \div 12 = \frac{5}{6}$. (See § 350 for the practical work.)

That is, in this case, *dividend and divisor are reduced to like numbers, or denominations, before dividing.*

345. Similar classes in the second case. In the second of these cases the similarity for various classes of numbers appears from the following examples:

\$10 ÷ .8 = \$100 ÷ 8, by multiplying each by 10;
\$10 ÷ $\frac{4}{5}$ = \$50 ÷ 4, " " " " 5,

although we have a simpler method for dividing common fractions, as explained in § 350, where it is shown that $10 \div \frac{4}{5} = 10 \times \frac{5}{4} = 12\frac{1}{2}$.

ORAL EXERCISE

Divide as indicated in Exs. 1–30:

1. $355 \div 5$.	2. $424 \div 4$.	3. $126 \div 3$.
4. $275 \div 5$.	5. $536 \div 4$.	6. $315 \div 3$.
7. $636 \div 6$.	8. $637 \div 7$.	9. $728 \div 8$.
10. $189 \div 9$.	11. $217 \div 7$.	12. $640 \div 8$.
13. $819 \div 9$.	14. $504 \div 7$.	15. $656 \div 8$.
16. $121 \div 11$.	17. $144 \div 12$.	18. $156 \div 12$.
19. $143 \div 13$.	20. $154 \div 14$.	21. $315 \div 15$.
22. $320 \div 16$.	23. $510 \div 17$.	24. $360 \div 18$.
25. $620 \div 20$.	26. $420 \div 21$.	27. $660 \div 22$.
28. $253 \div 23$.	29. $144 \div 24$.	30. $625 \div 25$.

Divide as indicated in Exs. 31–45, stating the remainders:

That is, $186 \div 17 = 10$, and 16 remainder.

31. $\$145 \div 12$.	32. $\$136 \div 13$.	33. $\$150 \div 14$.
34. $\$157 \div 15$.	35. $\$330 \div 16$.	36. $\$181 \div 17$.
37. $\$193 \div \$18$.	38. $\$200 \div \$19$.	39. $\$219 \div \$20$.
40. $\$430 \div \$21$.	41. $\$444 \div \$22$.	42. $\$691 \div \$23$.
43. 723 ft. $\div 24$.	44. 630 yd. $\div 25$.	45. 631 in. $\div 30$.

Divide as indicated in Exs. 46–63, including fractions in the quotients:

That is, $186 \div 17 = 10\frac{16}{17}$.

46. $1526 \div 5$.	47. $2751 \div 5$.	48. $6017 \div 5$.
49. $2419 \div 2$.	50. $3221 \div 2$.	51. $1683 \div 2$.
52. $3631 \div 3$.	53. $1234 \div 3$.	54. $6094 \div 3$.
55. $4805 \div 4$.	56. $8005 \div 4$.	57. $6013 \div 6$.
58. $7146 \div 7$.	59. $8009 \div 8$.	60. $8183 \div 9$.
61. $6477 \div 10$.	62. $5239 \div 10$.	63. $1211 \div 11$.

346. General principle of division. We have learned how to divide by *long division* and by *short division.* The general principle is, however, the same for both. Consider the case of $625 \div 25$.

$$\begin{array}{rl} 25 & \\ 25\overline{)625} & \\ \underline{500} & = 20 \text{ times } 25 \\ 125 & \\ \underline{125} & = \quad 5 \quad \text{“} \quad 25 \end{array} \qquad \begin{array}{l} 25\overline{)625} \\ \underline{25} \\ \\ \text{is the same as} \\ 25\overline{)500 + 125} \\ \underline{\quad 20 + \quad 5 = 25} \end{array}$$

In each case we see that $600 \div 25 =$ no hundreds; $620 \div 25 = 2$ tens, for 2 tens $\times 25 = 500$, and there is 125 left to be divided. Also, $125 \div 25 = 5$. Therefore the result is 25.

347. Division by a decimal. *In dividing by a decimal fraction always multiply both dividend and divisor by such a number as shall make the divisor a whole number.*

Thus, in the case of $62.5 \div 0.25$, multiply both by 100 and divide 6250 by 25.

WRITTEN EXERCISE

1. Divide 987 by 35 and write out the explanation.
2. Divide 575 by 25 in the three ways shown above.
3. Divide 506 by 22 in the three ways shown above.
4. At $26\frac{1}{2}$ ct. a yard, how many yards of cloth can be bought for \$36.57? If 138 yd. cost \$36.57, what will 1 yd. cost?
5. Derby hats cost from \$24 to \$42 a dozen. Make up two problems, similar to those in Ex. 4, state them, and solve.
6. Name and define the four terms used in division.
7. Write out a rule for dividing one number by another.

Divide as indicated, seeing how many problems you can solve in five minutes; in ten minutes:

8. $1554 \div 37$.	9. $1924 \div 52$.	10. $2793 \div 57$.
11. $3844 \div 62$.	12. $5593 \div 49$.	13. $2701 \div 73$.
14. $5402 \div 37$.	15. $3402 \div 81$.	16. $1225 \div 18$.
17. $1089 \div 33$.	18. $6822 \div 36$.	19. $8888 \div 808$.
20. $5203 \div 121$.	21. $20{,}273 \div 97$.	22. $17{,}841 \div 57$.
23. $2541 \div 121$.	24. $20{,}698 \div 98$.	25. $22{,}248 \div 72$.
26. $10{,}449 \div 81$.	27. $13{,}024 \div 407$.	28. $14{,}838 \div 209$.
29. $30{,}092 \div 121$.	30. $27{,}968 \div 304$.	31. $17{,}094 \div 777$.
32. $80{,}091 \div 809$.	33. $19{,}908 \div 237$.	34. $14{,}985 \div 666$.
35. $4291\overline{)55{,}783}$.	36. $2075\overline{)47{,}725}$.	37. $6772\overline{)277{,}652}$.
38. $3227\overline{)203{,}301}$.	39. $7407\overline{)674{,}037}$.	40. $2469\overline{)224{,}679}$.
41. $\$569.25 \div \$24.75$.	42. $\$570.24 \div \$17.82$.	
43. $\$1309.00 \div \$29.75$.	44. $\$2521.48 \div \$48.49$.	

45. If 35 floor tiles together weigh 113.75 lb., what is the average weight of each?

46. If \$1548 is divided equally among a dozen persons, how much is the share of each?

47. If \$2193.75 is divided equally among 15 persons, how much is the share of each?

48. If a piece of land containing 12.75 acres is cut into 17 equal building lots, what is the area of each?

49. If the dividend is 420, the quotient 32, and the remainder 4, what is the divisor?

50. If the dividend is 73 times the remainder, and the quotient is 24, and the divisor is 12, what is the remainder?

348. Division of denominate numbers by abstract numbers. Divide 33 ft. 9 in. by 15.

We divide 33 ft. by 15 and have 2 ft., with a remainder of 3 ft., which, reduced to inches (36 in.) and added to 9 in., equals 45 in., which is still to be divided. Again, 45 in. ÷ 15 = 3 in.

	2 ft.	3 in.
15)	33 ft.	9 in.
	30 "	
	3 ft. =	36 in.
		45 in.
		45 "

349. Division of denominate numbers by denominate numbers. Divide 23 ft. 4 in. by 3 ft. 4 in. Here either

1. 23 ft. 4 in. ÷ 3 ft. 4 in. = $23\frac{1}{3}$ ft. ÷ $3\frac{1}{3}$ ft. = $\frac{7}{3}^{0}$ ft. ÷ $\frac{1}{3}^{0}$ ft. = 7; or
2. 23 ft. 4 in. ÷ 3 ft. 4 in. = 280 in. ÷ 40 in. = 7.

In the first case we reduce to feet; in the second, to inches.

WRITTEN EXERCISE

1. 2043 gal. 3 qt. ÷ 25.
2. 430 yd. 10 in. ÷ 11.
3. 213 ft. 9 in. ÷ 23 ft. 9 in.
4. 652 ft. ÷ 81 ft. 6 in.
5. 518 yd. ÷ 42 yd. 3 ft. 6 in.
6. 1587 lb. ÷ 132 lb. 4 oz.
7. 355 hr. 33 min. 45 sec. ÷ 15.
8. 4868 cu. ft. 1200 cu. in. ÷ 23.
9. 22 hr. 40 sec. ÷ 5 hr. 30 min. 10 sec.
10. How much is an eighth of 116 lb. 8 oz.?
11. 4557 sq. ft. 54 sq. in. ÷ 147 sq. ft. 90 sq. in.
12. How much is half of 2 hr. 53 min. 18 sec.?
13. How much is a quarter of 11 yd. 2 ft. 8 in.?
14. If a cubic foot of gold weighs 1187 lb. 8 oz., and if gold is 19 times as heavy as water, what does a cubic foot of water weigh?
15. If it takes a man 8 hr. 31 min. 24 sec. to walk a certain distance, how long will it take an automobile to travel the same distance, if it goes 12 times as fast?

ORAL EXERCISE

1. Tell why $\frac{2}{3} \div 2 = \frac{1}{3}$; also why $\frac{1}{3} \div 2 = \frac{1}{6}$.

2. Tell why $1 \div \frac{1}{3} = 3$; also why $2 \div \frac{1}{3} = 6$, and $5 \div \frac{1}{3} = 15$.

3. Because $5 \div \frac{1}{3} = 15$, tell why $5 \div \frac{2}{3} = \frac{1}{2}$ of 15, or $1\frac{5}{2}$.

4. Because $5 \div \frac{2}{3} = 1\frac{5}{2}$, tell why $\frac{5}{7} \div \frac{2}{3} = \frac{1}{7}$ of $1\frac{5}{2}$, or $1\frac{5}{14}$.

5. Because $\frac{5}{7} \div \frac{2}{3} = 1\frac{5}{14}$, and because $\frac{3}{2}$ of $\frac{5}{7} = 1\frac{5}{14}$, what operation may be performed in place of dividing one fraction by another?

350. Dividing fractions. In dividing $\frac{5}{7}$ by $\frac{2}{3}$, we may

1. *Reduce to a common denominator and divide the new numerators.* $\frac{15}{21} \div \frac{14}{21} = 1\frac{5}{14}$, just as $\$15 \div \$14 = 1\frac{5}{14}$, or 15 ft. $\div$ 14 ft. $= 1\frac{5}{14}$. Or we may

2. *Invert the divisor and multiply.* That is, $\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \times \frac{3}{2} = 1\frac{5}{14}$. We use this operation rather than the other, because it is easier and gives the same result. Always indicate the multiplication first and cancel if possible.

WRITTEN EXERCISE

1. $\frac{7}{8} \div \frac{3}{4}$.
2. $\frac{3}{5} \div \frac{2}{15}$.
3. $1\frac{5}{12} \div \frac{7}{8}$.
4. $\frac{9}{16} \div \frac{3}{8}$.
5. $1\frac{5}{16} \div \frac{5}{4}$.
6. $\frac{2}{15} \div \frac{7}{5}$.
7. $\frac{3}{11} \div 1\frac{5}{13}$.
8. $1\frac{5}{11} \div 1\frac{7}{12}$.
9. $1\frac{5}{16} \div 3\frac{3}{32}$.
10. $\frac{14}{15} \div 1\frac{1}{12}$.
11. $\frac{7}{18} \div 2\frac{1}{22}$.
12. $\frac{19}{27} \div 3\frac{8}{81}$.
13. $\frac{15}{39} \div 1\frac{0}{13}$.
14. $1\frac{7}{52} \div 5\frac{1}{52}$.
15. $\frac{9}{16} \div \frac{5}{32}$.
16. $1\frac{5}{7} \div \frac{5}{14}$.
17. $\frac{7}{144} \div \frac{23}{60}$.
18. $2\frac{09}{21} \div 1\frac{7}{15}$.
19. $1\frac{37}{138} \div 3\frac{2}{33}$.
20. $1\frac{6}{41} \div 1\frac{8}{123}$.
21. $1\frac{43}{323} \div 1\frac{7}{13}$.
22. $1\frac{123}{322} \div \frac{63}{144}$.
23. $3\frac{03}{122} \div 1\frac{01}{244}$.
24. $1\frac{17}{211} \div \frac{99}{422}$.

351. Dividing mixed numbers. In dividing $15\frac{1}{2}$ by $7\frac{3}{4}$, we might

1. *Reduce to a common denominator, and divide the new numerators.* $\frac{62}{4} \div \frac{31}{4} = 62 \div 31 = 2$. Or

2. *Multiply both dividend and divisor by a common denominator, and then divide.* That is, $15\frac{1}{2} \div 7\frac{3}{4} = 62 \div 31$, by multiplying by 4.

But practically we

3. *Reduce to fractional forms and multiply by the inverted divisor.* That is, $15\frac{1}{2} \div 7\frac{3}{4} = \frac{31}{2} \div \frac{31}{4} = \frac{31}{2} \times \frac{4}{31} = 2$.

WRITTEN EXERCISE

1. $21 \div 3\frac{1}{4}$.	2. $83 \div 9\frac{8}{9}$.	3. $62 \div 5\frac{3}{4}$.
4. $38 \div 5\frac{5}{8}$.	5. $32 \div 4\frac{2}{3}$.	6. $17 \div 2\frac{7}{8}$.
7. $59 \div 8\frac{6}{7}$.	8. $41 \div 8\frac{7}{8}$.	9. $18\frac{5}{8} \div 4\frac{7}{8}$.
10. $33 \div 7\frac{4}{5}$.	11. $15\frac{2}{3} \div 6\frac{1}{3}$.	12. $17\frac{1}{8} \div 2\frac{1}{2}$.
13. $12\frac{1}{2} \div 3\frac{1}{2}$.	14. $15\frac{1}{2} \div 3\frac{1}{3}$.	15. $78 \div 9\frac{5}{12}$.
16. $16\frac{5}{9} \div 4\frac{1}{9}$.	17. $61\frac{4}{5} \div 8\frac{1}{7}$.	18. $324 \div 7\frac{1}{3}$.
19. $15\frac{7}{8} \div 5\frac{1}{3}$.	20. $82 \div 8\frac{3}{16}$.	21. $61\frac{7}{8} \div 5\frac{3}{16}$.
22. $75 \div 6\frac{5}{12}$.	23. $27\frac{1}{2} \div 33\frac{1}{3}$.	24. $32\frac{5}{12} \div 6\frac{1}{3}$.
25. $51\frac{1}{16} \div 2\frac{1}{8}$.	26. $29\frac{1}{5} \div 59\frac{3}{5}$.	27. $62\frac{7}{8} \div 15\frac{1}{3}$.
28. $31\frac{5}{8} \div 15\frac{3}{16}$.	29. $13\frac{9}{10} \div 4\frac{2}{5}$.	30. $258 \div 15\frac{3}{17}$.
31. $776.5 \div 129\frac{5}{12}$.	32. $120\frac{1}{16} \div 14\frac{7}{16}$.	33. $52\frac{1}{11} \div 17\frac{3}{11}$.

34. How many strips of cloth, each $5\frac{3}{8}$ yd. long, can be cut from a strip $37\frac{5}{8}$ yd. long?

35. How many city lots, each $31\frac{3}{4}$ ft. front, can be cut from a piece having $222\frac{1}{4}$ ft. front?

36. How many yards of cloth at $31\frac{1}{3}¢$ a yard can be bought for a dollar and a half?

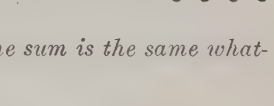
GENERAL PRINCIPLES OF THE OPERATIONS

ORAL EXERCISE

1. If I put 4 ct. with 3 ct., do I have the same result as if I put 3 ct. with 4 ct.? What does this tell about the order of adding numbers?

2. If I wish to add 4, 3, and 5, shall I get the same result by first adding $4 + 3$ and then 5, as by first adding $3 + 5$ and then 4? What does this tell about grouping numbers?

3. How does 3×4 compare with 4×3? What does this tell about the order of multiplying numbers?



352. Law of order in addition. *The sum is the same whatever the order of the addends.*

That is, $2 + 3 = 3 + 2$.

353. Law of grouping in addition. *The sum is the same however the addends are grouped.*

That is, $(2 + 3) + 5 = 2 + (3 + 5)$.

354. Law of order in multiplication. *The product is the same whatever the order of the factors.*

That is, $2 \times 3 = 3 \times 2$.

WRITTEN EXERCISE

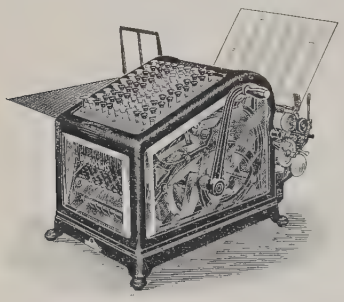
1. Make a rectangle $\frac{5}{8}$ in. by $\frac{3}{8}$ in., and separate it into $\frac{1}{8}$-in. squares, and show that $5 \times 3 = 3 \times 5$.

2. Show that $(2 \times 3) \times 4 = 2 \times (3 \times 4)$, and state the law of grouping in multiplication.

3. We have found that $2 + 3 = 3 + 2$, and $2 \times 3 = 3 \times 2$. Does $2 \div 3 = 3 \div 2$? Write out a statement about it.

THE PRACTICAL SHORT METHODS

355. But few practical short methods. There are many short methods of operation, and of these a few are really practical. People who have a great amount of computing now use machines, just as those who have much writing to do now use a typewriter. Here is a picture of an adding machine such as is used by most of the large banks. There are also machines that multiply, divide, and perform other operations.



356. Short methods in addition. 1. Learn to read the columns like a word, without pronouncing, even to yourself, the separate numbers any more than necessary.

2. Accustom yourself to "catch the tens" as much as possible in adding. Thus, in the annexed column, the eye should at once catch the two tens, and see that the result is 22. There is danger of mistakes in skipping numbers to catch the tens. A good computer, however, will know and detect the other combinations as well as he does the tens.

4
3
3
5
2
3
2
22

3. The shortest way of checking the work is to add in the opposite direction.

4. In adding several long columns, keep the sums of each order separate, on a slip of paper, and add these partial sums. At least keep a note of the figures "carried." This enables you to check more readily.

ORAL EXERCISE

Look at the example and state only the sum:

1. 17 32	2. 23 27	3. 42 69	4. 61 71	5. 15 38
6. 15 13 22	7. 21 12 31	8. 32 14 42	9. 18 6 61	10. 21 10 73
11. 26 25 32	12. 33 12 26	13. 42 63 48	14. 81 48 53	15. 16 17 82
16. 21 35 42	17. 32 76 63	18. 41 88 82	19. 19 73 35	20. 28 29 46
21. $75 32 61	22. $31 92 86	23. 48 ft. 26 35	24. 29 in. 36 27	25. 32 yd. 48 62
26. $32 41 62	27. $21 23 49	28. 36 yd. 28 42	29. 63 ft. 21 42	30. 81 lb. 28 96
31. $12 42 37	32. $26 47 35	33. 26 lb. 81 29	34. 82 yd. 63 48	35. 29 ft. 27 36

WRITTEN EXERCISE

See how long it takes to copy these numbers, and to add and check each result. Write the time with the results.

1.	2.	3.	4.
$147.50	$748.06	$375.42	$298.04
35.70	126.53	104.41	127.63
61.80	39.40	630.67	701.96
130.50	86.51	49.60	872.37
16.70	28.42	131.49	286.32

5.	6.	7.	8.
$359.68	$496.48	$396.04	$982.69
298.76	127.92	72.63	604.05
400.92	600.00	129.87	81.97
640.32	503.52	871.64	631.83
701.24	872.08	280.93	400.00
248.72	172.96	371.40	180.02

9.	10.	11.	12.
$1482.63	$4761.42	$3147.29	$4268.75
689.74	3917.63	2873.41	3942.35
2908.37	5238.58	4192.64	2065.08
8517.37	6082.37	128.46	5095.02
9310.26	4098.07	3029.00	2711.40
7091.63	5901.93	7081.00	1029.60
328.04	670.29	283.96	409.62

13.	14.	15.
127 ft. 9 in.	12 lb. 8 oz.	3 yr. 8 mo. 6 da.
37 6	9 15	5 2 27
41 8	17 12	3 10 14
32 5	15 11	1 0 19
15 8	21 7	3 7 0

16. A merchant has the following sums owing him: $147.60, $21.30, $29.47, $63.82, $173.46, $391.14, $2.94, $6.37, $98.71. What is the total amount?

17. What was the total number of dwellings in the following states in a certain year? Illinois, 845,836; Indiana, 552,495; Michigan, 521,648; Missouri, 593,528; New York, 1,035,180; Ohio, 857,636; Pennsylvania, 1,236,238; Texas, 575,734.

18. What was then the total population of the following cities? New York, 3,437,202; Chicago, 1,698,575; Philadelphia, 1,293,697; St. Louis, 575,238; Boston, 560,892; Baltimore, 508,957; Cleveland, 381,768; Buffalo, 352,387; San Francisco, 342,782; Cincinnati, 325,902; Pittsburg, 321,616.

19. What was then the total number of school children in the following states? Alabama, 733,222; Arkansas, 529,375; Georgia, 885,725; Illinois, 1,589,915; Indiana, 843,885; Iowa, 767,870; Kansas, 527,560; Kentucky, 798,027; Louisiana, 538,267; Massachusetts, 778,110; Michigan, 790,275; Minnesota, 612,990; Mississippi, 633,026; Missouri, 1,105,258; New Jersey, 572,923; New York, 2,146,764; North Carolina, 753,826; Ohio, 1,338,345; Pennsylvania, 2,031,171; South Carolina, 560,773; Tennessee, 780,421; Texas, 1,215,634; Wisconsin, 730,685.

20. The cost of some of our largest war ships is as follows: *Georgia*, $3,590,000; *Indiana*, $3,063,000; *Iowa*, $3,010,000; *Massachusetts*, $3,063,000; *Nebraska*, $3,733,600; *New Jersey*, $3,405,000; *Oregon*, $3,222,810; *Rhode Island*, $3,405,000; *Virginia*, $3,590,000; *California*, $3,800,000; *Colorado*, $3,780,000; *Maryland*, $3,775,000; *Pennsylvania*, $3,890,000; *South Dakota*, $3,750,000; *West Virginia*, $3,885,000. What is their total cost?

357. Short methods in subtraction. The shortest plan of subtraction is the "making change" method. It is better, however, to follow the method with which the class is familiar. All work should be checked by adding the remainder and the subtrahend, the sum being the minuend.

ORAL EXERCISE

Imagine yourself buying goods at a store, and handing the clerk the first amount given, the goods costing the second amount. State the amount of change due you.

1. \$1, 72¢.	2. \$2, \$1.37.	3. \$2, \$1.67.
4. \$3, \$2.17.	5. \$4, \$3.41.	6. \$5, \$1.27.
7. \$5, \$2.40.	8. \$5, \$3.05.	9. \$5, \$1.70.
10. \$10, \$6.55.	11. \$10, \$7.05.	12. \$10, \$6.05.
13. \$10, \$5.70.	14. \$10, \$3.70.	15. \$10, \$8.67.
16. \$15, \$12.75.	17. \$15, \$13.60.	18. \$15, \$11.42.
19. \$20, \$16.92.	20. \$20, \$17.30.	21. \$20, \$16.05.
22. \$25, \$19.60.	23. \$35, \$32.70.	24. \$40, \$37.06.

25. Subtract 78 from the sum of 45 and 57.

26. Subtract 49 from the difference between 37 and 88.

27. Subtract the difference between 69 and 72 from their sum.

28. The difference is 48 and the minuend is 63. What is the subtrahend?

29. How much less is the difference between 68 and 81 than that between 49 and 68?

30. How much greater is the difference between 37 and 64 than that between 37 and 19?

WRITTEN EXERCISE

See how long it takes to copy these numbers, and to subtract and check. Write the time with the last result.

1. \$175.62 81.98	2. \$129.87 67.98	3. \$208.09 98.78	4. \$400.12 69.84
5. \$170.08 69.99	6. \$370.42 91.64	7. \$209.18 87.69	8. \$302.78 62.95
9. \$802.65 69.81	10. \$200.00 93.74	11. \$608.75 35.82	12. \$672.93 98.94
13. \$409.98 23.49	14. \$682.70 93.81	15. \$200.00 187.60	16. \$325.34 128.92
17. \$429.81 149.92	18. \$308.72 129.85	19. \$298.05 109.72	20. \$408.00 129.60
21. \$1087.50 963.90	22. \$2023.40 1285.42	23. \$3029.62 2172.58	24. \$3278.48 1284.69

25. 37 ft. 8 in. 29 9	26. 41 lb. 8 oz. 29 12	27. 39 yd. 9 in. 27 16
28. 9 mo. 8 da. 6 15	29. 12 yr. 4 da. 8 92	30. 6° 9′ 21″ 1 27 42
31. 15 mi. 125 ft. 8 650	32. 134 ft. 3 in. 69 9	33. 42° 6′ 23″ 29 18 50
34. 67 gal. 1 qt. 19 3	35. 5 yr. 2 mo. 2 11	36. 75° 18′ 48″ 29 37 59

37. A man deposited $42.96 and $125 in a bank, and drew out $103.75. How much was left?

38. A man had $287.50 in a bank, and he drew out $12.75 and $37.63. How much was left?

39. A man deposited $15 a week in a bank, for 13 weeks. During this time he drew out $7.29, $6.35, $14.20, and $5.75. How much had he left at the end of the time?

40. In a year when this country produced 522,229,505 bu. of wheat, and 800,125,989 bu. of oats, what was the difference in amount?

41. There were 841,201,546 acres of farm land in this country in a certain year, 414,793,191 being improved. How many acres were unimproved?

42. Of the total number of farms in this country, 5,739,657, in a certain year, there were 5,537,731 with buildings on them. How many had no buildings?

43. The value of this farm property was then $20,514,001,838, of which $13,114,492,056 was the value of the land. What was the value of the buildings, implements, and stock?

44. By how much did the population of London, 4,536,063, in a certain year, exceed that of New York, 3,437,202? of Chicago, 1,698,575? of Boston, 560,892?

45. When the irrigation systems of California had cost $19,181,610, and those of Colorado $11,758,703, how much more had California spent on irrigation than Colorado?

46. The states having the greatest railroad mileage in a certain year were: Illinois, 11,116.18 mi.; Pennsylvania, 10,480.35 mi.; and Texas, 10,189.04 mi. By how much did that of Illinois exceed each of the other two?

358. Short methods in multiplication. There are a few multipliers which are used so frequently in business that it is desirable to know the shortest methods of multiplying by them. These include the numbers 5, 10, 25, 125 (with 12.5 or $12\frac{1}{2}$), $33\frac{1}{3}$, and their various per cents, together with 9 and 11.

ORAL EXERCISE

1. Multiply by 10: 43, 6.7, 0.42, 300, $260.10.

2. State the short method of multiplying by 10.

3. Multiply by 100: 75, 8.2, 5.2, $600, $425.75.

4. State the short method of multiplying by 100.

5. Since $5 = \frac{10}{2}$, to multiply by 5 why may we annex a 0, or move the decimal point one place to the right, and then divide by 2? Illustrate by an example.

6. Multiply by 5: 40, 66, 84, 98, 104, 124, 666, 42.2.

7. Since $25 = \frac{100}{4}$, what is the short method of multiplying by 25? Illustrate by an example.

8. Multiply by 25: 48, 64, 82, 34, 81, 13, 448, 204, 216.

9. Since $33\frac{1}{3} = \frac{100}{3}$, what is the short method of multiplying by $33\frac{1}{3}$? Illustrate by an example.

10. Multiply by $33\frac{1}{3}$: 60, 36, 42, 51, 66, 123, 312, 111, 66.3, 42.6, 15.63, 3.3333, 12,000, 93,300, 690,000.

11. Since $125 = \frac{1000}{8}$, what is the short method of multiplying by 125? Illustrate by an example.

12. Multiply by 125: 48, 56, 12, 36, 44, 8, 8.8, 4.8, 4.88, 1.688, 800, 8800, 1600, 16,000, 32,000.

13. 50×44.	14. 50×68.	15. 50×124.
16. 50×240.	17. 250×12.	18. 250×16.
19. 250×24.	20. 250×40.	21. 125×16.
22. 1250×16.	23. 1250×24.	24. 1250×32.

25. Since $5\% = \frac{5}{100} = \frac{1}{20}$, to multiply by 5% is to divide by 10 and by what other number? Illustrate.

26. How much is 5% of 40? of 70? of 90? of \$230? of \$350? of 400 ft.? of \$2.60? of 4220? of 84,000?

27. Since $25\% = \frac{1}{4}$, what is the short method of finding 25% of a number? Illustrate.

28. How much is 25% of 620? of 460? of \$30? of \$924? of \$424? of \$4.32? of 1600? of 32,000?

29. Since $33\frac{1}{3}\% = \frac{1}{3}$, what is the short method of finding $33\frac{1}{3}\%$ of a number? Illustrate.

30. How much is $33\frac{1}{3}\%$ of 48? of \$63? of \$720? of \$1.23? of 456 ft.? of 1230 mi.? of 72,000? of 15.63?

31. Since $12\frac{1}{2}\% = \frac{1}{8}$, what is the short method of finding $12\frac{1}{2}\%$ of a number? Illustrate.

32. How much is $12\frac{1}{2}\%$ of 48? of 56? of \$720? of \$6.40? of 800 ft.? of \$1600? of 72,000? of 16.08?

33. Increase \$48 by 25% of itself.

34. Decrease \$800 by 5% of itself.

35. Increase \$160 by $12\frac{1}{2}\%$ of itself.

36. Decrease \$720 by $33\frac{1}{3}\%$ of itself.

37. What will 125 erasers cost at 5¢ each?

38. What will 72 doz. eggs cost at $12\frac{1}{2}$¢ a dozen?

39. What will 24 yd. of silk cost at \$1.25 a yard?

40. What will 36 yd. of cloth cost at 25¢ a yard?

41. What will 16 yd. of cloth cost at $12\frac{1}{2}$¢ a yard?

42. What will \$8.40 worth of goods cost at 25% off?

43. What will \$3.60 worth of goods cost at $33\frac{1}{3}\%$ off?

44. Find 25% of $33\frac{1}{3}\%$ of 12; of 600; of 1200; of 1500.

45. Find $12\frac{1}{2}\%$ of 50% of 16; of 160; of 1600; of 4800.

46. Find 5% of 10% of $12\frac{1}{2}\%$ of 80; of 1600; of 3200.

359. Summary of short methods of multiplying.

1. *To multiply by* 10, *move the decimal point one place to the right, annexing zero if necessary.*

2. *To multiply by* 100, *move the decimal point two places to the right, annexing zeros if necessary.*

3. *To multiply by* 5, *multiply by* 10 *and divide by* 2.

4. *To multiply by* 25, *multiply by* 100 *and divide by* 4.

5. *To multiply by* 125, *multiply by* 1000 *and divide by* 8.

6. *To multiply by* $33\frac{1}{3}$, *multiply by* 100 *and divide by* 3.

7. *To find* 5%, *divide by* 10 *and by* 2.

8. *To find* 25%, *divide by* 4 ; 50%, *by* 2 ; 20%, *by* 5.

9. *To find* $33\frac{1}{3}$%, *take* $\frac{1}{3}$; $66\frac{2}{3}$%, *take* $\frac{2}{3}$.

10. *To find* $12\frac{1}{2}$%, *take* $\frac{1}{8}$; $16\frac{2}{3}$%, *take* $\frac{1}{6}$.

11. *To multiply by* 9, *which is* $10 - 1$, *multiply by* 10 *and then subtract the multiplicand.*

Thus, 9 times $476 = 4760 - 476 = 4284$.

12. *To multiply by* 11, *which is* $10 + 1$, *multiply by* 10 *and then add the multiplicand.*

WRITTEN EXERCISE

1. Multiply by 125 : 4832, 5275, 6892, \$49,365.
2. Multiply by $33\frac{1}{3}$: 1011, 7227, 3102, \$43,392.
3. Find $12\frac{1}{2}$% of \$6848, \$3272, \$42.48, \$35,284.
4. Find $16\frac{2}{3}$% of 600, \$33.42, \$14,286.66, \$32,331.36.
5. Find $66\frac{2}{3}$% of \$41.10, \$33.66, \$14,283.12, \$68,391.21.
6. Find 25% of $33\frac{1}{3}$% of 12, \$298.32, \$3802.80, \$296.64.
7. Multiply by 9 : 329, 746, \$981, 628 ft., 1476, \$23,481.
8. Multiply by 11 : 6123, 3762, \$4837, 3972 ft., \$31,287.

ORAL EXERCISE

Find the cost of the following:

Do not analyze aloud, at least until after the answer is given. Use short processes (page 301).

1. $3\frac{3}{4}$ yd. of felt @ \$1.60.
2. 16 yd. of denim @ 15¢.
3. $7\frac{1}{2}$ yd. of dimity @ 16¢.
4. $3\frac{1}{4}$ lb. of feathers @ 48¢.
5. 32 yd. of cambric @ 11¢.
6. 9 yd. of cashmere @ 75¢.
7. 16 yd. of gingham @ 11¢.
8. 22 yd. of buckram @ 15¢.
9. 17 yd. of crinoline @ 11¢.
10. 15 yd. of percaline @ 23¢.
11. 7 pieces of beading @ 23¢.
12. 8 yd. of eider down @ 65¢.
13. 12 yd. of chambray @ 25¢.
14. $22\frac{1}{2}$ yd. of nainsook @ 20¢.
15. 16 yd. of linen lawn @ 75¢.
16. A dozen whalebones @ 24¢.
17. 35 yd. of cheese cloth @ 5¢.
18. 12 yd. of jute burlap @ 35¢.
19. $3\frac{1}{2}$ doz. Clark's cotton @ 42¢.
20. $7\frac{3}{4}$ yd. of torchon lace @ 12¢.
21. $4\frac{1}{2}$ doz. basting cotton @ 24¢.
22. 15 yd. of cotton scrim @ 22¢.
23. $5\frac{1}{2}$ doz. darning cotton @ 24¢.
24. 7 gross of bone buttons @ 22¢.

25. 16 gross of shoe buttons @ 7¢.
26. $8\frac{1}{2}$ yd. of farmer's satin @ 44¢.
27. $3\frac{1}{2}$ yd. of damask linen @ 98¢.
28. $5\frac{3}{4}$ yd. of cotton flannel @ 16¢.
29. 8 yd. of cotton burlap @ $1.12.
30. $7\frac{1}{2}$ lb. of hammock cord @ 22¢.
31. $22\frac{1}{2}$ yd. of Persian lawn @ 30¢.
32. 11 yd. of striped flannel @ 65¢.
33. 14 yd. of plain gingham @ 25¢.
34. 15 pieces of mohair braid @ 9¢.
35. 6 balls of crochet cotton @ 35¢.
36. 8 pieces of seam binding @ 12¢.
37. 19 skeins of Prisma cotton @ 5¢.
38. $17\frac{1}{2}$ yd. of butcher's linen @ 50¢.
39. 15 yd. of huckaback linen @ 45¢.
40. 17 yd. of unbleached muslin @ 7¢.
41. 17 skeins of Poseidon cotton @ 3¢.
42. 7 yd. of arras-cloth burlap @ $1.25.
43. 16 yd. of Alpine rose muslin @ $9\frac{1}{4}$¢.
44. 15 yd. of Java cream canvas @ 70¢.
45. 25 yd. of Java colored canvas @ 45¢.
46. 15 yd. of single-thread canvas @ 60¢.
47. 11 yd. of white narrow canvas @ 38¢.
48. 9 yd. of basket-weave burlap @ $1.15.
49. $\frac{1}{2}$ doz. boxes for materials @ 35¢ each.
50. A gross of pearl buttons @ 13¢ a dozen.
51. 7 pieces of feather-stitched braid @ 24¢.
52. 16 balls of knitting cotton @ 4 for a quarter.

53. If 6 lb. of coffee cost \$1.86, what will 5 lb. cost?

54. If 12 lb. of butter cost \$3, what will 18 lb. cost?

55. If $\frac{4}{5}$ of a building lot is worth \$2200, what is the lot worth?

56. At \$24 a month, what is the rent of a house for $3\frac{1}{2}$ mo.?

57. If 3% of a number is 36, what is 5 times the number?

58. If 7% of a number is 63, what is 20 times the number?

59. If the divisor is 13 and the quotient is 12, what is the dividend?

60. If a man buys a horse for \$150 and sells it at an advance of 30%, what is the selling price?

61. A grocer bought coffee at 25¢ a pound and sold it at a profit of 20%. What was the selling price?

62. If a man can build a fence in 32 days, in how many days should he build $12\frac{1}{2}$% of it?

63. The product of three numbers is 242. Two of the numbers are 11 and 2. What is the third number?

64. If a man owned $\frac{3}{4}$ of the stock of a certain company and sold $\frac{8}{9}$ of his share, how much did he have left?

65. A man buys \$11.88 worth of goods and sells $\frac{1}{3}$ of them at the same rate. How much does he receive?

66. How many square rods are there in a field 20 rd. square? What is the land worth at \$$\frac{3}{8}$ a square rod?

State the profit in buying the following goods at the price first given, and selling at the second price:

67. 12 lb. mixed candy, \$1.92; 25¢ a pound.

68. 25 bars peanut brittle, \$1.56; 10¢ a bar.

69. 216 chocolate bars, 45¢; 3 bars for a cent.

70. 1 doz. boxes chocolate chips, \$1.92; 25¢ a box.

71. 6 lb. fine chocolate creams, \$2.40; 60¢ a pound.

72. 75 packages peanut brittle, \$4.68; 10¢ a package.

73. 40 packages chocolate drops, \$1.25; 5¢ a package.

74. 12 packages chocolate creams, 72¢; 10¢ a package.

75. 2 boxes chocolate cream bars, 48 to the box, 76¢; 1¢ a bar.

76. 4 boxes gum drops, 24 packages to the box, \$3; 5¢ a package.

77. 3 boxes caramels, 40 packages to the box, \$3.75; 5¢ a package.

78. 5 boxes licorice drops, 48 packages to the box, \$7.20; 5¢ a package.

79. 4 boxes chocolate almonds, 40 packages to the box, \$5; 5¢ a package.

80. 9 cases lemon soda, 2 doz. bottles to the case, \$4.50; 5¢ a bottle.

81. 6 boxes salted peanuts, 24 packages to the box, 75¢ a box; 5¢ a package.

82. 4 boxes pepsin chewing gum, 20 packages to the box, \$2.20; 5¢ a package.

83. 3 boxes vanilla chocolates, 2 doz. packages to the box, \$2.25; 5¢ a package.

360. Short methods in division. There are a few divisors which are used so frequently in business that it is desirable to know the shortest methods of dividing by them. These include the numbers 5, 10, 25, 100, 125 (with 12.5 or $12\frac{1}{2}$), and $33\frac{1}{3}$.

ORAL EXERCISE

1. Divide by 10: \$420, \$5700, 330, \$425, 375 lb., 6 mi.

2. State the short method of dividing by 10.

3. Since $\frac{1}{5} = \frac{2}{10}$, we may take $\frac{1}{5}$ of a dollar by taking how many tenths of it? State a short method of dividing by 5, or of taking $\frac{1}{5}$ of a number.

4. Divide by 5: \$1110, \$3240, 5280 ft., 4230 mi., 3221.

5. Divide by 100: \$600, 7000, 4800, \$560, \$275, \$48.

6. State the short method of dividing by 100.

7. State the short method of finding $\frac{1}{25}$ (or $\frac{4}{100}$) of a dollar; of dividing a number by 25.

8. Divide by 25: \$2200, \$1200, \$3100, 4100, \$32, \$11.

9. State the short method of dividing by 125. ($1\frac{1}{25} = 1\frac{8}{000}$.)

10. Divide by 125: \$11,000, 2000, \$4000, \$70, \$600.

11. State the short method of dividing by $33\frac{1}{3}$. (Take $1\frac{3}{00}$.)

12. Divide by $33\frac{1}{3}$: \$200, 500, \$7, \$12, \$30,000, \$11,000.

13. In a certain year Delaware had 346.2 mi. of railroad, and the District of Columbia had $\frac{1}{10}$ as many. How much had the latter?

14. France then had 26,600 mi. of railroad, and Korea had 0.001 as many. How many had Korea?

15. The United States then had 193,300 mi. of railroad, and Ecuador had 0.001 as many. How many had Ecuador?

361. Summary of short methods of division.

1. *To divide by* 10, *move the decimal point one place to the left.*

2. *To divide by* 100, *move the decimal point two places to the left, and so on for other powers of* 10.

3. *To divide by* 5, *multiply by* 2 *and divide by* 10.

4. *To divide by* 25, *multiply by* 4 *and divide by* 100.

5. *To divide by* 125, *multiply by* 8 *and divide by* 1000.

6. *To divide by* $33\frac{1}{3}$, *multiply by* 3 *and divide by* 100.

WRITTEN EXERCISE

1. State a short method of dividing by $66\frac{2}{3}$, or $\frac{200}{3}$.

2. State a short method of dividing by $16\frac{2}{3}$, or $\frac{100}{6}$.

3. State a short method of dividing by $12\frac{1}{2}$, or $\frac{100}{8}$.

4. $6484 \div 66\frac{2}{3}$. 5. $7280 \div 66\frac{2}{3}$. 6. $9276 \div 66\frac{2}{3}$.

7. $2937 \div 16\frac{2}{3}$. 8. $4831 \div 16\frac{2}{3}$. 9. $9134 \div 16\frac{2}{3}$.

10. $9237 \div 12\frac{1}{2}$. 11. $7396 \div 12\frac{1}{2}$. 12. $8187 \div 12\frac{1}{2}$.

13. $6889 \div 33\frac{1}{3}$. 14. $4959 \div 33\frac{1}{3}$. 15. $8892 \div 33\frac{1}{3}$.

16. $4936 \div 125$. 17. $67{,}400 \div 125$. 18. $39{,}888 \div 125$.

19. Divide by 25: 13,428, \$37,642, \$135,621, \$427,693.

20. Divide by 125: 17,200, \$68,300, \$276,400, \$317,500.

21. Divide by $33\frac{1}{3}$: 69,720, \$32,960, \$48,750, \$276,320.

22. At 25¢ a yard, how many yards of cloth can be bought for \$37.25?

23. At $33\frac{1}{3}$¢ a yard, how many yards of cloth can be bought for \$98?

24. Divide 3,742,697 by 25, by long division, and see how many seconds it takes. Do the same by the short method stated above. Write the time with the answer.

ORAL EXERCISE

1. To multiply by $\frac{1}{2}$ is the same as to divide by what number? To divide by $\frac{1}{2}$ is the same as to multiply by what number? Illustrate on the blackboard.

2. Multiply by $\frac{1}{2}$: 48, 250, 380, 460, 666, 1110, 1234.

3. Divide by $\frac{1}{2}$: 70, 35, 60, 300, 450, 700, 1450, 8112.

4. To multiply by $\frac{1}{3}$ is the same as to divide by what number? To divide by $\frac{1}{3}$ is the same as to multiply by what number? Illustrate on the blackboard.

5. Multiply by $\frac{1}{3}$: 36, 90, 63, 45, 75, 300, 123, 321.

6. Divide by $\frac{1}{3}$: 30, 70, 50, 25, 80, 400, 2000, 4211.

7. Multiply by $\frac{2}{3}$: 30, 60, 90, 33, 63, 300, 600, 1230.

8. Divide by $\frac{2}{3}$: 20, 12, 16, 10, 60, 100, 200, 500, 1000.

9. Multiply by $\frac{1}{4}$: 8, 16, 32, 48, 64, 84, 100, 200, 1000.

10. Divide by $\frac{1}{4}$: 9, 11, 21, 25, 60, 70, 91, 125, 250, 800.

11. Multiply by $\frac{3}{4}$: 8, 16, 20, 32, 40, 44, 48, 60, 80, 444.

12. Divide by $\frac{3}{4}$: 9, 12, 15, 21, 30, 60, 66, 90, 120, 300.

13. Since $\frac{1}{5} = \frac{2}{10}$, what is the short method of multiplying by $\frac{1}{5}$? of dividing by $\frac{1}{5}$?

14. Multiply by $\frac{1}{5}$: 21, 31, 34, 123, 112, 321, 132, 125.

15. Divide by $\frac{1}{5}$: 22, 300, 34, 160, 444, 664, 486, 1010.

16. Since $\frac{2}{5} = .4$, state a short method of multiplying and of dividing by $\frac{2}{5}$. Do the same for $\frac{3}{5}$ and $\frac{4}{5}$.

17. Multiply by $\frac{5}{6}$; 30, 60, 66, 120, 600, 720, 840, 1200.

18. Because $.12\frac{1}{2} = \frac{1}{8}$, what is the short method of multiplying by $.12\frac{1}{2}$?

19. At $12\frac{1}{2}$¢ a yard, what will 16 yd. cost? 72 yd.?

20. At 48¢ a dozen, what will $\frac{1}{2}$ doz. eggs cost?

21. At \$1.28 a yard, what will $\frac{5}{8}$ yd. of silk cost?

22. At \$1.20 a yard, what will $\frac{3}{4}$ yd. of silk cost?

23. At \$2.40 a yard, what will $\frac{3}{8}$ yd. of lace cost?

24. At \$1.68 a yard, what will $\frac{3}{4}$ yd. of lace cost?

25. At 78¢ a yard, what will $\frac{1}{3}$ yd. of velvet cost?

26. At \$18.60 a dozen, what will $\frac{2}{3}$ doz. spoons cost?

27. At $12\frac{1}{2}$¢ a yard, how much cloth will \$1.50 buy?

28. How much will $12\frac{1}{2}$ doz. forks cost at \$16 a dozen?

29. How much will $6\frac{1}{4}$ doz. eggs cost at 16¢ a dozen?

30. At 16¢ a pound, how much will $3\frac{1}{4}$ lb. of meat cost?

31. At 24¢ a pound, how much will $2\frac{3}{4}$ lb. of meat cost?

32. At 64¢ a pound, what will $\frac{3}{4}$ lb. shelled almonds cost? What will 2 lb. 4 oz. cost?

33. At 36¢ a dozen, how much must a dealer pay for $2\frac{3}{4}$ doz. small bottles of mucilage?

34. At \$19.20 a dozen, how much must a dealer pay for $1\frac{1}{2}$ doz. fountain pens? for 2 doz.?

35. At 15 mi. an hour, how far will an automobile travel in 20 min.? in 40 min.? in 30 min.?

36. At $12\frac{1}{2}$¢ a dozen, how many dozen eggs will \$9.25 buy? How many dozen will \$12.50 buy?

37. At 35 mi. an hour, how far will a train travel in 12 min.? in 24 min.? in 36 min.? in 48 min.?

38. At 47 mi. an hour, how far will a train travel in 12 min.? (12 min. $= \frac{1}{5}$ hr.) How far in 15 min.?

WRITTEN EXERCISE

1. $\frac{1}{2}$ of \$73.10.	2. $\frac{1}{2}$ of $3\frac{2}{3}$.	3. $\frac{1}{2}$ of 375 ft.
4. $\frac{1}{4}$ of \$61.20.	5. $\frac{1}{4}$ of $5\frac{1}{8}$.	6. $\frac{1}{4}$ of 239 ft.
7. $\frac{3}{4}$ of \$27.40.	8. $\frac{3}{4}$ of $6\frac{3}{5}$.	9. $\frac{3}{4}$ of 297 lb.
10. $\frac{1}{3}$ of \$71.40.	11. $\frac{1}{3}$ of $4\frac{7}{8}$.	12. $\frac{1}{3}$ of 237 yd.
13. $\frac{2}{3}$ of \$62.10.	14. $\frac{2}{3}$ of $2\frac{3}{8}$.	15. $\frac{2}{3}$ of 128 mi.
16. $\frac{1}{5}$ of \$67.30.	17. $\frac{2}{5}$ of $4\frac{2}{3}$.	18. $\frac{3}{5}$ of 293 yd.
19. $12\frac{3}{4} \times 16\frac{4}{5}$.	20. $32\frac{1}{2} \times 63\frac{1}{3}$.	21. $41\frac{2}{3} \times 33\frac{1}{5}$.
22. $12\frac{1}{2}\%$ of \$72.40.	23. $12\frac{1}{2}\%$ of $6\frac{1}{2}$.	24. $12\frac{1}{2}\%$ of 375 ft.
25. 275 in. $\times \frac{1}{3}$ in.	26. 268 ft. $\div \frac{1}{3}$ ft.	
27. 826 yd. $\div \frac{1}{3}$ yd.	28. 2372 ft. $\div \frac{1}{5}$ ft.	
29. 1635 yd. $\div \frac{1}{5}$ yd.	30. \$12.60 $\div$ \$0.12$\frac{1}{2}$.	
31. \$32.40 $\div$ \$0.12$\frac{1}{2}$.	32. 268 yd. $\div 12\frac{1}{2}$ yd.	

33. At $12\frac{1}{2}$¢ a yard, how much cloth will \$876 buy?

34. At $12\frac{1}{2}$¢ a gross, how many gross of paper fasteners will \$17.75 buy?

35. At $33\frac{1}{3}$¢ a hundred, how many photograph cards can be bought for \$1.83?

36. At $66\frac{2}{3}$¢ a gross, how many gross of pens can be bought for \$24?

37. At $\frac{3}{8}$ yd. each, how many strips of ribbon can be cut from a piece 27 yd. long?

38. When a train is traveling at the rate of 32 mi. in 40 min., what is its rate per hour?

39. When a horse is trotting at the rate of $\frac{1}{2}$ mi. in 3 min., what is his rate per hour?

MEASURES

ORAL EXERCISE

1. Express 640 rd. as miles.
2. Express 64 oz. as pounds.
3. Express 2 A. as square rods.
4. Express 33 ft. as rods; as yards.
5. Express 54 cu. ft. as cubic yards.
6. Express 9 ft. as inches; as yards.
7. Express 11 da. as hours; as weeks.
8. Express 17 qt. as pints; as gallons.
9. Express 9 pk. as quarts; as bushels.
10. How many cubic inches in 10 cu. ft.?
11. How many rods in 3 mi.? in 10 mi.?
12. How many yards in 8 rd.? in 10 rd.?
13. How many cubic feet in 2 cd. of wood?
14. Express 300 min. as hours; as seconds.
15. How many quarts in a barrel of $31\frac{1}{2}$ gal.?
16. How many pints in a 60-gallon hogshead?
17. Express 80 sq. rd. as a fraction of an acre.
18. How many right angles in 270°? in 180°?
19. How many days from May 23 to June 25?
20. How many days from June 11 to July 17?
21. What part of a 16-carat ring is pure gold?
22. Express 11 yd. as rods; as feet; as inches.
23. Express 30 pt. as quarts; as gills; as gallons.
24. How many feet in 2 rd. 8 ft.? in 2 rd. 10 ft.?
25. How many feet in 10 rd.? in 5 rd.? in 2 rd.?
26. How many degrees in 540′? in 420′? in 660′?

27. How many mills in $2.75?

28. How many days from August 15 to October 15?

29. How many acres in a tract of land 10 mi. square?

30. How many days from January 17 to February 17?

31. How many square feet in 10 sq. yd.? in 5 sq. yd.?

32. At 6 qt. a day, how long will 3 bu. of oats last a horse?

33. What is the perimeter of a square that is 4 ft. 9 in. on a side?

34. What is the side of a square whose perimeter is 14 ft. 4 in.?

35. Since a gallon contains 231 cu. in., how many gallons in 693 cu. in.?

36. How many square feet in a surface having an area of 720 sq. in.?

37. Express a long ton as a short ton and hundredths of a short ton.

38. What is the perimeter of an equilateral triangle that is 3 ft. 8 in. on a side?

39. If school closes June 21, how many days from that day to the fourth of July?

40. How many quarts will a 10-bushel bin hold? a 5-bushel bin? How many pecks in each?

41. A man buys a building lot 2 rd. front. How many square feet of sidewalk 5 ft. wide must he have?

42. A cellar $13\frac{1}{2}$ ft. by 20 ft. is to be excavated to a depth of 6 ft. How many loads (cubic yards) of earth must be removed?

43. If a man buys a load of coal weighing 1 T. 600 lb., how many pounds does he buy? Express this in tons and tenths of a ton.

State which is the greater:

44. 750 da. or 2 yr.
45. 15° or 800′.
46. $7\frac{1}{2}$ gal. or 31 qt.
47. 95 oz. or 6 lb.
48. 13 wk. or 91 da.
49. 3 rd. or 50 ft.
50. 160 rd. or 2640 ft.
51. 16 yd. or 50 ft.
52. 256 cu. ft. or 2 cd.
53. 95 pk. or 24 bu.
54. 6 ft. 8 in. or 75 in.
55. 200 da. or 7 mo.
56. 5 lb. 4 oz. or 85 oz.
57. 3 yd. 4 in. or 41 in.
58. 5 ft. 8 in. or 67 in.
59. 9 gal. 2 qt. or 37 qt.
60. 1300 A. or 2 sq. mi.
61. 3500 cu. in. or 2 cu. ft.
62. 290 sq. in. or 2 sq. ft.
63. 12 sq. rd. or 100 sq. ft.

64. A rectangle is 1 ft. 6 in. long and $1\frac{1}{4}$ ft. wide. What is the perimeter?

65. How many inches in each side of a square whose perimeter is a yard?

66. How many inches in each side of a square whose perimeter is 1 yd. 8 in.?

67. The perimeter of a rectangle is 5 ft. The rectangle is 17 in. long. How wide is it?

68. The sides of a triangle are 17 in., 12 in., 19 in. How many feet in the perimeter?

69. If John weighs 62 lb. 8 oz. and Fred weighs 5 lb. more, how much do the two weigh?

70. How many inches in each side of an equilateral triangle whose perimeter is 2 ft. 3 in.?

71. Suppose your schoolroom to be 29 ft. 6 in. long and 20 ft. 6 in. wide, how many feet in the perimeter?

72. The perimeter of a triangle is 1 yd. 3 in. Two of the sides are 13 in. and 14 in. How long is the third side?

362. Illustrative problems. 1. Express $35\frac{1}{2}$ yd. as feet.

Since 1 yd. = 3 ft.,

$35\frac{1}{2}$ yd. = $35\frac{1}{2}$ times 3 ft., or $106\frac{1}{2}$ ft.

2. Express 81 oz. as pounds and ounces.

Since 1 oz. = $\frac{1}{16}$ lb.,

81 oz. = 81 times $\frac{1}{16}$ lb. = $5\frac{1}{16}$ lb., or 5 lb. 1 oz.

3. Express 2 ft. 8 in. as inches.

Since 1 ft. = 12 in.,

2 ft. = 2 times 12 in. = 24 in.

2 ft. 8 in. = 24 in. + 8 in. = 32 in.

WRITTEN EXERCISE

1. Express 92 rd. 3 ft. as feet; as inches.

2. Express 32 yd. 2 ft. as feet; as inches.

3. Express $3\frac{1}{2}$ A. as acres and square rods.

4. Express 21 sq. ft. 82 sq. in. as square inches.

5. Express 95 cu. ft. as cubic yards; as cubic inches.

6. Express 27 sq. rd. 4 sq. yd. as square yards; as square feet.

7. Express 8 sq. yd. 7 sq. ft. as square feet; as square inches.

8. Express 35 cu. yd. 15 cu. ft. as cubic yards; as cubic feet; as cubic inches.

9. Express 4 T. 1735 lb. as tons and the decimal part of a ton; as pounds; as ounces.

10. Express 39,426 lb. as tons and the decimal part of a ton; as tons and pounds; as ounces.

11. Express 27 qt. 1 pt. as gallons and the decimal part of a gallon; as quarts and the decimal part of a quart; as pints.

363 Surveyors' table of length:

7.92 inches = 1 link (li.).
100 links = 4 rods = 1 chain (ch.).
80 chains = 5280 ft. = 1 mile.

City property is usually measured by feet and decimal fractions of a foot; farm property, by rods or chains.

WRITTEN EXERCISE

1. Express 1 mi. as chains; as rods; as links.
2. Express 142 ch. as miles and chains; as rods.
3. Express 17 rd. as chains; as links; as inches.
4. How many feet in 60 li.? in 10 ch.? in 125 li.?
5. Express 17 mi. 32 ch. as miles and the decimal part of a mile; as chains; as rods.
6. It is 30 ch. 44 li. around a square field. How long is each side? Suppose it were 97 ch. 64 li. around?
7. An equilateral triangle is 4.72 ch. on a side. How many links in the perimeter? How many chains?
8. By using the surveyors' table, find how many inches there are in a mile. Check by using the common table.

Add:

9.	10.	11.
4 ch. 3 rd.	38 ch. 93 li.	9 mi. 275 rd.
9 2	19 78	8 192

Subtract:

12.	13.	14.
26 ch. 1 rd.	19 ch. 15 li.	16 mi. 12 rd.
17 3	13 87	7 234

Multiply:

15.	16.	17.
4 ch. 2 rd.	29 ch. 78 li.	8 mi. 192 rd.
27	16	37

364. Surveyors' table of square measure:

16 sq. rd. = 1 square chain (sq. ch.).
10 sq. ch. = 1 acre (A.).

The square rod is sometimes called a *perch*. The word *rood* is sometimes met in reading, meaning 40 sq. rd. or $\frac{1}{4}$ acre. 640 acres = 1 sq. mi., and 36 sq. mi. = 1 township in the government surveys.

ORAL EXERCISE

1. Add 6 ch. 2 rd. and 3 ch. 3 rd.
2. Express 3 A. as square chains.
3. From 7 ch. subtract 4 ch. 2 rd.
4. From 5 ch. subtract 1 rd. 40 li.
5. Add 7 ch. 90 li. and 6 ch. 70 li.
6. Express 2 mi. as rods; as chains.
7. Multiply 1 ch. 2 rd. by 2; by 3; by 4.
8. Express as square rods: 3 sq. ch., 5 sq. ch.
9. How many acres in 30 sq. ch.? 50 sq. ch.?
10. How many square chains in 7 A.? in 17 A.?
11. How many feet in 10 mi.? How many chains?
12. Express as rods: 3 ch., 5 ch., 7 ch., 16 ch., 30 ch.
13. How many square chains in a field 6 ch. by 9 ch.? How many acres?
14. How many square chains in a field 15 ch. by 11 ch.? How many acres?
15. How many square chains in a field 5 ch. by 4 ch.? How many acres?
16. Divide 15 ch. by 2, expressing the result as chains; as rods; as chains and rods.

365. Illustrative problem. How many acres in a field 32 ch. long and 6 ch. 30 li. wide?

6 ch. 30 li. = 6.30 ch.

32 times 6.30 times 1 sq. ch. = 201.6 sq. ch.

Since 1 sq. ch. = $\frac{1}{10}$ A.,

201.6 sq. ch. = 201.6 times $\frac{1}{10}$ A. = 20.16 A.

Actual measurements of this kind in connection with the school ground and other pieces of land are valuable.

WRITTEN EXERCISE

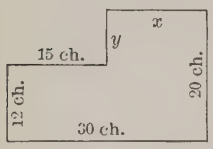
Find the areas in acres:

1. 16 ch. by 14 ch.
2. 8 ch. by 23 ch.
3. 23 ch. by 29 ch.
4. 32 ch. by 34 ch.
5. 5 ch. 2 rd. by 11 ch.
6. 1 ch. 1 rd. by 2 rd.
7. 5 ch. 30 li. by 17 ch.
8. 6 ch. 48 li. by 9 ch.
9. 6 ch. 80 li. by 13 ch.
10. 5 ch. 18 li. by $3\frac{1}{2}$ ch.
11. 8 ch. 10 li. by $2\frac{1}{2}$ ch.
12. 8 ch. 42 li. by 12 ch.
13. 11 ch. 75 li. by 6 rd.
14. 8 ch. 30 li. by 10 ch. 40 li.
15. 12 ch. 45 li. by 15 ch. 75 li.
16. 13 ch. 80 li. by 14 ch. 65 li.
17. 32 ch. 75 li. by 27 ch. 50 li.
18. 27 ch. 42 li. by 31 ch. 60 li.
19. 34 ch. 27 li. by 42 ch. 83 li.
20. 14 ch. 15 li. by 13 ch. 17 li.
21. Express 1 A. as square yards; as square feet.
22. Express 1 sq. rd. as square feet; as square inches.
23. Express 1 sq. mi. as square rods; as square yards; as square feet.
24. Express 3 sq. mi. 32 A. as square miles and the decimal part of a square mile; as acres.

25. Express 7 A. 8 sq. ch. as acres and the decimal part of an acre; as square chains; as square rods.

26. How many square feet in a city lot 31.8 ft. wide by 107.6 ft. long?

27. What is the difference in area between a piece of ground 20 rd. square and one containing 20 sq. rd.?



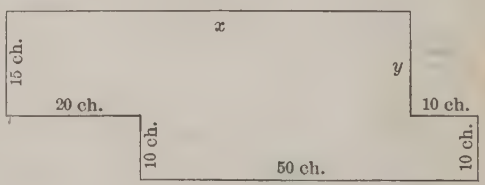
28. In the field here shown find the length of x and y; the perimeter; the number of acres.

29. How many square feet in a school playground $8\frac{1}{2}$ rd. wide and 160 ft. long?

30. A piece of land 34 ch. by 15 ch. is sold for \$75 an acre. What is the price?

31. Which costs the more, a piece of land 42 ch. by 27 ch. at \$65 an acre, or a piece 37 ch. by 29 ch. at \$70 an acre? How much more?

32. In the field here shown find the length of x and y; the perimeter; the number of acres.



33. Two fields have equal perimeters, 80 ch. One is a square and the other a rectangle 30 ch. long. Having equal perimeters, have they equal areas? What is the area of each?

Find the value of the following pieces of land:

34. 24 ch. by 30 ch. at \$75 an acre.

35. 42 ch. by 65 ch. 50 li. at \$85 an acre.

36. 37 ch. 50 li. by 42 ch. 75 li. at \$60 an acre.

ORAL EXERCISE

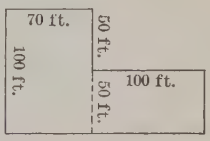
1. If a rectangle has an area of 50 sq. in., and is 10 in. long, how wide is it? What is its perimeter?

State only the answers on this page. Reserve explanations for written work.

State the lengths of rectangles whose areas and widths are as follows:

2. 72 sq. ft., 8 ft.
3. $\frac{1}{3}\frac{5}{2}$ sq. in., $\frac{3}{4}$ in.
4. $\frac{5}{1\frac{1}{2}}$ sq. in., $\frac{2}{3}$ in.
5. 45 sq. rd., 5 rd.
6. 56 sq. ch., 7 ch.
7. 54 sq. ch., 6 ch.
8. 132 sq. ft., 11 ft.
9. 143 sq. in., 11 in.
10. 108 sq. ch., 9 ch.
11. 750 sq. in., 20 in.

12. How many square feet in the area of this city lot? (Take the parts on opposite sides of the dotted line, find the area of each separately, and add.)



13. Find the areas of squares whose sides are 6 in., 12 in., 4 in., 11 in., 20 in., 40 in., 100 in.

14. A square has an area of 25 sq. in. What is the length of each side?

State the lengths of the sides of squares whose areas are as follows:

15. 49 sq. ch.
16. 81 sq. ft.
17. 64 sq. in.
18. 100 sq. mi.
19. 144 sq. in.
20. 121 sq. ft.
21. 36 sq. mi.
22. 400 sq. mi.
23. 900 sq. ft.
24. 1600 sq. ft.
25. 2500 sq. ft.
26. 3600 sq. ft.

366. Illustrative problems. 1. If a rectangle has an area of 15 sq. in. and is 5 in. long, how wide is it?

1. If a rectangle is 5 in. long and 1 in. wide, area = 5 sq. in.
2. Therefore the number of inches in width is 15 sq. in. ÷ 5 sq. in. = 3. Therefore it is 3 in. wide.

Pupils should construct the figure by paper folding or cutting, or should draw it, if necessary. Another form of solution is given in Ex. 2.

2. A rectangle $32\frac{1}{2}$ in. long has an area of $227\frac{1}{2}$ sq. in. What is the width?

Work:

$32\frac{1}{2})227\frac{1}{2}$

$65)455$

455

7

1. If w = the number of inches in width, then w times $32\frac{1}{2}$ times 1 sq. in. = $227\frac{1}{2}$ sq. in.
2. Therefore $w = 227\frac{1}{2}$ sq. in. ÷ $32\frac{1}{2}$ sq. in.
3. Therefore $w = 7$, the number of inches in width.

367. We therefore see that

The number of units of area of a rectangle, divided by the number of units of length, equals the number of units of width.

Or, briefly, *Area divided by length equals width.*

WRITTEN EXERCISE

1. What is the area of a rectangle $5' \times 4'$? Draw the figure and explain all of your work.

2. How wide is a 12.95-acre farm 17 ch. long? Consider such farms as rectangular.

3. How long is a 272-acre farm 40 ch. wide?

4. A room having a floor area of $205\frac{5}{9}$ sq. ft. is 16 ft. 8 in. long. How wide is it?

5. A school playground 63 ft. wide has an area of 4473 sq. ft. How long is it?

6. How wide is a 295.2-acre farm 72 ch. long? Draw to scale a plan of the farm.

7. What is the area of a rectangle 3.25 in. by 5.75 in.?

8. A rectangle has an area of 294 sq. ft. It is 17 ft. 6 in. long. How wide is it?

9. A floor has an area of $358\frac{7}{8}$ sq. ft. The room is 21 ft. 9 in. long. What is the width?

10. A floor has an area of 560 sq. ft. The width of the room is $17\frac{1}{2}$ ft. What is the length?

11. Find the cost of a farm 120 rd. by 242 rd., at \$75 an acre. Draw to scale a plan of the farm.

12. A floor has an area of 288 sq. ft. 80 sq. in. The room is 17 ft. 8 in. long. What is the width?

13. A floor has an area of 788 sq. ft. 18 sq. in. The room is 24 ft. 3 in. wide. What is the length?

14. A rectangular table top is $42\frac{3}{4}$ in. long, and it has an area of $1296\frac{3}{4}$ sq. in. What is the width?

15. A page of a book is $7\frac{1}{8}$ in. by $4\frac{5}{8}$ in., and the book has 400 pages. How many square feet of page surface?

16. A rectangular playground has an area of 188 sq. yd. 540 sq. in. It is 14 yd. 9 in. long. What is the width?

17. How many square yards of oilcloth are needed for a kitchen 15 ft. by 21 ft., allowing 5% more than the area for matching?

18. Two fields have each a perimeter of 376 rd. One is 71 rd. wide and the other is 93 rd. wide. Find the length and area of each, and draw a plan.

19. Two fields have each an area of 4750 sq. rd. One is 40 rd. wide and the other 50 rd. What is the length of each? the perimeter? Draw each to a scale.

20. A city building lot, rectangular in shape, is advertised as having an area of 2107 sq. ft. The lot has a frontage of 21 ft. 6 in. What is the depth of the lot?

Farm Measures

WRITTEN EXERCISE

1. A farmer buys three tracts of land of the following dimensions: 51 ch. 12 li. × 20 ch., 51 ch. 13 li. × 40 ch., 52 ch. × 21 ch. How many acres in each? in all?

51 ch. 12 li. × 20 ch. means 51 ch. 12 li. by 20 ch.

2. He paid $48.50 an acre for the land. What was the total cost?

3. A piece of the property 7 ch. long and 2 ch. 28 li. wide was not tillable. What was the area of this part? of the tillable part?

4. The farmer grew wheat on a piece $66\frac{2}{3}$ ch. long and 15 ch. wide, and the yield was 13 bu. to the acre. He sold the wheat at 85¢ a bushel. How much did he receive for it?

5. He grew oats on a piece 15 ch. long and 10 ch. wide, and the yield was 42 bu. to the acre. He sold the oats at 59¢ a bushel. How much did he receive for it?

6. He grew corn on a piece 20 ch. long and 15 ch. wide, and the yield was 34 bu. to the acre. He sold the corn at 56¢ a bushel. How much did he receive for it?

7. He used an acre for his house and lawn, this piece having a frontage of 120 ft. What was the depth of the lot? What was the length of fence necessary to inclose it?

8. The house is 50 ft. long and 30 ft. wide, and is rectangular in ground plan. How many square feet are left from the acre?

9. The farmer took for a vegetable garden a rectangular piece of land 60 rd. in perimeter, the length being twice the width. What was the area of the piece?

ORAL EXERCISE

State the areas of the parallelograms whose bases and altitudes are given in Exs. 1–8. (*See page 155.*)

1. 160 ft., 25 ft.
2. 66 ft., $16\frac{2}{3}$ ft.
3. 64 in., $12\frac{1}{2}$ in.
4. 150 ft., $33\frac{1}{3}$ ft.
5. 444 in., 25 in.
6. 303 ft., $33\frac{1}{3}$ ft.
7. 800 in., 125 in.
8. 300 ft., $66\frac{2}{3}$ ft.
9. State a rule for finding the area of a parallelogram.

368. Parallelogram. Illustrative problem. What is the area of a parallelogram of base 10 ft. and altitude 6 ft.?

1. A rectangle 10 ft. by 6 ft. has an area 10 times 6 sq. ft.
2. Therefore the area of the parallelogram is also 60 sq. ft. (See page 155.)

WRITTEN EXERCISE

Find the areas of parallelograms of base and altitude as follows:

1. 6.7 in., 3.9 in.
2. 42.5 ft., 27.8 ft.
3. 16.9 ft., 7.2 ft.
4. $3\frac{1}{8}$ yd., 2 ft. 7 in.
5. 5 rd. 6 ft., $2\frac{1}{2}$ yd.
6. 17 ft. 8 in., 27 in.
7. 9 ft. 6 in., 6 ft. 9 in.
8. 345.25 ft., 127.37 ft.
9. 2 yd. 17 in., 2 ft. 3 in.
10. 17 yd. 23 in., 5 yd. 2 ft.

Find the altitudes of parallelograms of bases and areas as follows:

11. $15\frac{1}{2}$ in., 108.5 sq. in.
12. 25 ft., 275 sq. ft.
13. 22 ft. 4 in., 268 sq. ft.
14. 26.46 in., 588 sq. ft.
15. 129.9 ft., 303.1 sq. yd.
16. 31 ft. 2 in., 187 sq. ft.
17. 16 ft. 8 in., $95\frac{5}{6}$ sq. ft.
18. 17 ft. 6 in., $157\frac{1}{2}$ sq. ft.

ORAL EXERCISE

State the areas of the triangles whose bases and altitudes are given in Exs. 1–6. (*See page 154.*)

1. 80 in., 75 in.
2. 80 in., 25 in.
3. 16 in., 125 in.
4. 44 in., 50 in.
5. 60 in., $33\frac{1}{3}$ in.
6. 120 in., $16\frac{2}{3}$ in.
7. State a rule for finding the area of a triangle.

369. Triangle. Illustrative problem. What is the area of a triangle of base 18 in. and altitude 7 in.?

$\frac{1}{2}$ of 18 times 7 sq. in. = 63 sq. in. (See page 154.)

WRITTEN EXERCISE

Find the areas of the triangles whose bases and altitudes are given in Exs. 1–8:

1. 43 ft., 32 ft.
2. 9.3 ft., 4.9 ft.
3. 7.9 in., 8.7 in.
4. $37\frac{1}{4}$ ft., $42\frac{1}{2}$ ft.
5. 8.7 in., 6.3 in.
6. 2 yd. 32 in., 27 in.
7. 324 in., 47 in.
8. 3 ft. 4 in., 2 ft. 8 in.

9. What is the area of an equilateral triangle whose perimeter is 37 ft. 6 in. and whose altitude is 10.8 ft.?

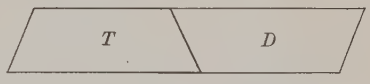
10. What is the base of a triangle whose altitude is 6.4 rd. and area $\frac{1}{2}$ acre? altitude 7.2 in. and area $\frac{1}{2}$ sq. ft.?

11. A right-angled triangle has the two sides forming the right angle 30 rd. and 40 rd. What is its area in square rods? in acres?

12. What is the number of square feet of slating required for a steeple, the surface being made up of 6 triangles, each with a base 20 ft. and altitude 65 ft.?

370. Trapezoid. A four-sided figure with two parallel sides is called a *trapezoid.*

371. Area of a trapezoid. If a trapezoid T have a duplicate cut from paper and turned over and fitted to it, like D, the two together make a parallelogram. (Illustrate by paper cutting.) Therefore



The area of a trapezoid equals half that of a rectangle with the same altitude and with a base equal to the sum of the two parallel sides.

372. Illustrative problem. What is the area of a trapezoid of altitude 4 in. and parallel sides 8 in. and 10 in.?

As above explained, the area is $\frac{1}{2}$ of 4 times (8 + 10) sq. in., or 36 sq. in.

ORAL EXERCISE

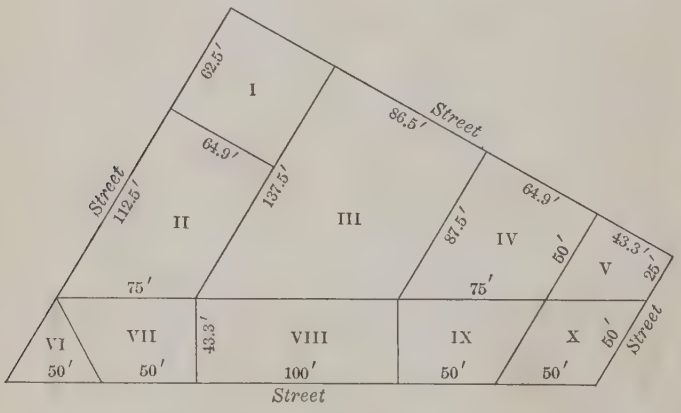
State the areas of the trapezoids whose altitudes are first given in Exs. 1–10, followed by the two parallel sides:

1. 6 in., 7 in. and 13 in.
2. 8 in., 3 in. and 12 in.
3. 10 in., 12 in. and 13 in.
4. 12 in., 7 in. and 8 in.
5. 14 in., 14 in. and 16 in.
6. 13 in., 6 in. and 14 in.
7. 22 in., 21 in. and 29 in.
8. 15 in., $7\frac{1}{2}$ in. and $12\frac{1}{2}$ in.
9. 17 in., $8\frac{1}{3}$ in. and $11\frac{2}{3}$ in.
10. 11 in., $11\frac{1}{4}$ in. and $18\frac{3}{4}$ in.
11. If the area of a trapezoid is 48 sq. in., and the parallel sides are 5 in. and 7 in., what is the altitude?
12. If the area of a trapezoid is 48 sq. in. and the altitude is 6 in., what is the sum of the parallel sides? If one of these sides is 9 in., what is the other?

WRITTEN EXERCISE

Find the areas of the trapezoids whose altitudes are first given in Exs. 1–7, followed by the two parallel sides:

1. 5 rd., 6 rd. 7 ft. and 9 rd.
2. 322 ft., 427 ft. and 534 ft.
3. 127 ft., 129 ft. and 148 ft.
4. 236 in., 208 in. and 235 in.
5. 34 ft., 27 ft. 8 in. and 33 ft.
6. 62 ft. 3 in., 59 ft. and 78 ft.
7. 4 yd., 2 yd. 27 in. and 6 yd.
8. If the area of a trapezoid is 21 sq. ft. 16 sq. in., the altitude 5 ft. 4 in., and one of the parallel sides 4 ft. 9 in., how long is the other parallel side?



An irregular city block is divided into lots as shown. Find the area, in square feet, of lots numbered as follows:

9. I. 10. II. 11. III. 12. IV. 13. V.
14. VI. 15. VII. 16. VIII. 17. IX. 18. X.

373. Laying out public lands. In the more recently settled parts of the country land is laid out as here described.

374. Principal meridian. Through a given tract a meridian is chosen as the *principal meridian.*

375. Base line. An east and west line is chosen as the *base line.*

The principal meridian and base line are here shown.

376. Townships. Lines are run parallel to the principal meridian and base line, at intervals of 6 mi. This divides the land into *townships.*

377. Ranges. The north and south rows of townships are called *ranges.*

X on the first map is numbered T. 2 N., R. 3 W.; that is, the second township north of the base line, in the third range west of the meridian.

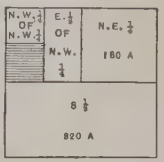
			N			
	6	5	4	3	2	1
	7	8	9	10	11	12
W	18	17	16	15	14	13	*E*
	19	20	*Y*	22	23	24
	30	29	28	27	26	25
	31	32	33	34	35	36
			S			

378. Sections. A township is divided into *sections,* each 1 mi. square.

This map shows the method of numbering these sections.

Each section is then divided as shown in the third map.

If this map represents the shaded part *Y* of the second map, and that represents the shaded part *X* of the first map, the shaded part here shown would be thus described: S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 21, T. 2 N., R. 3 W. This means the southwest quarter of the northwest quarter of section 21, second township north, third range west.



In sections of the country where land is not laid out in this way little attention should be given to this subject.

WRITTEN EXERCISE

Write the description, plot, and find the area:

1. S.E. $\frac{1}{4}$, Sec. 5, T. 3 S., R. 3 W.
2. N.E. $\frac{1}{4}$, Sec. 8, T. 2 N., R. 2 W.
3. E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 2, T. 2 N., R. 3 E.
4. S. $\frac{1}{2}$ of S.E. $\frac{1}{4}$, Sec. 20, T. 2 S., R. 3 W.
5. N. $\frac{1}{2}$ of S.W. $\frac{1}{4}$, Sec. 10, T. 3 S., R. 2 E.
6. E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 30, T. 2 S., R. 3 E.
7. N.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$, Sec. 5, T. 1 N., R. 1 W.
8. S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 32, T. 1 S., R. 3 E.
9. N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 6, T. 3 N., R. 3 W.

10. How much is this farm worth at \$65 an acre? W. $\frac{1}{2}$ of S. $\frac{1}{2}$, Sec. 3, T. 2 N., R. 2 W.

11. How much is this farm worth at \$75 an acre? N.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$, Sec. 5, T. 2 S., R. 2 E.

12. Find the area of this farm: S.W. $\frac{1}{4}$, Sec. 10, T. 2 S., R. 2 E. Draw a plan of a township and locate the farm.

13. Find the area of this farm: N. $\frac{1}{2}$, Sec. 6, T. 1 N., R. 1 E. Draw a plan of a township and locate the farm.

14. Mr. Simmons owns the S. $\frac{1}{2}$, N.E. $\frac{1}{4}$, Sec. 3, T. 2 N., R. 3 E. How many rods of fence are needed to inclose it?

15. How far is it from Mr. Taylor's farm, N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 16, T. 1 N., R. 3 W., to Mr. Hunt's farm, S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 28, T. 1 N., R. 3 W.? Draw the map.

16. A road running straight through a farm is $\frac{1}{2}$ mi. long and 3 rd. wide. How many acres in the road? If hay can be cut from the sides, averaging $\frac{1}{4}$ of the area, and the amount of hay averages 2 tons to the acre and is worth \$9.50 a ton, what is gained by attending to this crop?

II. MEASURES. PERCENTAGES. PROPORTION

VOLUMES

ORAL EXERCISE

1. Give the table of dry measure.

2. Give the table of cubic measure.

3. Give the table of liquid measure.

4. A cellar is $21' \times 18'$, and $9'$ deep. Express the dimensions in yards; the volume in cubic yards.

5. At 50 ct. a cubic yard, how much will it cost to excavate a cellar containing 126 cu. yd.? 175 cu. yd.?

$21' \times 18'$ means 21 ft. by 18 ft. $6'' \times 5''$ means 6 in. by 5 in.

6. What is the volume of a box $7'' \times 3'' \times 8''$?

7. What is the volume of a box $6'' \times 5'' \times 10''$?

8. How many cubic feet in a room $10' \times 12' \times 9'$?

9. How many cubic feet in a room $12' \times 12' \times 10'$?

Find the volumes of solids of the following dimensions:

10. $3'' \times 4'' \times 7''$.
11. $2'' \times 8'' \times 9''$.
12. $6' \times 8' \times 10'$.
13. $5' \times 9' \times 11'$.
14. $20' \times 30' \times 5'$.
15. $3' \times 8' \times 11'$.
16. $11'' \times 11'' \times 10''$.
17. 6 ft. $\times$ 8 ft. $\times$ 11 ft.
18. 6 yd. $\times$ 8 yd. $\times$ 2 yd.
19. 4 ft. $\times$ 9 ft. $\times$ 11 ft.
20. 21 in. $\times$ 20 in. $\times$ 10 in.
21. 5 yd. $\times$ 2 yd. $\times$ 7 yd.
22. 11 in. $\times$ 12 in. $\times$ 10 in.
23. 2 yd. $\times$ 7 yd. $\times$ 11 yd.
24. 12 in. $\times$ 12 in. $\times$ 10 in.
25. 3 yd. $\times$ 9 yd. $\times$ 11 yd.
26. 23 in. $\times$ 20 in. $\times$ 10 in.
27. 5 yd. $\times$ 8 yd. $\times$ 12 yd.
28. 42 ft. $\times$ 11 ft. $\times$ 10 ft.
29. 5 rd. $\times$ 16 rd. $\times$ 20 rd.

379. Illustrative problem. What is the volume of a box 3 ft. wide, $6\frac{1}{3}$ ft. long, 2 ft. high?

If it were 1 ft. each way, the volume would be 1 cu. ft.

But it is 3 times as wide, $6\frac{1}{3}$ times as long, and 2 times as high.

Therefore the volume is

3 times $6\frac{1}{3}$ times 2 times 1 cu. ft., or 38 cu. ft.

If there is any difficulty in understanding this fact, in this review, inch cubes may be used as in the earlier classes.

WRITTEN EXERCISE

1. How many cords in a pile of 4-ft. wood, 5 ft. high and 56 ft. long? 6 ft. high and 128 ft. long?

2. How many loads (cubic yards) of earth must be taken out in excavating a cellar $24' \times 18'$, and $7'$ deep?

3. How much will it cost to excavate a ditch $150' \times 2' \times 6'$, at $50¢$ per cubic yard? at $62\frac{1}{2}¢$ per cubic yard?

4. A schoolroom is 32 ft. 8 in. long, $18\frac{1}{3}$ ft. wide, and $12\frac{1}{2}$ ft. high. What is the number of cubic feet in the room?

5. A tunnel 720 ft. long has a cross-section area of 180 sq. ft. How much earth and rock were excavated? (Imagine the cross section $10' \times 18'$.)

6. In preparing a flower bed in a park it was necessary to fill in a space $60' \times 100'$ to an average depth of $1\frac{1}{2}'$. The earth cost $33\frac{1}{3}¢$ a load. What was the total cost of the earth used?

7. A freight car is $8' \times 34'$, and the interior is $7'$ high. How many cubic feet does it contain? If it is filled with grain to a height of $5'$, what is the weight of the grain at 60 lb. to the bushel, allowing $1\frac{1}{4}$ cu. ft. to the bushel?

8. A fish pond has been excavated to a depth of 5 ft. The dimensions of the bottom are 40 ft. and 30 ft. How much earth was excavated?

9. The size of an ordinary Nebraska farm-wagon box is 10 ft. by 3 ft., and 24 in. to 26 in. deep. Find the contents in cubic feet for each of these depths; also in cubic inches.

10. Such a wagon box 24 in. deep contains 50 bu. of shelled corn. How many bushels of shelled corn will such a box 26 in. deep contain?

11. If a wagon box contains 50 bu. of wheat worth 89¢ a bushel, how many loads must be drawn to carry $311.50 worth? to carry $133.50 worth?

12. Prairie hay is heavier than eastern meadow hay, a cube of the former 7 ft. on an edge weighing a ton (the measurement taken 30 days after the hay is stacked). If it takes $33\frac{1}{3}\%$ more in bulk of a certain eastern meadow hay to make a ton, how many cubic feet to a ton of the latter?

13. What is the weight of prairie hay that will fill a space 30 ft. long, 17 ft. 6 in. wide, and 11 ft. deep, taking the number of cubic feet to the ton suggested in Ex. 12?

14. What is the weight of the eastern meadow hay mentioned in Ex. 12 required to fill the space mentioned in Ex. 13?

15. A haystack is estimated to contain 2600 cu. ft. If prairie hay, what does it weigh, to the nearest half ton? If eastern meadow hay (Ex. 12)?

16. A schoolroom is 30 ft. long, 20 ft. wide, and 15 ft. high. If it is occupied by 29 children and the teacher, how many cubic feet of air are allowed to each? If 2400 cu. ft. of fresh air per hour should be allowed to each person, how many times an hour should the air be changed?

Problems in Excavation

WRITTEN EXERCISE

1. How many cubic yards of earth must be removed in digging a tunnel 492 ft. long, 39 ft. wide, and 19 ft. 6 in. high?

2. If a dirt car is 27 ft. long, 6 ft. wide, and $3\frac{1}{4}$ ft. deep, how many cubic yards will it carry? How many will 12 cars carry?

3. If one laborer can shovel a cubic yard of earth into a car in 30 min., how long will it take him to load a car? How long will it take 15 men? (See Ex. 2.)

4. How many cars of the above dimensions will it take to remove the earth from the tunnel mentioned in Ex. 1? How many trains of 9 cars each? Allowing 30 ft. for the total length of a car, how long a train of cars would be needed?

5. If one laborer can remove the earth from a space 20 ft. long, 6 ft. wide, and 6 ft. deep in a day, how long will it take 100 men to remove the earth from the tunnel above mentioned?

6. If a laborer is paid at the rate of 10¢ a load (cubic yard) of earth removed, what will he earn in a day, removing the amount specified in Ex. 5?

7. In the first great subway in New York City 9733 cu. yd. of brick were used. Taking the size of the brick as $8\frac{1}{4}'' \times 4'' \times 2''$, how many bricks were used?

8. What will be the earnings of a laborer who can excavate a space a yard wide, a yard deep, and 12 ft. long, in solid rock, in one day, at 75¢ per cubic yard? How much at $87\frac{1}{2}$¢ per cubic yard?

ORAL EXERCISE

In Exs. 1–12, x *represents a missing number. What is the number?*

1. $2 \times 6 \times x = 24$.
2. $2 \times 3 \times x = 42$.
3. $3 \times 4 \times x = 84$.
4. $5 \times 6 \times x = 90$.
5. $2 \times 6 \times x = 60$.
6. $8 \times 8 \times x = 64$.
7. $5 \times 6 \times x = 210$.
8. $6 \times 9 \times x = 108$.
9. $5 \times 5 \times x = 125$.
10. $2 \times 6 \times x = 144$.
11. x times 13 cu. ft. = 130 cu. ft.
12. x times 16 cu. ft. = 320 cu. ft.

13. If the product of three numbers is 400, and two of the numbers are 5 and 10, what is the third one?

14. If the product of three numbers is 540, and the product of two of them is 90, what is the third number?

15. If a box is 9 in. by 10 in. by 11 in., what is its cubic contents? If the cubic contents of a box 9 in. wide and 11 in. long is 99 cu. in., how deep is the box?

16. If a box contains 54 cu. in., and is 9 in. long and 3 in. wide, how deep is it?

17. If a box contains 24 cu. in., and the area of the base is 6 sq. in., how deep is the box?

Given the following volumes of boxes, and the length and breadth, find the depth:

18. 60 cu. in., 5 in., 4 in.
19. 64 cu. in., 8 in., 4 in.
20. 18 cu. in., 3 in., 2 in.
21. 108 cu. in., 9 in., 6 in.
22. 63 cu. in., 7 in., 3 in.
23. 160 cu. in., 8 in., 4 in.
24. 240 cu. in., 10 in., 6 in.
25. 350 cu. in., 10 in., 7 in

380. Illustrative problems. 1. How thick is a block 5 ft. long and 4 ft. wide, containing 60 cu. ft.?

Analysis:

If it is x ft. thick, x times 5 times 4 cu. ft. = 60 cu. ft. and $x = \frac{60 \text{ cu. ft.}}{5 \text{ times } 4 \text{ cu. ft.}} = 3$. Therefore it is 3 ft. thick.

Work:

$$\frac{60}{5 \times 4} = 3$$

We may, if we prefer, give this analysis:

Since it is 5 ft. long and 4 ft. wide, the area of the base is 20 sq. ft.

If it were 1 ft. high, the volume would be 20 cu. ft.

Therefore it is as many times 1 ft. high as 60 cu. ft. ÷ 20 cu. ft., or 3.

381. Hence we see that

The number of cubic units of volume, divided by the product of the numbers of units of two dimensions, equals the number of units of the third dimension.

This is sometimes less accurately expressed: *The volume divided by two dimensions equals the third dimension.*

2. What is the area of the base of a block $2\frac{1}{2}$ in. high, containing 70 cu. in.?

If it were 1 in. high, the volume would be 70 cu. in. ÷ $2\frac{1}{2}$ = 28 cu. in. But the base of such a block is 28 sq. in.

$$70 \div \frac{5}{2} = \frac{2 \times 70}{5} = 28$$

This might be briefly stated as in § 381: *The volume divided by the height equals the area of the base.* This has a meaning only as we think of the measures as abstract numbers.

3. What is the thickness of a block containing 100 cu. in., the base containing 30 sq. in.?

If it were 1 in. thick, it would contain 30 cu. in. Therefore it is as many times 1 in. thick as 100 cu. in. ÷ 30 cu. in. = $3\frac{1}{3}$. Therefore it is $3\frac{1}{3}$ in. thick.

$$\frac{100}{30} = 3\frac{1}{3}$$

WRITTEN EXERCISE

1. How many cubic feet in a hall $80' \times 60' \times 23'\ 4''$?

2. A tank $9' \times 16\frac{2}{3}'$ contains 900 cu. ft. How deep is it?

3. A box $13'' \times 20''$ contains 3900 cu. in. How deep is it?

4. A tank 4′ deep contains 450 cu. ft. What is the area of the base?

5. A hall 22′ high and 30′ wide contains 28,600 cu. ft. Find the length.

6. A block containing 700 cu. ft. has a base area of 56 sq. ft. How thick is it?

7. A hall contains 41,341.3 cu. ft. It is 41.3 ft. long and 36.4 ft. wide. How high is it?

8. A box contains 42.159 cu. in. The area of the bottom is 10.81 sq. in. How deep is the box?

9. A block of granite is 14 in. long and 8.3 in. wide. It contains 836.64 cu. in. How thick is it?

10. The floor of a hall contains 1386 sq. ft. The hall contains 15,246 cu. ft. Required the height.

11. A storeroom is 14.3 ft. long, 13 ft. 6 in. wide, and has a capacity of 2316.6 cu. ft. How high is the room?

12. The floor of a room is square and contains 144 sq. ft. The room contains 1368 cu. ft. Required the three dimensions of the room.

13. A room 24 ft. long and 9 ft. high, containing 3240 cu. ft., is carpeted with plain ingrain carpet 1 yd. wide. What is the cost of the carpet at 75¢ a yard?

14. A room contains 2110.68 cu. ft. The room is 16.4 ft. long, and the floor area is 234.52 sq. ft. What are the three dimensions of the room?

15. Express in bushels the volume of a bin 6 ft. by 4 ft. by 3 ft. $6\frac{1}{2}$ in., allowing 2150 cu. in. to the bushel.

16. How many bars of soap, each 4 in. long, $2\frac{3}{4}$ in. wide, and $1\frac{1}{2}$ in. thick, can be packed in a box 16 in. long, $13\frac{3}{4}$ in. wide, and 1 ft. deep?

17. What is the value of the bricks in a kiln 60 ft. long, 25 ft. wide, and 9 ft. high, each brick being 8 in. by 4 in. by 2 in., at \$9 a thousand?

18. The length of a tank is 100% greater than its width, and its width is 200% of its depth. If the width is 2 yd., how many gallons does it hold?

19. A cord of stone having the same volume as a cord of wood, what is a pile of stones 27 ft. long, 5 ft. wide, and 6 ft. high worth at \$4.20 a cord?

20. If it takes 550 cu. ft. of clover hay to make a ton, how many tons will fill a mow 32 ft. long, the width being double the depth and half the length?

21. Allowing 231 cu. in. to the gallon, how many gallons in a watering trough that is 6 ft. long and 16 in. wide, the ratio of its depth to its width being 3 : 4?

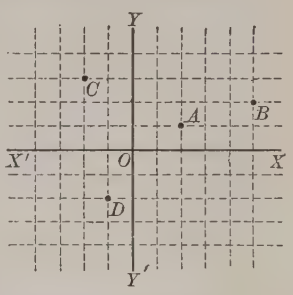
22. A bin 21 ft. 6 in. long and 10 ft. wide is filled with wheat to the depth of 5 ft. Allowing 2150 cu. in. to the bushel, what is the wheat worth at 80¢ a bushel?

23. Find the cost of the carpet needed for a room 11 ft. 3 in. wide, 10 ft. high, containing 2278 cu. ft. 216 cu. in., the carpet being 27 in. wide and costing \$1.35 a yard, allowing 9 in. for matching on each strip except the first.

24. A cellar is 32 ft. long, 17 ft. wide, and 8 ft. 6 in. deep. How much will it cost to build the wall, 1 ft. thick, at 35¢ per cubic foot, no allowance being made for openings, and the length of the wall being determined by the outside measure (thus doubling the corners)?

LONGITUDE AND TIME

382. Axes. A point on any surface may be located by *two* measures taken from two intersecting lines XX' (read "XX prime") and YY'. These lines are called *axes*.



In this figure the point A is 2, 1, and the point B is 5, 2.

Distances to the left of YY', or below XX', may be marked $-$. Thus, C is $-2, 3$, and D is $-1, -2$.

383. Prime meridian. An arc on the earth's surface, from the north pole to the south pole, is called a *meridian.* The one through the Royal Observatory at Greenwich, England, is taken by most nations as the *prime meridian.*

384. Points on a map. On a map the lines taken for locating points are the *equator* and the *prime meridian.*

385. Latitude and longitude. Instead of giving the distances from these lines in miles, they are given in degrees. Thus, St. Louis is located when we say that it is 90° 12′ 17″ W., 38° 38′ 3.6″ N. The distance in degrees east and west from the prime meridian is called *longitude;* north and south from the equator, *latitude.*

WRITTEN EXERCISE

1. Draw two axes and indicate the points 2, 4; 7, 6.
2. In the same way, indicate $-2, 4$; $-6, 1$; $-5, 5$.
3. In the same way, indicate the points $-2, -3$; $-1, -4$.
4. In the same way, indicate the points $3, -2$; $2, -1$.
5. Draw a rough map, indicating a place 40° N., 75° W.

386. Correspondence of longitude to time. Since the earth turns about on its axis once every 24 hours, the place in which we live will pass through 360° between now and this time to-morrow. That is, to the time 24 hours will correspond the longitude 360°. Therefore

360° of long. correspond to 24 hr.
1° " " corresponds " $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.
1′ " " " " $\frac{1}{60}$ " 4 min., " $\frac{1}{15}$ min., " 4 sec.
1″ " " " " $\frac{1}{60}$ " 4 sec., " $\frac{1}{15}$ sec.

387. Apparent motion of the sun. Because the earth *really* turns from west to east, the sun *appears* to pass from east to west, as trees *seem* to move when we are on the cars.

388. Earlier west, later east. Because the earth revolves 360° in 24 hours, the sun appears to pass through 15° an hour. Therefore when it is noon here it is an hour later, or 1 P.M., 15° east of here; an hour earlier, or 11 A.M., 15° west of here; and 6 hours earlier, or 6 A.M., 90° west of here.

This work belonging partly to geography, no illustrations are here introduced. The teacher should use the globe as needed.

ORAL EXERCISE

1. When it is 9 A.M. here, what time is it 15° east of here? 30° west of here? directly south of here?

2. When it is noon here, what time is it 30° east of here? 90° west of here? 150° east of here? directly north of here?

3. What difference of time corresponds to a difference of longitude of $7\frac{1}{2}$°? 45°? 60°? 105°? 120°? 165°? 180°?

4. Since 1′ of longitude corresponds to 4 sec. of time, to what does 7′ correspond? 11′? 15′? 30′? 45′? 60′? 120′?

5. Since 1″ of longitude corresponds to $\frac{1}{15}$ sec. of time, to what does 30″ correspond? 60″? 45″? 5″? 3″? 150″?

389. Illustrative problem. Two ships at sea are 65° 7′ 30″ of longitude apart. What is their difference in time?

1. Since 1° corresponds to $\frac{1}{15}$ hr., 65° correspond to 65 times $\frac{1}{15}$ hr. $= 4\frac{1}{3}$ hr. $= 4$ hr. 20 min.

2. Since 1′ corresponds to $\frac{1}{15}$ min., 7′ correspond to 7 times $\frac{1}{15}$ min. $= \frac{7}{15}$ min. $= 28$ sec.

3. Since 1″ corresponds to $\frac{1}{15}$ sec., 30″ correspond to 30 times $\frac{1}{15}$ sec. $= 2$ sec.

4. Therefore the difference in time is 4 hr. 20 min. 30 sec.

It is apparent that the mere figures of the answer could be obtained by this division, although the reason would not be so clear.

$$15^\circ)\overline{65^\circ\ 7'\ 30''}$$
$$\underline{\quad}$$
$$4\ 20\ 30$$

WRITTEN EXERCISE

1. 27° 4′ 15″ of longitude corresponds to what difference in time?

2. Two ships are 75° 30′ 30″ of longitude apart. What is their difference in time?

3. A ship at 62° 3′ 40″ W. receives a wireless telegram from one at 60° 1′ 10″ W. at 11 A.M. When was it sent?

4. For time purposes the longitude of Berlin is 15° E., and that of Chicago 90° W. What is the difference in longitude? Illustrate by a rough map. When it is 2 P.M. in Chicago, what time is it in Berlin?

5. For time purposes the longitude of San Francisco is taken as 120° W. When it is noon in London (on the prime meridian), what time is it in San Francisco? When it is noon in San Francisco, what time is it in London?

6. A steamer 68° 10′ 30″ W. sends a wireless telegram to the Nantucket light-ship, 70° W., at 8 : 30 A.M. At what time is it received? If transmitted to New York, 75° W., without loss of time, when is it received there?

390. Correspondence of time to longitude. Since 24 hours correspond to 360° of longitude, we have the following:

24 hr. correspond to 360°.
1 hr. corresponds to $\frac{1}{24}$ of 360°, or 15°.
1 min. " " $\frac{1}{60}$ " 15°, " $\frac{1}{4}$°, or 15′.
1 sec. " " $\frac{1}{60}$ " 15′, " $\frac{1}{4}$′, " 15″.

391. Illustrative problem. The difference in time between two ships is 3 hr. 7 min. 3 sec. What is the difference in longitude?

1. Since 1 hr. corresponds to 15°, 3 hr. correspond to 3 times 15° = 45°.

2. Since 1 min. corresponds to 15′, 7 min. correspond to 7 times 15′ = 105′ = 1° 45′.

3. Since 1 sec. corresponds to 15″, 3 sec. correspond to 3 times 51″ = 45″.

4. Therefore the difference in longitude is 46° 45′ 45″.

Evidently the mere figures of the answer could be obtained by calling 3 hr. 7 min. 3 sec. 3° 7′ 3″ and multiplying by 15, although the reason would not be so clear.

3°	7′	3″
		15
46°	45′	45″

WRITTEN EXERCISE

1. The difference of time between two ships is 2 hr. 3 min. 10 sec. What is the difference in longitude?

2. When it is noon at Denver, it is 7 P.M. at Greenwich. What longitude does Denver use for its time purposes?

3. The difference in time between the Harvard and Columbia observatories is 8 min. 22.7 sec. What is the difference in longitude?

4. At 10:4 A.M. a steamer in longitude 26° 30′ W. sends a wireless message to another steamer. It is received at 10:19 A.M. What is the longitude of the second steamer?

5. When it is noon in Greenwich it is 9 P.M. in Japan. What longitude does Japan use for its time purposes?

6. When it is noon in Melbourne it is 2 A.M. in Greenwich. What longitude does Melbourne use for its time purposes?

7. A traveler sails from New York with his watch set by the time of 75° W. If the watch indicates 2:30 P.M. when it is 5 P.M. by the ship's time, in what longitude is he?

8. A traveler sails from England with his watch set by Greenwich time. When he reaches 52° 30′ 45″ W., is his watch faster or slower than the time of that place, and how much?

9. Suppose a telegram sent from an observatory 121° 32′ 48″ W. at 8:40 A.M. to another observatory 73° 8′ 16″ W., and to require 16 min. for transmission. At what time will it be received?

10. Suppose a telegram sent at 4 P.M. from a place in Germany 15° E. to a place in California 120° W. At what time will it reach its destination, allowing 30 min. for repeating at various terminal offices?

11. A ship's captain observing the sun finds that when the ship is on the meridian (that is, at noon) the time is 3 hr. 7 min. 35 sec. P.M. by a chronometer set by Greenwich time. What is the ship's longitude?

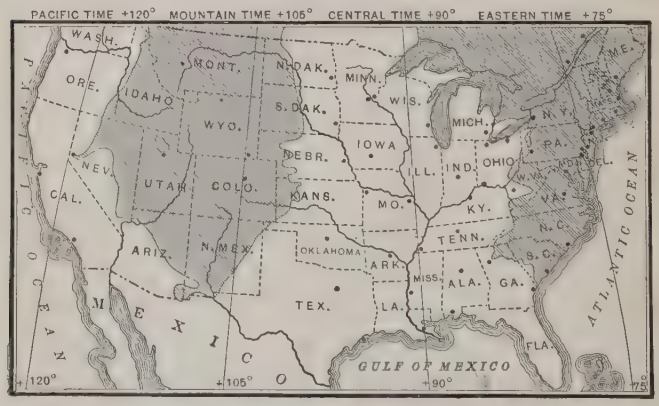
12. Two observatories A and B are connected by a telegraph line. A telegram sent from A at 2 hr. 4 min. 6 sec. P.M. reaches B at 4 hr. 8 min. P.M. The longitude of B is 79° 12′ 42.2″ W. What is the longitude of A?

13. The officers of a steamer in the Mediterranean Sea find that when the sun is on the meridian the time is 10 o'clock 50 min. 15 sec. A.M. by a chronometer set by Greenwich time. What is the ship's longitude?

392. Standard time. It is so much trouble to think of differences of time like 1 hr. 2 min. 17.5 sec., that most civilized countries have adopted a system of standard time. They have considered all places in one section as having the same longitude, some multiple of 15°, so that the differences in time from Greenwich (the prime meridian) shall be exact hours, and therefore all differences in time shall also be exact hours.

For this reason New York (73° 58′ 25.5″ W.) is considered to have the longitude 75° W., and Chicago (87° 36′ 42″ W.) the longitude 90° W. Then when it is noon in England (which is all considered of longitude 0°) it is 5 hours earlier in New York, or 7 A.M., and it is 6 A.M. in Chicago.

393. Standard time map. This map shows the standard time sections in the United States, the irregularities of



the divisions being due to the position of railway termini nearly 7½° from the *time meridians*.

When it is noon in New York it is 11 A.M. in the Central Time section, 10 A.M. in the Mountain Time section, and 9 A.M. in the Pacific Time section.

394. Standard time problems. Since for practical purposes standard time is generally used, the exercises refer to that time unless the contrary is stated. The cities selected are near the time meridians. Cities on the line dividing two time sections usually adopt one or the other.

Questions concerning the date line and the change of day do not involve arithmetic, but belong rather to geography.

ORAL EXERCISE

1. When it is 10 A.M. in Iowa, what time is it in New York? in Wisconsin? in Louisiana? in Ohio?

2. What time is it now in your school? in Boston? in Florida? in Arkansas? in Colorado? in California?

3. When it is 10 A.M. in San Francisco, what time is it in Denver? in St. Louis? in Cleveland? in Albany?

4. When it is 9 A.M. in Denver, what time is it in Los Angeles? in New Orleans? in Cincinnati? in Boston?

5. When it is noon in California, what time is it in Illinois? in Massachusetts? (Consult the map, page 342.)

6. When it is 3 P.M. in Milwaukee, what time is it in Oregon? in Louisville? in New York? in Philadelphia?

7. When it is 1 P.M. in New York, what time is it in Richmond? in Philadelphia? in Chicago? in California?

8. When it is 9:30 A.M. in New Orleans, what time is it in Alabama? in Maine? in Wyoming? in San Francisco?

9. Suppose a telegram sent from New York at 1 P.M. to San Francisco. If it goes without loss of time, what time will it be when it crosses the Mississippi River? when it crosses Colorado? when it reaches San Francisco?

WRITTEN EXERCISE

1. France uses the time of Paris (2° 20′ 15″ E.). When it is noon, standard time, in Iowa, what is the time in France?

2. Japan uses the time of 135° E. When it is noon at Greenwich, what is the time in Tokyo? in St. Louis? in Portland, Oregon?

3. All of Great Britain uses Greenwich time. If it is 5 : 30 P.M. at Liverpool, what is the local time of a ship in 17° 16′ 30″ W. longitude?

4. The longitude of Chicago is 87° 36′ 42″ W. What is the difference between the local (that is, the real meridian) and standard time?

5. Italy uses the Central European time (15° E.). When it is 7:30 A.M. in Springfield, Illinois, what time is it in Rome (7 hr. later)? in London?

6. When it is 1 P.M. in London, what is the time in Paris? (See Exs. 1, 3.)

7. The longitude of San Francisco is 122° 25′ 40.8″ W. Which is the later, local or standard time, and how much?

8. Berlin uses the standard time of 15° E. The longitude of Berlin is 13° 23′ 43.5″. What is the difference between the local and standard time?

9. What is the difference in standard and in local time between Berlin and San Francisco? (See Exs. 7, 8.)

10. At midnight, December 31, 75° time, the United States Naval Observatory sends an electric time signal to various parts of the world, indicating that a new year is beginning at Washington. When does the signal reach California? Colorado? Alabama? England? Germany?

395. Table of longitudes, for reference:

Albany	73° 44′ 48″ W.	New York	73° 58′ 25½″ W.
Berlin	13° 23′ 43½″ E.	Paris	2° 20′ 15″ E.
Chicago	87° 36′ 42″ W.	San Francisco	122° 25′ 40½″ W.

WRITTEN EXERCISE

1. What is the difference between the local and standard time of New York? of Albany?

2. What is the difference in local time between Berlin and Paris? Berlin and New York?

3. What is the difference in local time between Albany and San Francisco? Albany and Paris?

4. What is the difference in local time between New York and Chicago? Chicago and San Francisco?

5. When it is 1 P.M., standard time, at Chicago, what is the standard time at San Francisco? the local time?

6. When it is noon, local time, at Albany, what is the standard time there? What is then the standard time at Chicago?

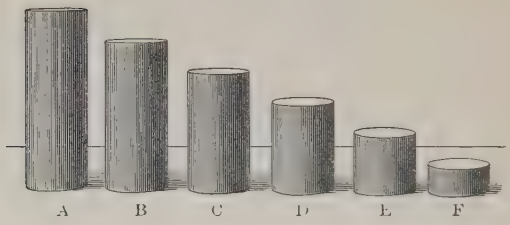
What differences of local time correspond to these differences of longitude?

7. 8° 8′ 8″.	8. 15° 27′ 30″.	9. 7° 23′ 48″.
10. 24° 10′ 3″.	11. 17° 42′ 15″.	12. 8° 19′ 32″.
13. 25° 16′ 9″.	14. 32° 48′ 19″.	15. 6° 17′ 19″.
16. 48° 16′ 17″.	17. 63° 19′ 30″.	18. 9° 13′ 45″.

What differences of longitude correspond to these differences of local time?

19. 3 hr. 4 min. 8 sec.	20. 10 hr. 4 min. 4 sec.
21. 5 hr. 17 min. 5 sec.	22. 7 hr. 9 min. 18 sec.
23. 8 hr. 19 min. 7 sec.	24. 5 hr. 50 min. 55 sec.

PERCENTAGE REVIEWED



ORAL EXERCISE

1. F is what part as large as E? what per cent?
2. F is what part as large as D? what per cent?
3. F is what part as large as C? what per cent?
4. F is what part as large as B? what per cent?
5. F is what part as large as A? what per cent?

Tell what per cent

6. E is of C.
7. E is of D.
8. D is of C.
9. E is of A.
10. E is of B.
11. C is of B.
12. E is of F.
13. D is of F.
14. C is of F.

E is twice F, or $\frac{200}{100}$ of F, or 200% of F.

15. A is of D.
16. C is of E.
17. D is of E.
18. Find 10%, or $\frac{1}{10}$, of 50, 175, 212, \$327, \$475.50.
19. Find 50%, or $\frac{1}{2}$, of 86.4, \$16.44, \$244.50, \$862.20.
20. Find $12\frac{1}{2}$%, or $\frac{1}{8}$, of 16, 80, 96, 728, 488, 648, \$840.
21. Find $16\frac{2}{3}$%, or $\frac{1}{6}$, of 36, 66, 90, 720, 366, 540, \$636.
22. Find 20%, or $\frac{1}{5}$, of 75, 90, 250, 305, 555, 750, \$3.50.
23. Find 25%, or $\frac{1}{4}$, of 64, 96, 324, 4.40, 2.40, 320, \$840.
24. Find $33\frac{1}{3}$%, or $\frac{1}{3}$, of 63, 96, 3.33, 1.50, 630, 900, 324.

Sam is 6 years old, Nora is 9, and Tom is 12. Tell the following per cents:

25. Sam's age is what per cent of Tom's?

26. Sam's age is what per cent of Nora's?

27. Nora is what per cent older than Sam?

28. Sam is what per cent younger than Nora?

29. Nora's age is what per cent of Tom's? of Sam's?

30. Tom is what per cent older than Sam? than Nora?

31. Nora's age is what per cent of the sum of Sam's and Tom's?

32. Nora's age is what per cent of the product of Sam's and Tom's?

33. Sam's age is what per cent of the difference between Nora's and Tom's?

It is 3000 miles from London to New York, 1000 miles from New York to Chicago, and 2000 miles from Chicago to San Francisco. Tell the following per cents:

34. The distance from Chicago to San Francisco is what per cent of that from London to Chicago?

35. The distance from New York to Chicago is what per cent of that from London to San Francisco?

36. The distance from New York to Chicago has what ratio to that from Chicago to San Francisco?

37. The distance from New York to San Francisco has what ratio to that from London to San Francisco?

38. The distance from London to New York is what per cent greater than that from New York to Chicago?

39. The distance from London to New York is what per cent less than that from London to San Francisco?

396. Terms used. When we say that 5% of \$400 is **\$20**, we call 5% the *rate*, \$400 the *base*, and \$20 the *percentage*, terms already defined in the earlier treatment of the subject.

397. *To find any per cent of a number, multiply the number by the rate.*

That is, *the percentage is the product of the base and rate.*

398. The most important per cents. The per cents studied on page 346 are among the most important of all, and their common-fraction values must be thoroughly known before taking up the business arithmetic on page 400.

WRITTEN EXERCISE

1. How much is \$448.64 plus 25% of itself?
2. How much is \$369.36 less $33\frac{1}{3}$% of itself?
3. Express as per cents: $\frac{4}{7}$, $\frac{5}{8}$, $\frac{3}{11}$, $\frac{4}{15}$, $\frac{5}{32}$, $\frac{5}{12}$.
4. How much is 125% of \$246.80? of \$1234.40?
5. What is the interest on \$750 for 1 yr. at $3\frac{1}{2}$%?
6. How much is 93% of \$623? of \$275? of \$350?
7. What is the discount on \$950 worth of goods at 15%?
8. If a village formerly of 1460 inhabitants has gained 5% in population, what is its population now?

Express as common fractions in their simplest forms:

9. 48%.	10. 40%.	11. 85%.	12. $28\frac{4}{7}$%.
13. $16\frac{3}{4}$%.	14. $22\frac{1}{2}$%.	15. $55\frac{5}{9}$%.	16. $37\frac{1}{2}$%.
17. $77\frac{7}{9}$%.	18. $27\frac{3}{11}$%.	19. $14\frac{2}{7}$%.	20. $17\frac{4}{5}$%.
21. $32\frac{3}{8}$%.	22. $27\frac{9}{10}$%.	23. $67\frac{4}{7}$%.	24. $53\frac{9}{32}$%.

Using common-fraction equivalents, find:

25. $16\frac{2}{3}$% of 102.
26. $33\frac{1}{3}$% of \$111.21.
27. $66\frac{2}{3}$% of \$729.
28. $12\frac{1}{2}$% of \$336.48.

ORAL EXERCISE

1. Since $125\% = 1\frac{1}{4}$, find 125% of \$400.
2. Find 150% of \$200, \$300, \$500, \$1000.
3. Find $133\frac{1}{3}\%$ of \$300, \$36, \$360, \$450, \$600.
4. Find $116\frac{2}{3}\%$ of \$60, 600 ft., 360 mi., 420 lb., 48 yd.
5. Since $75\% = 1 - \frac{1}{4}$, find 75% of 40, 400, 200, \$360.
6. Since $66\frac{2}{3}\% = 1 - \frac{1}{3}$, find $66\frac{2}{3}\%$ of 30, 66, 45, \$300.
7. Since $87\frac{1}{2}\% = 1 - \frac{1}{8}$, find $87\frac{1}{2}\%$ of 8, 16, 32, 56, 88.
8. Since $80\% = 1 - \frac{1}{5}$, find 80% of 50, 45, 75, \$5000.
9. Since $83\frac{1}{3}\% = 1 - \frac{1}{6}$, find $83\frac{1}{3}\%$ of 600, 60, 66, \$36.

Find 125% *of the following:*

10. 44 ft.	11. 50 ft.	12. \$120.	13. \$8.40.
14. 30 in.	15. 84 yd.	16. \$150.	17. \$1.80.
18. \$300.	19. 500 ft.	20. \$1000.	21. \$3000.

Find 150% *of the following:*

22. 66 ft.	23. 82 in.	24. \$150.	25. \$2.40.
26. 35 in.	27. 45 in.	28. \$250.	29. \$1.10.

Find $133\frac{1}{3}\%$ *of the following:*

30. \$330.	31. \$630.	32. \$120.	33. \$2.40.
34. \$1200.	35. 36 in.	36. 48 ft.	37. 75 rd.

Find $116\frac{2}{3}\%$ *of the following:*

38. \$1200.	39. \$660.	40. 2400.	41. \$3.60.

Find 75% *of the following:*

42. \$440.	43. \$300.	44. \$150.	45. \$220.

Find $66\frac{2}{3}\%$ *of the following:*

46. \$3.30.	47. \$7.50.	48. \$1.20.	49. \$5.40.

WRITTEN EXERCISE

1. If 20% of the days of a common year are stormy, how many days are not stormy?

2. A dealer bought \$167.40 worth of clocks and sold them at a profit of $33\frac{1}{3}\%$. How much did he gain?

3. How much is the interest on \$642.25 for 1 yr. at 4%? How much is it for 2 yr.? for $\frac{1}{2}$ yr.? for 2 yr. 6 mo.?

4. A village of 3600 inhabitants increased in population 15%. It then decreased 5%. What was its population then?

5. If in a period of 72 days $33\frac{1}{3}\%$ are cloudy, and if it rains on $16\frac{2}{3}\%$ of the cloudy days, how many of the days are rainy?

6. A dealer bought \$1260 worth of goods and marked them $33\frac{1}{3}\%$ above cost. He sold them at 10% less than the marked price, at "bargain sales." What did he make?

7. A dealer bought \$644.80 worth of shoes and sold them at a profit of 25%. How much did he gain? If he failed to collect 10% of this gain, how much did he really make?

8. A dry goods dealer bought \$2460.30 worth of Persian lawn, and marked it $33\frac{1}{3}\%$ above cost. He sold half of it, and then sold the rest at 10% off the marked price. How much did he gain?

9. The number of pupils in a school three years ago was 235. The next year there were 20% more. Last year there were $33\frac{1}{3}\%$ less than two years ago. This year there are 25% more than last year. How many are there this year?

ORAL EXERCISE

1. $33\frac{1}{3}$ is what per cent of $66\frac{2}{3}$? of 100?

2. 50 is what part of 100? what per cent?

3. \$75 is what per cent of \$150? of \$37.50?

4. 1 ft. equals what per cent of 1 yd.? of 4 ft.?

5. 40 is what per cent of 80? of 100? of 60? of 40?

6. 120 is what per cent of 100? of 60? of 40? of 120?

7. If a quarter of the days of last month were rainy, what per cent were not rainy?

8. 25 is what per cent of 50? of 75? of 100? of 125? of 150? of 200? of 2500? of 5000?

9. \$11 is what part of \$33? what per cent? what per cent of \$22? of \$55? of \$1100? of \$2200?

The first of these numbers is what per cent of the second?

10. 50, 40.	11. 60, 20.	12. 75, 15.	13. 90, 60.
14. 15, 20.	15. 21, 28.	16. 63, 21.	17. 48, 12.
18. 51, 17.	19. 96, 24.	20. 11, 55.	21. 81, 27.
22. 57, 57.	23. 12, 24.	24. 80, 120.	25. 16, 24.
26. 75, $37\frac{1}{2}$.	27. $12\frac{1}{2}$, $6\frac{1}{4}$.	28. $66\frac{2}{3}$, $33\frac{1}{3}$.	29. 90, 150.

The first of these numbers is what per cent greater than the second?

30. 75, 50.	31. 60, 40.	32. 44, 33.	33. 99, 66.
34. 66, 55.	35. 36, 27.	36. 93, 62.	37. 21, 14.

The first of these numbers is what per cent less than the second?

38. 21, 28.	39. 28, 32.	40. 44, 48.
41. 42, 63.	42. 300, 400.	43. 210, 240.
44. 52, 104.	45. 108, 120.	46. 150, 200.

399. Illustrative problem. What per cent of \$35 is \$11.66⅔?

Work in steps:

1. If $x\%$ of \$35 = \$11.66⅔,
2. Then $x\%$ = \$11.66⅔ ÷ \$35
   = 0.33⅓.
3. Therefore \$11.66⅔ is 33⅓% of \$35.

Or we may simply note that \$11⅔ is $\frac{11\frac{2}{3}}{35}$ of \$35, which is 33⅓%.

Actual work:

$$\begin{array}{r} 0.33\frac{1}{3} \\ \$35\overline{)\$11.66\frac{2}{3}} \\ 10\ 5 \\ \hline 1\ 16\frac{2}{3} \\ 1\ 05 \\ \hline 11\frac{2}{3} \end{array}$$

$$\frac{11\frac{2}{3}}{35} = \frac{35}{3 \times 35} = \frac{1}{3}$$

400. *The percentage (part) divided by the base (whole) equals the rate.*

WRITTEN EXERCISE

1. \$82.67 is what per cent of \$248.01?

2. \$17.42 is what per cent of \$156.78?

3. The 45 pupils in our grade are what per cent of the 225 in school?

4. If a man saves \$187.50 out of his salary of \$1250, what per cent does he save?

5. The 45 minutes devoted to arithmetic to-day are what per cent of the 4½ hours that we spend in school?

6. By how much does the area of Texas, 265,780 sq. mi., exceed 7% of that of the United States, 3,616,484 sq. mi.?

7. The 9,789,012 tons of sugar produced in the world in a certain year is what per cent of the 10,876,680 tons produced the following year?

8. The 6.27 inches of rainfall here last September was what per cent of the total rainfall of 41.8 inches for the year ending with that month?

Dressmaking Problems

WRITTEN EXERCISE

1. A dressmaker bought 16 yd. of velvet at \$3 a yard, selling 9 yd. at a profit of $16\frac{2}{3}\%$ and the rest at a rate of profit half as great. What was the rate of gain on the whole?

2. She bought a 25-yd. box of chiffon velvet at \$4 a yard, with 10% off for cash, selling it at \$4.35 a yard. What was her gain per cent?

3. She bought a 75-yd. piece of silk skirt lining at 65 ct. a yard. She sold 28 yd. at 90 ct., 15 yd. at 95 ct., and the remainder, at the close of the season, at 70 ct. What was her per cent of gain?

4. She bought a 50-yd. piece of silk waist lining at 75 ct. a yard. She sold 12 yd. at \$1 and 10 yd. at 95 ct., but the remainder, being kept in stock over the season, had to be sold at 65 ct. What was her per cent of gain or loss?

5. She bought a 20-yd. silk dress pattern at \$2.10 a yard, being allowed, as a dressmaker, a discount of 5%, and 6% off for cash. She charged her customer the marked price, \$2.10. What was her per cent of profit?

6. She charged her customer \$25.50 for 3 yd. of Honiton lace, which had cost her \$7 a yard. What was her per cent of profit?

7. She charged her customer \$2 for findings for the dress. These consisted of 4 spools of silk at 10 ct. each, 1 spool of thread at 5 ct., 3 yd. of featherbone at 10 ct., a card of hooks and eyes at 8 ct., skirt braid 16 ct., plaiting 30 ct., waist binding 30 ct., and collar 10 ct. What was her gain per cent on the findings?

Grocery Problems

WRITTEN EXERCISE

1. If a grocer pays 15¢ a dozen for eggs, how many must he sell for 25¢ to gain $33\frac{1}{3}\%$ on the cost?

2. A grocer buys eggs at 20¢ a dozen, and sells them at the rate of 10 for 25¢. What is his gain per cent?

3. If a man buys sugar at 5¢ a pound, what must he charge for $3\frac{1}{2}$ lb. in order to gain 20% on the cost?

4. A grocer buys sugar at $12 per barrel of 300 lb., and sells it at the rate of 3 lb. 8 oz. for 17¢. What was his gain per cent?

5. A grocer buys canned fruit at the rate of 17¢ a can, and sells it at the rate of $5.10 per case of 24 cans. What is his gain per cent?

6. My butcher's bill was $8 last week, my grocer's bill was 25% more, and my baker's bill was 80% less than my grocer's bill. What was the total of the three bills?

7. My grocer offers me 60 cakes of one kind of soap for $3.24, or 100 cakes of another kind for $4.38. Which is the more expensive per cake, and what per cent more expensive is it?

8. A grocer buys a crate of eggs containing 40 doz., paying 15¢ a dozen. He drops the crate and breaks 5 doz. How many must he sell for 25¢ so as still to gain $16\frac{2}{3}\%$ on the total cost?

9. A boy in a grocery store receives $6 a week. He spends 25% of it for board, 20% of the remainder for clothes, and $1 in other ways. If he saves the rest, how much will he save in a year (52 wk.)?

ORAL EXERCISE

1. 25 is $\frac{1}{2}$ of what number? 50% of what number?

2. 40 is $\frac{1}{3}$ of what number? $33\frac{1}{3}\%$ of what number?

3. Of what number is 50 one half? one third? two thirds? 10%? 25%? 50%? 100%? 125%? 200%?

The following numbers are the given per cents of what other numbers?

Think of the rate as a common fraction. If 7 is 10% of a number, it is $\frac{1}{10}$ of the number. Hence the number is 70. If 20 is $66\frac{2}{3}\%$ of a number, it is $\frac{2}{3}$ of the number, and $\frac{1}{3}$ is 10. Hence the number is 30.

4. 21, 10%.	5. 17, 50%.	6. 32, $33\frac{1}{3}\%$.
7. 49, 50%.	8. 63, $33\frac{1}{3}\%$.	9. 48, $66\frac{2}{3}\%$.
10. 150, 75%.	11. 400, 80%.	12. 12, $16\frac{2}{3}\%$.
13. 100, $8\frac{1}{3}\%$.	14. 21, $12\frac{1}{2}\%$.	15. 55, 125%.
16. 330, 150%.	17. 720, 200%.	18. 350, $166\frac{2}{3}\%$.
19. 99, $112\frac{1}{2}\%$.	20. 81, $112\frac{1}{2}\%$.	21. 180, $112\frac{1}{2}\%$.
22. 500, $166\frac{2}{3}\%$.	23. 250, $166\frac{2}{3}\%$.	24. 276, $110\frac{1}{2}\%$.

25. If \$6 is 3% of a number, what is 1% of the number? What is the number?

26. If \$9.12 is the interest on a certain sum for 1 yr. at 3%, what is the sum?

27. If \$4.50 is 5% of a certain sum, what is the sum? If it is $2\frac{1}{2}\%$, what is the sum?

28. If a piece of cloth shrinks 2 yd., the shrinkage being 5%, what was the original length?

29. A kettle of water loses 25% of its contents in boiling, and then contains 2 qt. less than it did at first. How much did it contain at first?

What is the number which, if decreased by 25% of itself, becomes the following?

If 150 is $\frac{3}{4}$ of a number, what is $\frac{1}{4}$? $\frac{4}{4}$?

30. 150. 31. 600. 32. 450. 33. 750.

34. 3750. 35. 2250. 36. 9000. 37. 6300.

Also, if decreased by $33\frac{1}{3}\%$ *of itself, becomes the following?*

38. 240. 39. 400. 40. 500. 41. 600.

42. 1000. 43. 1200. 44. 1600. 45. 2000.

Also, if decreased by 20% of itself, becomes the following?

46. 480. 47. 840. 48. 640. 49. 320.

50. \$80. 51. \$60. 52. \$24. 53. \$36.

Also, if increased by $16\frac{2}{3}\%$ *of itself, becomes the following?*

If 140 is $\frac{7}{6}$ of a number, what is $\frac{1}{6}$? $\frac{6}{6}$?

54. 140. 55. 350. 56. 280. 57. 210.

58. \$42. 59. \$70. 60. \$63. 61. \$49.

Also, if increased by $12\frac{1}{2}\%$ *of itself, becomes the following?*

62. $\frac{8}{8}$. 63. 90. 64. 63. 65. 81.

66. \$27. 67. \$18. 68. \$45. 69. \$99.

Also, if increased by $66\frac{2}{3}\%$ *of itself, becomes the following?*

70. $\frac{5}{3}$. 71. 25. 72. 45. 73. 75.

74. \$60. 75. \$80. 76. \$100. 77. \$500.

401. Illustrative problem. \$13.80 is 6% of what amount?

	Work in steps:	*Actual work:*
1. Since	6% of x = \$13.80,	.06)\$13.80
2. Therefore	x = \$13.80 ÷ 0.06,	6) \$1380
by dividing equals by 6%.		\$230
3. That is,	x = \$230.	

Or we may think that since \$13.80 is 6% of some number, 1% is $\frac{1}{6}$ of \$13.80, or \$2.30, and 100% is 100 times \$2.30, or \$230.

402. *The percentage (part) divided by the rate equals the base (whole).*

WRITTEN EXERCISE

1. \$0.85 is 2% of what sum? 5% of what sum?
2. \$1.43 is 4% of what sum? 5% of what sum?
3. \$1.75½ is 6% of what sum? 10% of what sum?
4. \$22.68 is 7% of what sum? 20% of what sum?
5. On what sum is \$7 the interest for 1 yr. at 4%?
6. If \$13.50 is the interest on a certain sum for 2 yr. at 3%, what is the interest for 1 yr.? What is the principal?
7. A class measuring a certain distance on a wall made an error of 0.28 ft., which was 0.07% of the total distance. What was the distance?
8. A certain city has increased 4230 in population in ten years, which is 15% of the population ten years ago. What was the population then?
9. A merchant gained \$4375 last year, which was 17½% of his capital. This year he has lost \$2750. What per cent of his capital has he lost this year?
10. A man lost \$247.50 in business last year, or 15% of his capital at the beginning of the year. How much was his capital at the beginning of the year?

403. Illustrative problems. 1. A man gained $16\frac{2}{3}\%$ on his capital of \$18,000. How much did he then have?

1. His capital $+ 16\frac{2}{3}\%$ of his capital $= 116\frac{2}{3}\%$ of his capital.
2. $116\frac{2}{3}\%$ of \$18,000 = \$21,000, the amount after the increase.

2. A village having a population of 1900 ten years ago has since lost 13%. What is its present population?

1. Population $-$ 13% of the population = 87% of the population.
2. 87% of 1900 = 1653, the present population.

In the first of these examples we might, of course, find $16\frac{2}{3}\%$, or $\frac{1}{6}$, and add, and in the second we might find 13% and subtract.

WRITTEN EXERCISE

1. Increase \$2750 by 15% of itself; by 12% of itself.

2. Decrease \$3500 by 12% of itself; by 8% of itself.

3. What is the amount of \$350 and a year's interest at 6%? at 5%? at $3\frac{1}{2}\%$? at $4\frac{1}{2}\%$?

4. A school had 204 pupils last year and has $16\frac{2}{3}\%$ more this year. How many has it now?

5. What is the amount of \$825 and two years' interest, the annual rate being 4%? 5%? 6%? $4\frac{1}{2}\%$?

6. A merchant starts the year with \$15,500, and loses 15% in his business. What does he then have?

7. Decrease \$750 by 50% of itself; that result by 50% of itself; and that result by 50% of itself. The result is what per cent of the \$750?

8. A man started in business with \$8500. The first year he gained 15%, and the next year he lost 15% of what he then had. How much had he then?

9. Write out a rule for finding the base increased or decreased by a certain per cent of itself, given the base and rate.

The Cattle Industry

WRITTEN EXERCISE

1. A ranchman owned 960 head of cattle in the fall, but the losses by storms and disease brought the number down to 928 in the spring. What was his per cent of loss?

2. In a certain year cattle on the range in the Southwest were worth \$8.40 a head; a few years later they were worth \$13.44 a head. What was the rate of increase in value?

3. In another year cattle in the same locality were selling for \$22.50 a head, an increase of 80% in their value of five years before. What was their value then?

4. On the upland pastures in the Southwest it averages about 20 acres per head to pasture a steer through the winter. How many square miles for 1000 head?

5. A western farmer bought in November of one year 30 steers at \$48.14 each, and fed them six months, or exactly 183 days. The corn cost him \$9 a day for all the steers, and the hay 10¢ a day for each. He sold them for \$121.06 each. How much did he lose on the lot, not considering the interest on his investment?

Oyster Industry

WRITTEN EXERCISE

Small "seed oysters" are taken from natural beds off the coast. They are "planted" in shallow water in "beds" rented from the state, and there are fattened. The large ones are called "box" oysters; the average or small ones, "cullings."

1. A man gathered 90 bu. of box oysters and 150 bu. of cullings. Each was what per cent of the other? of all?

2. There are 250 box oysters to a bushel, and $33\frac{1}{3}\%$ more cullings to a bushel. How many box oysters did the man gather? How many cullings?

3. He sold the box oysters for \$6.75 a thousand, which was $192\frac{6}{7}\%$ as much as he received per thousand for cullings. How much did he receive for all of his cullings?

4. He paid \$66 rent for one oyster bed and 12% less for another. What did he pay for both?

5. The oyster man paid 15¢ a bushel for shifting his oysters to another bed, and $66\frac{2}{3}\%$ as much for freight. What was the cost for both items on 250 bu.? He also paid a man \$2.50 a day for 12 days for dredging (gathering) and culling (sorting). What was the total of these three items of expense?

SIMPLE INTEREST REVIEWED

ORAL EXERCISE

Find the interest on the following sums for the times and at the rates specified:

1. \$2 for 1 yr. at 5%; at 6%.
2. \$25 for 1 yr. at 4%; at 2%.
3. \$300, at 5%, for 1 yr.; for 2 yr.
4. \$400, at 5%, for 1 yr.; for 3 yr.
5. \$500, at 6%, for 1 yr.; for 6 mo.
6. \$700, at 6%, for 1 yr.; for $1\frac{1}{2}$ yr.
7. \$400, at 6%, for 2 yr.; for 2 yr. 6 mo.
8. \$250, at 4%, for 1 yr.; for 1 yr. 6 mo.
9. \$1000, at 5%, for 3 yr.; for 3 yr. 6 mo.
10. \$2500, at 4%, for 1 yr.; for 2 yr.; for $2\frac{1}{4}$ yr.
11. \$3000, at 6%, for 1 yr. 6 mo.; for 1 yr. 4 mo.
12. \$2000, at 5%, for 1 yr. 6 mo.; for 2 yr. 6 mo.
13. \$1500, at 3%, for 1 yr.; for 2 yr.; for 1 yr. 4 mo.
14. \$5000, at 6%, for 1 yr.; for 3 yr.; for 6 mo.; for 3 mo.; for 1 mo.; for 15 da.

Find the amount of principal and interest together, given the following principals, rates, and times:

15. \$100, at 6%, for 1 yr.; for 2 yr.
16. \$300, at 5%, for 1 yr.; for 2 yr.
17. \$250, at 4%, for 1 yr.; for 3 yr.
18. \$500, at 4%, for 1 yr.; for 5 yr.
19. \$1000, at $4\frac{1}{2}$%, for 1 yr.; for 2 yr.
20. \$2000, at $5\frac{1}{2}$%, for 1 yr.; for 2 yr.

404. Illustrative problem. Find the interest on \$500, at 6%, for 2 yr. 3 mo. 6 da.

1. The interest for 1 yr. is 6% of \$500, or \$30.
2. For 2 yr. it is 2 times \$30 = \$60.
   " 3 mo., or $\frac{1}{4}$ yr., it is $\frac{1}{4}$ of \$30 = 7.50
   " 6 da., or $\frac{1}{5}$ of $\frac{1}{12}$ yr., it is $\frac{1}{5}$ of $\frac{1}{12}$ of \$30 = .50
3. Therefore the total interest is \$68.00

405. How to find interest. Therefore, to find interest,

Find the interest for one year, and multiply by the number of years or the part of a year.

WRITTEN EXERCISE

1. Find the interest on \$275 for 3 yr. 6 mo. at 5%.
2. Find the interest on \$1250 for 7 yr. 11 mo. at 6%.
3. Find the interest on \$630 for 2 yr. 4 mo. 5 da. at 6%.
4. Find the interest on \$345 for 2 yr. 3 mo. 3 da. at 5%.
5. Find the interest on \$290 for 1 yr. 6 mo. 5 da. at 5%.
6. Find the interest on \$350 for 1 yr. 9 mo. 15 da. at 4%.
7. Find the interest on \$650 for 2 yr. 8 mo. 10 da. at 6%.
8. Find the interest on \$1575 for 3 yr. 1 mo. 6 da. at $5\frac{1}{2}$%; at $4\frac{1}{2}$%; at 6%.
9. A man borrowed \$250 on January 15, at 6%. He paid the principal and interest on October 15. How much did he pay?
10. A man borrowed \$350 on January 27, at 6%. He paid the principal and interest on February 27 of the following year. How much did he pay?
11. A man bought 12 head of cattle on February 1, at \$45 a head, giving his promissory note at 6%. He paid it on the 16th of the following April. What was the amount of principal and interest?

406. Cancellation used in interest. It is often advisable to shorten the work by cancellation. Find the interest on \$256.75 for 2 yr. 2 mo. 10 da. at $4\frac{1}{2}\%$.

Since $4\frac{1}{2}\% = \frac{9}{2}\% = \frac{9}{200}$, and 2 yr. 2 mo. 10 da. = (730+60+10) da. = 800 da., we have

$$\frac{\overset{4}{\cancel{800}} \text{ times } 9 \text{ times } \overset{51.35}{\cancel{\$256.75}}}{\underset{73}{\cancel{365}} \text{ times } \cancel{200}} = \$25.32.$$

When the time exceeds 1 yr., and cancellation is used, it is better to take 365 da. to the year. Of course this gives exact interest, as described on page 370.

WRITTEN EXERCISE

Find the interest, given the principal, time, and rate:

1. \$172.60, 3 yr. 2 mo. 7 da., 4%.
2. \$391.75, 3 yr. 8 mo. 17 da., 4%.
3. \$235.50, 3 yr. 1 mo. 9 da., 6%.
4. \$175.50, 2 yr. 8 mo. 10 da., 6%.
5. \$142.80, 1 yr. 3 mo. 15 da., 5%.
6. \$273.40, 2 yr. 8 mo. 20 da., 5%.
7. \$172.60, 2 yr. 5 mo. 6 da., $4\frac{1}{2}\%$.
8. \$182.60, 4 yr. 8 mo. 17 da., $5\frac{1}{4}\%$.
9. \$163.40, 3 yr. 9 mo. 15 da., $5\frac{1}{2}\%$.
10. \$625.50, 2 yr. 11 mo. 25 da., $5\frac{3}{4}\%$.
11. \$475.50, 3 yr. 10 mo. 15 da., $4\frac{1}{4}\%$.
12. \$265.50, 2 yr. 10 mo. 20 da., $4\frac{3}{4}\%$.
13. \$3562.50, 3 yr. 2 mo. 5 da., $5\frac{1}{2}\%$.

407. The Six Per Cent Method. The following short method, known as the Six Per Cent Method, is often convenient.

Required the interest on \$420 for 5 mo. 10 da. at 6%

1. Since 2 mo. = $\frac{1}{6}$ yr., the rate for 2 mo. = $\frac{1}{6}$ of 6% = 1%.
2. Therefore the int. for 2 mo. = 1% of \$420 = \$4.20

and for	2 mo. more,		4.20
" "	1 mo. "	$\frac{1}{2}$ of \$4.20,	2.10
" "	10 da. "	$\frac{1}{3}$ of \$2.10,	.70

3. Therefore the total interest is \$11.20

Therefore the interest at 6% for 60 days is 0.01 of the principal, for 6 days 0.001 of the principal, and for other periods the interest can be found from the interest for these periods.

Various other forms of this method are given, but none is so simple as the above. This is sometimes spoken of as the Banker's Method. The following method may be taken, if desired, in which case the above should not be taken.

Since the rate for 1 yr. is $\frac{6}{100}$ of the principal,

and for 1 mo. is $\frac{1}{12}$ as much, or $\frac{\frac{1}{2}}{100}$ of the principal,

and for 1 da. is $\frac{1}{30}$ of this, or $\frac{\frac{1}{6}}{1000}$ of the principal, therefore

408. *Multiply the principal by 6 times the number of years and $\frac{1}{2}$ the number of months as hundredths, adding the product of the principal by $\frac{1}{6}$ the number of days as thousandths.*

Thus, to find the interest, at 6%, on \$120, for 2 yr. 3 mo. 9 da.:

First method:

\$7.20	int. for	1 yr.
7.20		" "
1.20	(1% of \$120)	2 mo.
.60	($\frac{1}{2}$ of \$1.20)	1 "
.18	($\frac{3}{10}$ of \$0.60)	$\frac{9}{30}$ "
\$16.38		

Second method:

$$\frac{2\times 6+\frac{3}{2}}{100}\text{ of }\$120+\frac{\frac{9}{6}}{1000}\text{ of }\$120$$

$$=\left(\frac{27}{200}+\frac{9}{6000}\right)\text{of }\$120=\$16.38.$$

409. Further uses of the Six Per Cent Method. After finding the interest at 6%, the interest for other rates is easily found. Thus, take the interest at 6% and

Subtract $\frac{1}{6}$ of itself to find the interest at 5%.
" $\frac{1}{3}$ " " " " " " " 4%.
" $\frac{1}{4}$ " " " " " " " $4\frac{1}{2}$%.
Divide by 2 " " " " " 3%.
Add $\frac{1}{6}$ of itself " " " " " 7%.
" $\frac{1}{3}$ " " " " " " " 8%.

ORAL EXERCISE

Find the interest at 6% :

1. On \$750, 60 da.
2. On \$275, 60 da.
3. On \$350, 60 da.
4. On \$425, 60 da.
5. On \$300, 30 da.
6. On \$250, 30 da.
7. On \$460, 30 da.
8. On \$840, 30 da.
9. On \$200, 90 da.
10. On \$400, 90 da.
11. On \$500, 12 da.
12. On \$400, 12 da.
13. On \$400, 15 da.
14. On \$600, 15 da.
15. On \$600, 10 da.
16. On \$720, 10 da.

Find the interest at 5% :

17. On \$600, 60 da.
18. On \$120, 60 da.
19. On \$660, 60 da.
20. On \$120, 30 da.
21. On \$1200, 30 da.
22. On \$720, 90 da.

Find the interest at 4% :

23. On \$300, 60 da.
24. On \$120, 60 da.
25. On \$600, 60 da.
26. On \$900, 60 da.
27. On \$1200, 30 da.
28. On \$900, 90 da.

WRITTEN EXERCISE

Find the interest:

1. \$320, 8 mo., 6%.
2. \$240, 7 mo., 5%.
3. \$175, 9 mo., 6%.
4. \$185, 2 yr. 6 mo., 5%.
5. \$330, 7 mo., 8%.
6. \$240, 3 yr. 8 mo. 2 da., 4%.
7. \$1200, 2 yr. 6 mo. 3 da., 3%.
8. \$475, 2 yr. 2 mo. 2 da., 4½%.
9. \$2435, 2 yr. 3 mo. 9 da., 4%.
10. \$1765, 3 yr. 2 mo. 5 da., 4½%.
11. \$4200, 1 yr. 3 mo. 15 da., 5%.
12. \$381.25, 1 yr. 6 mo. 18 da., 3%.
13. \$426.40, 1 yr. 6 mo. 20 da., 5%.
14. \$692.50, 2 yr. 8 mo. 15 da., 4½%.

Find the amount of principal and interest:

15. \$175, 1 yr. 2 mo. 6 da., 6%.
16. \$342, 2 yr. 3 mo. 4 da., 5%.
17. \$1725, 2 yr. 8 mo. 6 da., 4½%.
18. \$2175, 3 yr. 6 mo. 15 da., 5%.
19. \$312.50, 2 yr. 7 mo. 10 da., 4%.
20. \$427.25, 1 yr. 8 mo. 3 da., 5½%.
21. \$375.50, 2 yr. 6 mo. 4 da., 3½%.
22. \$137.50, 1 yr. 7 mo. 2 da., 4½%.
23. \$243.75, 3 yr. 2 mo. 8 da., 2½%.

24. A man can buy \$1750 worth of goods on 90 days' credit, or he can get them for \$1720 in cash, which he can borrow for the 90 days at 6%. Which is the better plan, and how much better is it?

410. Difference in time. In working without tables, 360 days are usually considered a year, and 30 days a month.

Thus, to find the time from July 15 to October 5 it is customary to subtract as here shown, and say that the difference is 2 mo. 20 da., or 80 da. A banker, however, will see by his tables that July 15 is the 196th day of the year and October 5 is the 278th day, and that the difference in time is 82 da.

10 mo.	5 da.
7	15
2 mo.	20 da.

278
196
82

411. Table of time. The following table may be used:

	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March . . .	306	337	365	31	61	92	122	153	184	214	245	275
April . . .	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June . . .	214	245	273	304	334	365	30	61	92	122	153	183
July . . .	184	215	243	274	304	335	365	31	62	92	123	153
August . .	153	184	212	243	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . .	92	123	151	182	212	243	273	304	335	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December .	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from any day of any month to the corresponding day of any month within a year is found opposite the first month and under the second. For example, from November 5 to May 5 is 181 days; to May 17, 12 days more.

WRITTEN EXERCISE

Find from the table the number of days from:

1. July 7 to November 7.
2. October 8 to March 15.
3. January 20 to May 10.
4. November 25 to July 7.
5. September 7 to January 7.
6. August 5 to December 10.

Using the table on page 367 to find the difference in time, find the interest:

7. On \$275, from May 2 to August 17, at 6%.
8. On \$650, from July 14 to October 29, at 6%.
9. On \$675, from March 25 to October 5, at 5%.
10. On \$675.50, from March 1 to July 25, at 3%.
11. On \$645.50, from June 2 to October 19, at 3%.
12. On \$185, from February 10 to August 2, at 5%.
13. On \$725, from October 3 to December 27, at 6%.
14. On \$420.50, from May 15 to November 2, at $4\frac{1}{2}$%.
15. On \$350.75, from April 13 to September 20, at $4\frac{1}{2}$%.

Find the amount of principal and interest:

16. On \$980, from January 7 to July 2, at 5%.
17. On \$145, from March 3 to January 5, at 6%.
18. On \$75, from May 2 to September 17, at 6%.
19. On \$442.50, from May 9 to January 6, at $4\frac{1}{2}$%.
20. On \$275, from August 15 to February 7, at 6%.
21. On \$875, from February 12 to August 5, at 5%.
22. On \$630, from September 14 to March 9, at 5%.
23. On \$675.90, from June 6 to February 1, at $4\frac{1}{2}$%.
24. On \$825.30, from July 15 to January 20, at $4\frac{1}{2}$%.
25. On \$1027.50, from August 9 to February 2, at 3%.
26. On \$2075.50, from September 3 to January 15, at 3%.
27. What is the interest on a note for \$275, dated June 17, and due the following January 3, at 5%?
28. What is the amount of principal and interest on a note for \$450, dated September 21, and due the following August 1, at $5\frac{1}{2}$%?

412. Interest tables. Bankers generally use interest tables based on 360 days to the year, although some use those based on 365 days, the latter plan being the fairer but yielding less interest. The following shows part of an interest table.

3 MONTHS, 6%

Total Days	1000	2000	3000	4000	5000	6000	7000	8000	9000	Days over 3 Mo.
90	15.00	30.00	45.00	60.00	75.00	90.00	105.00	120.00	135.00	0
91	15.17	30.33	45.50	60.67	75.83	91.00	106.17	121.33	136.50	1
92	15.33	30.67	46.00	61.33	76.67	92.00	107.33	122.67	138.00	2
93	15.50	31.00	46.50	62.00	77.50	93.00	108.50	124.00	139.50	3
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

413. Illustrative problem. From the table find the interest on \$275 for 3 mo. 3 da. at 6%.

The int. on	\$200	is	0.1	that for	\$2000,	or	\$3.10
" " "	\$70	"	0.01	" "	\$7000,	"	\$1.09
" " "	\$5	"	0.001	" "	\$5000,	"	\$0.08
" " "	\$275	"					\$4.27

WRITTEN EXERCISE

Find the interest at 6%, in Exs. 1–6, using the table:

1. \$2500, 3 mo.
2. \$3450, 92 da.
3. \$250.75, 91 da.
4. \$5200, 93 da.
5. \$475.50, 3 mo. 3 da.
6. \$24,300, 3 mo.

Find the interest at 6%, taking the exact number of days from the table on page 367, and using the above table:

7. \$350.75, December 19 to March 21.
8. \$1250, November 17 to February 15.
9. \$650, March 1 to June 2; May 4 to August 5.
10. \$775, February 9 to May 13; to May 10; to May 12.

414. Exact interest. When 365 days are taken as a year, and the exact number of days between the dates is taken, the interest is called *exact interest.*

The government and some banks use exact interest.

415. Illustrative problem. What is the exact interest on \$1750 from July 5 to September 5 at 5%?

1. The exact difference in time is 62 da. (See table, page 367.)
2. The interest for 1 yr. is 5% of \$1750, or \$87.50.
3. The interest for 62 da. is $\frac{62}{365}$ of \$87.50, or \$14.86.

WRITTEN EXERCISE

1. Find the exact interest on \$4800 for 93 da. at 5%.

2. Find the amount of \$525 for 2 yr. 3 mo. 6 da. at 5%, exact interest; also at 6%, 4½%, 3½%.

3. Find the interest on \$1260 for 73 da. at 6%, 360 da. to the year; also the exact interest for 73 da. at 6%.

4. Find the exact interest on \$450 from January 3 to February 9 at 4%; from May 17 to July 20 at 5%.

5. What is the difference between the common interest at 360 days to the year, and the exact interest, on \$5000 for 60 days, at 6%?

Find the exact interest, and also the common interest at 360 days to the year, on each of the following:

6. \$420, 6%, 93 da.
7. \$3250, 2%, 192 da.
8. \$275, 5%, 63 da.
9. \$252.50, 6%, 60 da.
10. \$450, 5½%, 70 da.
11. \$1250, 4½%, 300 da.
12. \$3200, 4%, 3 mo. 9 da.
13. \$455.80, 5%, 275 da.
14. \$6000, 4%, from April 1 to June 1.
15. \$4850, 3%, from February 3 to March 10.

416. To find the rate. Illustrative problem. If the common interest on \$250 for 3 mo. 6 da. is \$4, what is the rate?

1. The interest for 1 da. is $\frac{\$4}{96}$, and the interest for 360 da. is 360 times $\frac{\$4}{96}$, or \$15.

2. \$15 is $\frac{15}{250}$ of \$250, and $\frac{15}{250} = 0.06$.

3. Or we may say that, since $r\%$ of \$250 = \$15, $r\% = \$15 \div \$250 = 0.06$.

That is,

417. *The rate equals the interest for one year divided by the principal.*

WRITTEN EXERCISE

Given the principal, common interest, and time, find the rate:

1. \$275, \$27.50, 2 yr.
2. \$250.50, \$30.06, 2 yr.
3. \$420, \$50.40, 3 yr.
4. \$480, \$36, 1 yr. 6 mo.
5. \$121.25, \$9.70, 2 yr.
6. \$240, \$24, 2 yr. 6 mo.
7. \$320.50, \$57.69, 3 yr.
8. \$325, \$45.50, 3 yr. 6 mo.
9. \$275.25, \$55.05, 5 yr.
10. \$240, \$10.50, 1 yr. 3 mo.
11. \$3500, \$87.50, 10 mo.
12. \$160, \$19.80, 2 yr. 3 mo.
13. \$220, \$9.35, 1 yr. 5 mo.
14. \$480, \$26.60, 1 yr. 7 mo.
15. \$31, \$0.93, 1 yr. 6 mo.
16. \$540, \$60.75, 2 yr. 6 mo.
17. \$720, \$58.40, 2 yr. 10 da.
18. \$360, \$22, 1 yr. 6 mo. 10 da.
19. \$360, \$29.22, 1 yr. 4 mo. 7 da.
20. \$1240, \$172.05, 2 yr. 9 mo. 9 da.

418. To find the time. Illustrative problem. How long will it take the interest on \$320 to amount to \$28 at 5%?

Since the interest for 1 yr. is 5% of \$320, or \$16, \$28 is the interest for $\frac{28}{16}$ of a year, or $1\frac{3}{4}$ yr.

Or since the interest for 1 yr. is \$16, for x years it is \$16 x.

Therefore $\$16\,x = \$28.$

$$x = \$28 \div \$16 = 1\tfrac{3}{4}.$$

Therefore the time is $1\frac{3}{4}$ yr., or 1 yr. 9 mo.

419. *The number of years equals the total interest divided by the interest for one year.*

WRITTEN EXERCISE

Find the time it will take to gain the common interest stated in Exs. 1–10:

1. \$240, \$36 int., 6%.
2. \$280, \$28 int., 4%.
3. \$3000, \$155 int., 2%.
4. \$350, \$17.50 int., 3%.
5. \$175, \$12.25 int., 4%.
6. \$525, \$47.25 int., 6%.
7. \$600, \$42.75 int., 5%.
8. \$230, \$20.70 int., $4\frac{1}{2}$%.
9. \$360, \$19.91 int., $5\frac{1}{2}$%.
10. \$2500, \$37.50 int., $4\frac{1}{2}$%.
11. How long will it take a sum to double itself at 6%?
12. How long will it take \$220, together with the interest, to amount to \$253, at 6%?
13. A girl had \$250 invested for her, at 6%, on her birthday. When she became 21 it amounted, with interest, to \$400. How old was she when it was invested?
14. A father gave his son his promissory note for \$225, due when the son became 21 years old. The rate of interest was 5%, and when the note became due the principal and interest together amounted to \$303.75. How old was the son when the note was given?

ORAL EXERCISE

State without explanation the interest on:

1. \$100, 2 yr., 6%.
2. \$200, 1 yr., 6%.
3. \$400, 1 yr., 3%.
4. \$200, 2 yr., 3%.
5. \$500, 2 mo., 6%.
6. \$400, 4 mo., 6%.
7. \$850, 1 mo., 6%.
8. \$750, 6 mo., 6%.
9. \$640, 3 mo., 6%.
10. \$880, 3 mo., 6%.
11. \$660, 2 mo., 5%.
12. \$800, 3 mo., 5%.
13. \$360, 2 mo., 4%.
14. \$900, 2 mo., 4%.
15. \$500, 4 mo., 3%.
16. \$650, 4 mo., 3%.
17. \$800, 2 yr., $2\frac{1}{2}$%.
18. \$700, 2 yr., $2\frac{1}{2}$%.

The rate of $2\frac{1}{2}$% for a year is the same as 5% if the time is 2 yr

19. \$650, 3 yr., $3\frac{1}{3}$%.
20. \$975, 3 yr., $3\frac{1}{3}$%.

Given the principal, interest, and time, find the rate:

21. \$400, \$20, 1 yr.
22. \$500, \$60, 2 yr.
23. \$600, \$72, 3 yr.
24. \$300, \$36, 4 yr.
25. \$1000, \$25, 6 mo.
26. \$200, \$6, 9 mo.
27. \$750, \$7.50, 2 mo.
28. \$640, \$3.20, 1 mo.

Given the interest, time, and rate, find the principal:

29. \$12, 1 yr., 6%.
30. \$9, 1 yr., 3%.

If \$9 is 3% of the principal, what is 1%? 100%?

31. \$32, 2 yr., 4%.
32. \$30, 2 yr., 3%.
33. \$12, 6 mo., 4%.
34. \$63, 3 yr., 3%.
35. \$4, 1 mo., 4%.
36. \$10, 2 mo., 5%.

Given the principal, interest, and rate, find the time:

37. \$400, \$20, 5%.
38. \$300, \$36, 6%.

WRITTEN EXERCISE

Find the common interest in Exs. 1–4:

1. \$3200, 93 da., 5%.
2. \$2100, $1\frac{1}{2}$ yr., 4%.
3. \$85.50, $6\frac{4}{15}$ mo., 6%.
4. \$175, 2 yr. $4\frac{1}{10}$ mo., 6%

Find the exact interest in Exs. 5–8:

5. \$3200, 128 da., 5%.
6. \$5000, 193 da., 4%.
7. \$2750, 227 da., 6%.
8. \$32,460, 230 da., 5%.

Given the principal and common interest, find the rate:

9. \$108, \$6, 1 yr. 1 mo. 10 da.
10. \$36, \$3.66, 2 yr. 6 mo. 15 da.
11. \$1440, \$77.70, 1 yr. 6 mo. 15 da.
12. \$2750, \$115.50, 1 yr. 2 mo. 12 da.

Find the time, given the principal, interest, and rate:

13. \$5500, \$230, $3\frac{1}{2}$%
14. \$375, \$54.25, 6%.
15. \$425, \$63.75, 3%.
16. \$2880, \$155.40, $3\frac{1}{2}$%.

17. How long must a man have \$2750 invested at 5% so that the principal and interest would amount to \$3231.25?

18. A man borrowed \$8000 in Boston at $4\frac{1}{2}$%, and loaned it in Oregon at 8%. How much did he gain in three years by the transaction?

19. A man has \$250 in a savings bank at 3%. He leaves the principal, but draws out the interest as it is due. How much does he draw out in four years?

20. A man having \$13,500 invested at 5% reinvests \$4000 of it at $5\frac{1}{2}$%, \$5000 of it at $3\frac{1}{2}$%, and leaves the rest invested as before. What is the annual difference in income?

21 Which yields the better income, \$1675 at 5% or \$1375 at 6%? What is the difference for 2 yr. 6 mo.?

22. A man receives \$1718.75 interest in 2 yr. 6 mo. on an investment of \$17,500. What is the rate of interest?

23. A boy has \$300 given to him the day he is 14 years old. His father invests it for him in a 5% promissory note, due on the day he is 21 years old. What is the amount on the day the note is due?

24. A real estate dealer buys 360 acres of farm land at \$30 an acre, and after keeping it 7 months sells it at an advance of \$3.75 an acre. The money being worth to him 5% a year, how much does he gain by the transaction?

25. A man having \$26,750 invested in business has found that his annual profits average 18% a year. He is offered \$35,000 for the business, which he can invest at $4\frac{1}{4}$%. If he sells out and retires, what will be his loss in income?

26. On April 10 a coal dealer borrowed \$33,250 at 5%, with which he purchased his summer's supply of coal at \$4.75 a ton. He sold the coal at \$5.65 a ton, the buyers paying for unloading and delivery, and paid his debt on November 16. How much did he gain?

27. I own a house which I rent at \$25 a month. My taxes are \$50 a year, my repairs \$50, and my insurance \$10. Would it be better for me to sell the house for \$5000 and invest the money at 4%, if my taxes would then be reduced to \$10? What is the difference in income?

28. On October 15 a dealer purchased \$1750 worth of goods for the holiday trade on 30 days credit. At the end of that time he gave his note for 2 months at 6% a year. He sold the goods for \$2105, and paid the note when due. What per cent did he make on the original purchase price?

Factory Problems

WRITTEN EXERCISE

1. A certain canning factory uses the product of 75 acres of peas during the three weeks' season. In this time it puts up 48,000 cans of peas. At 6 days to the week, how many acres of peas does it use a day? How many cans does it put up every 3 hours, allowing 8 hours to the working day?

2. A $\frac{1}{2}$-lb. stick of solder is used to seal 30 of these cans. How many pounds of solder are required for a week's work? This solder being 20% lead, how many pounds of lead are used in a day?

3. The steam kettle in which the peas are cooked will hold 3 baskets, each containing enough peas for 240 cans. How many cans of peas can be cooked in one forenoon (7.50 A.M. to 12 M.) if 25 min. are allowed to each kettleful?

4. A farmer owns 3 of the 75 acres mentioned in Ex. 1. He therefore furnishes the peas for how many cans? If he takes for his pay 50% of the canned goods for which he furnished the peas, how many cases of 24 cans each should he receive?

5. A workman in the factory puts together 200 of these cases a day. How long will it take him to put together cases enough for the season's output mentioned in Ex. 1? How much will he earn in that time, at 90¢ per hundred cases?

6. The factory runs from 8 A.M. to 5 P.M., with an hour out at noon. One day some machinery broke down, stopping all work from 10 A.M. to 2 : 30 P.M. What was the total output that day?

Problems of a Grocer

WRITTEN EXERCISE

1. Mr. F. T. Barker has \$20,000 invested in his building and grocery stock. Last year his profits were \$2500. What was his rate of profit?

2. Of the \$20,000 he owes \$8000, paying $5\frac{1}{2}\%$ interest. How much did this take from his \$2500 profit? What rate of income is he receiving on his own money?

3. Last summer he bought 100 muskmelons for \$6.50. He sold half of them at 9¢ each, 33 of them at the rate of 3 for 25¢, and the rest he had to throw away. Did he gain or lose, and what per cent?

4. He bought 300 bananas for \$4 and sold them at the rate of 22¢ a dozen. What was his gain per cent? At 25¢ a dozen, what would it have been?

5. He bought 500 lb. of clover honey for \$70 and sold all but 50 lb. of it at a profit of 32%. The rest he sold at a loss of 55%. Did he gain or lose, and how much?

6. He employs 4 order clerks, paying them \$12 a week and $\frac{1}{2}\%$ commission on all goods sold. Their average sales being \$280, \$220, \$240, and \$270 a week respectively, what is the average weekly income of each?

7. He buys sugar at $20 per barrel of 344 lb., and sells it at 6¢ a pound. How much does he gain on 135 lb.?

8. If he can sell a barrel of sugar (see Ex. 7) at the rate of 48¢ for 7 lb., how much will he make on a barrel?

9. If he buys a 45-lb. chest of tea for $16.20, and sells it at 50¢ a pound, what is his rate of profit?

10. If 12 chests of the above tea were damaged by being stored in a damp place, and brought 25¢ a pound, what was the loss? What was the per cent of loss?

11. He bought coffee by the 100-lb. bag at $20 a bag. At what price per pound must he sell this to make 45%?

12. If he buys flour at $4.75 a barrel and sells it at $5.25, what profit does he realize on 51 bbl.? What is the per cent of profit?

13. He pays $4 a box for laundry soap, 100 cakes to the box, and sells the soap at 6 cakes for a quarter. What is his gain on a dozen boxes? his per cent of gain?

14. He bought 15 boxes of starch at $1.60 a box. There being 40 lb. to the box, what is his gain if he sells it all at 5¢ a pound? What is the rate of gain?

15. He bought 9 cases of tomatoes, 2 doz. cans to the case, at 92¢ a dozen. He sold them at 3 cans for a quarter. What was his gain or loss? the per cent of gain or loss?

16. A box of condensed milk contains 4 doz. cans. If he buys 18 boxes at $3.84 each, and sells $\frac{1}{3}$ at 7¢ a can and the rest at a profit of $12\frac{1}{2}\%$, does he gain or lose on the lot, and how much?

17. He buys 100 5-lb. caddies of Hyson tea for $420. He sells 75 caddies at a profit of 15%, and the rest at a loss of 5%. How much did he gain? At what price apiece did he sell the 25 caddies?

RATIO AND PROPORTION REVIEWED

ORAL EXERCISE

1. What are the two common ways of expressing the ratio of 3 to 5? Write them on the blackboard.

2. What must be the nature of the terms of a ratio, as to similarity? Illustrate.

3. In the ratio 4 ft. : 20 ft., which term is the antecedent? What is the name of the other term?

4. What does the ratio of \$4 to \$2 equal? of 4 ft. to 2 ft.? What is the nature of the ratios of concrete numbers?

State the simplest values of the following ratios:

5. $\frac{2}{4}$. 6. $\frac{3}{6}$. 7. $\frac{12}{36}$. 8. $\frac{17}{51}$. 9. $\frac{21}{28}$.

10. $\frac{30}{40}$. 11. $\frac{25}{75}$. 12. $\frac{45}{60}$. 13. $\frac{15}{60}$. 14. $\frac{36}{12}$.

15. \$5 : \$10. 16. 6 in. : 3 in. 17. 10 ft. : 30 ft.

18. \$6 : \$42. 19. \$68 : \$17. 20. 13 mi. : 104 mi.

State the value of x *in each of the following:*

21. $\frac{x}{3}=7$. 22. $\frac{x}{5}=9$. 23. $\frac{x}{4}=7$. 24. $\frac{x}{6}=8$.

25. $\frac{x}{9}=3$. 26. $\frac{x}{5}=8$. 27. $\frac{x}{7}=6$. 28. $\frac{x}{8}=8$.

29. $x:3=4$. 30. $x:4=9$. 31. $x:9=4$. 32. $x:8=6$.

33. $x:7=7$. 34. $x:8=3$. 35. $x:3=11$. 36. $x:9=9$.

37. $\frac{5}{x}=5$. 38. $\frac{10}{x}=2$. 39. $\frac{12}{x}=4$. 40. $\frac{15}{x}=3$.

41. $\frac{20}{x}=10$. 42. $\frac{30}{x}=3$. 43. $\frac{45}{x}=15$. 44. $\frac{55}{x}=5$.

45. $\frac{x}{2}=\frac{3}{6}$. 46. $\frac{x}{4}=\frac{6}{8}$. 47. $\frac{x}{15}=\frac{3}{5}$. 48. $\frac{x}{20}=\frac{3}{10}$.

420. Ratio. The relation of one quantity to another of the same kind, as expressed by division, is called the *ratio* of the first to the second.

The ratio of \$4 to \$6 is written $\frac{\$4}{\$6}$ or \$4 : \$6. It equals $\frac{4}{6}$, or $\frac{2}{3}$. For the names of the terms see page 218.

421. Illustrative problems. 1. If $\frac{x}{3.7} = 2.7$, what does x equal?

Multiplying by 3.7, $3.7 \times \frac{x}{3.7} = 3.7 \times 2.7$,

or $x = 9.99$.

2. Find the value of x, given $\frac{5.44}{x} = 1.7$.

Multiplying by x, $5.44 = 1.7\,x$.

Hence $x = \frac{5.44}{1.7} = 3.2$.

WRITTEN EXERCISE

Find the value of x *in each of the following:*

1. $\frac{x}{4.9} = 6.7$.
2. $\frac{x}{71.2} = 6.42$.
3. $\frac{x}{31.2} = 0.61$.
4. $\frac{x}{12.6} = 17.3$.
5. $\frac{2\,x}{31} = 4.2$.
6. $\frac{3\,x}{1.7} = 4.2$.
7. $\frac{5\,x}{7.3} = 2.5$.
8. $\frac{7\,x}{9.3} = 6.3$.
9. $\frac{2.55}{x} = 5$.
10. $\frac{2.97}{x} = 2.7$.
11. $\frac{9.03}{x} = 4.3$.
12. $\frac{45.6}{x} = 5.7$.
13. $\frac{x}{17} = \frac{2.3}{4}$.
14. $\frac{x}{2.3} = \frac{4.1}{7}$.
15. $\frac{x}{1.6} = \frac{4}{7}$.
16. $\frac{2}{x} = \frac{3.5}{7}$.
17. $\frac{3\frac{1}{2}}{x} = \frac{5}{2.2}$.
18. $\frac{2\,x}{3\frac{1}{2}} = \frac{5}{7}$.
19. $\frac{3}{x} = \frac{22\frac{1}{2}}{7}$.
20. $\frac{3\,x}{7.1} = \frac{12}{14\frac{1}{5}}$.
21. $\frac{5\,x}{9.1} = \frac{6}{5.46}$.

422. Illustrative problem. Required to separate \$1540 into two parts having the ratio of 5 to 9.

Since one part contains \$5 when the other contains \$9, the two will then contain \$14.

Therefore $\frac{5}{14}$ belongs to the first part and $\frac{9}{14}$ to the second.

$\frac{5}{14}$ of \$1540 = \$550, and $\frac{9}{14}$ of \$1540 = \$990.

Check. \$550 + \$990 = \$1540.

423. Partitive proportion. The separation of a number into parts having a given ratio is called *partitive proportion.*

ORAL EXERCISE

Separate into parts having the ratios stated:

1. 20, 3 : 7.	2. 20, 2 : 3.	3. 26, 4 : 9.
4. 32, 5 : 11.	5. 32, 7 : 9.	6. 34, 6 : 11.
7. 18, 5 : 13.	8. 40, 7 : 13.	9. 39, 2 : 11.
10. 90, 13 : 17.	11. 26, 11 : 2.	12. 24, 11 : 13.

13. Two partners are to divide \$1500 of profits in the ratio of 1 to 2. What is the share of each?

14. A 20-ft. beam is to be divided in the ratio of 2 to 3. The point of division is how far from each end?

15. The ratio of cloudy to clear days in a certain November was 2 : 3. How many of the days were clear?

16. The ratio of cloudy to clear days in February, 1904, in a certain place, was 12 : 17. How many days were clear?

17. In a class of 30, 4 out of 5 were promoted. What was the ratio of promoted to nonpromoted? How many failed of promotion?

18. Out of 44 boys, some are defending a fort and some are attacking. The ratio of the defenders to the others is 1 to 3. How many are attacking?

WRITTEN EXERCISE

Separate into parts having the ratios stated:

1. 528, 17:31. 2. 702, 13:41. 3. 450, 21:29.

4. 1050, 34:41. 5. 13.23, 31:32. 6. 15.54, 21:53.

7. Separate 165 into 3 parts having the ratio of 3 to 5 to 7.

This means that the first is to the second as 3 to 5, and the second to the third as 5 to 7. That is, $\frac{3}{15}$ of 165 is the first, $\frac{5}{15}$ the second, $\frac{7}{15}$ the third, or $\frac{1}{5}$, $\frac{1}{3}$, $\frac{7}{15}$, making 33 the first part.

Separate into parts having the ratios stated:

8. 273, 5:7:9. 9. 651, 7:11:13.

10. 2173, 11:13:17. 11. 33.67, 9:11:17.

12. 4.455, 11:21:23. 13. 40.47, 17:19:21.

14. Water is composed of two gases, hydrogen and oxygen, combined in the proportion of 2 to 1, respectively. In 390 parts of water, how many parts of each gas?

15. Two partners agree to divide their profits in the ratio of the amounts invested in the business. One invests \$4000 and the other \$3000, and they make \$2800. What is the share of each?

16. Two farmers agree to pay for the rent of a pasture in the ratio of the number of head of cattle pastured. One pastures 23 head and the other 32, and they pay \$68.75. What is the share of each?

17. Two men receive \$25.90 for picking apples, to be divided in the ratio of the number of bushels picked by each. For every 66 bu. picked by the first the second picked $2\frac{1}{2}$ bu. less. How was the money divided?

The common expression "to divide in the proportion of 2 to 3" means the same as "to divide in the ratio of 2 to 3."

424. Proportion. An expression of the equality of two ratios is called a *proportion.*

425. Illustrative problems. 1. In the proportion $x : 17.2 = 4.3 : 8$, find the value of x.

$$\frac{x}{17.2} = \frac{4.3}{8}. \quad \text{Multiplying by } 17.2, \text{ as in § 421,} \quad x = \frac{17.2 \times 4.3}{8} = 9.245.$$

2. If $21.3 : 17.1 = 3 : x$, find the value of x.

1. Because $21.3 : 17.1 = 3 : x$, we must have $17.1 : 21.3 = x : 3$.

2. That is, $\frac{x}{3} = \frac{17.1}{21.3}$, and $x = \frac{3 \times 17.1}{21.3} = 2\frac{23}{71}$.

WRITTEN EXERCISE

Find the value of x :

1. $x : 18.2 = 9.1 : 7.3$.
2. $x : 4.1 = 82.7 : 82$.
3. $x : 21.7 = 2.4 : 3.3$.
4. $x : 1.37 = 2.4 : 13.7$.
5. $x : 3.21 = 4.12 : 6.6$.
6. $x : 0.32 = 7.21 : 1.8$.
7. $3.17 : x = 4.28 : 3.2$.
8. $2.4 : x = 3.8 : 123$.
9. $8.21 : 3 = 4.105 : x$.
10. $3.9 : 4 = x : 3200$.

11. A certain circle has a circumference of 182.73 in. What is the circumference of a circle whose radius is to that of the first as $1.7 : 2.91$?

12. A certain water pipe has a diameter of 1.91 in. What is the diameter of a pipe whose circumference is to that of the first as $2.6 : 3.42$?

13. A certain fountain pen will write on an average 4210 words with one filling. The amount of ink it holds is to that held by a similar pen as $10 : 13$. How many words will the second pen average at one filling?

426. Extremes and means. The first and last terms of a proportion are called the *extremes;* the second and third, the *means.*

427. It is an interesting property of proportion, as seen in a case like

$$\tfrac{3}{7} = \tfrac{15}{35},$$

where $3 \times 35 = 7 \times 15$, that

The product of the means equals the product of the extremes.

428. We also see that if $x : 21 = 35 : 105$,

$$x = \frac{21 \times 35}{105}.$$

Therefore, in any proportion,

The product of the means divided by the given extreme equals the required extreme.

429. Illustrative problem. If $x : 7 = 13 : 20$, what is the value of x?

By § 428, $$x = \frac{7 \times 13}{20} = 4\tfrac{11}{20}.$$

WRITTEN EXERCISE

1. A miller uses 18 bu. of wheat for 4 bbl. of flour. How many barrels can he make from 207 bu.?

2. He sells 25 bbl. of this flour for $112.50. At the same rate, what will he receive for 476 bbl.?

3. The Bennington Battle monument casts a shadow 125 ft. 10 in. long at the same time that a 3-ft. post casts a 15-in. shadow. What is the height of the monument?

4. To find the height of a church spire a boy measured its shadow, 152 ft. 3 in., at the same time that a post 3 ft. 6 in. high was casting a shadow 5 ft. 3 in. long. How high was the spire?

5. How long will it take a train to go 148 mi., at the rate of $55\frac{1}{2}$ mi. in 1 hr. 30 min.?

6. If 9 men can complete some work in 20 hr., how many less men can complete it in 16 hr. more time?

7. If a man can skate 75 yd. in $8\frac{1}{3}$ sec., how long would it take him to skate 200 yd. if he could maintain this rate?

8. If a man can skate 220 yd. in 19 sec., how long would it take him to skate 5 mi. if he could maintain this rate?

9. If 750 lb. can be transported 100 mi. for a certain sum, how far should $\frac{1}{2}$ T. be transported at the same rate?

10. If 6 men can complete some work in 15 hr., how many men must be added to complete it in 5 hr. less time?

11. If 10 men can complete some work in 30 hr., how many men must be added to complete it in 10 hr. less time?

12. If a man is rowing at the average rate of 1 mi. in 6 min. $23\frac{41}{105}$ sec., how long will it take him to row 1 mi. 550 yd.?

13. If a man can swim 45 yd. in 23 sec., how long would it take him to swim 2 mi. if he could maintain this rate?

14. When a sum of money is divided among 7 persons each receives \$16.80. How much would each receive if the same sum were divided among 8 persons?

15. One man can do a piece of work in 10 da., and another in 9 da. If the wages of the first are \$3.60 a day, what should be the wages of the second?

430. Compound proportion. An expression of the equality of the products of ratios is called a *compound proportion.*

For example, $\frac{2}{3}$ of $\frac{5}{7} = \frac{x}{6}$ of $\frac{3}{4}$. This may also be written thus:

$$\left.\begin{matrix} 2:3 \\ 5:7 \end{matrix}\right\} = \left\{\begin{matrix} x:6 \\ 3:4 \end{matrix}\right.$$

431. Inverse ratio. The ratio $2:3$ is called the *inverse* of the ratio $3:2$.

Problems formerly solved by compound proportion are now more frequently solved by analysis (see page 169). Where it is necessary to distinguish the ordinary proportion of four terms from compound proportion, the former is called *simple proportion.* Compound proportion may be omitted without interfering with the subsequent work.

432. Illustrative problems. 1. In the compound proportion here given, find the value of x.

$$\left.\begin{matrix} 2:3 \\ 5:7 \end{matrix}\right\} = \left\{\begin{matrix} x:6 \\ 3:4 \end{matrix}\right.$$

1. Writing this in the more familiar form of fractions, we have

$$\frac{2}{3} \text{ of } \frac{5}{7} = \frac{x}{6} \text{ of } \frac{3}{4}.$$

2. If we divide these equals by $\frac{3}{4}$, or multiply by $\frac{4}{3}$, they will still be equal. Therefore

$$\frac{4}{3} \text{ of } \frac{2}{3} \text{ of } \frac{5}{7} = \frac{x}{6} \text{ of } \frac{\cancel{3}}{\cancel{4}} \text{ of } \frac{\cancel{4}}{\cancel{3}}.$$

3. Since we now have $\frac{1}{6}$ of x, x equals 6 times as much. Therefore

$$6 \times \tfrac{4}{3} \text{ of } \tfrac{2}{3} \text{ of } \tfrac{5}{7} = x.$$

Therefore

$$\tfrac{80}{21} = x.$$

Such results should always be checked by being substituted for x in the compound proportion. Thus, $\frac{2}{3}$ of $\frac{5}{7} = \frac{1}{6}$ of $\frac{80}{21}$ of $\frac{3}{4}$, for each reduces to $\frac{10}{21}$.

2. If 4 loads of hay will last 6 horses 8 weeks, 8 loads will last how many horses 3 weeks?

1. Considering only the number of loads in relation to the number of horses, the ratio of the latter equals the ratio of the former, for twice as many horses will need twice as many loads, and so on. Therefore

x horses : 6 horses = 8 loads : 4 loads, or $x : 6 = 8 : 4$.

2. Considering only the number of weeks in relation to the number of horses, the ratio of the latter equals just the inverse ratio of the former; for if we have twice as many horses, the hay will last only half as long, the number of weeks diminishing in the same ratio that the number of horses increases. Therefore

x horses : 6 horses = 8 weeks : 3 weeks, or $x : 6 = 8 : 3$.

3. Finally, considering both the loads and the time, we have

$$x : 6 = \begin{cases} 8 : 4 \\ 8 : 3 \end{cases},$$

or, in the more familiar form of fractions,

$$\frac{x}{6} = \frac{8}{4} \text{ of } \frac{8}{3}, \qquad \text{whence } x = \overset{2}{\cancel{6}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{4}} \text{ of } \frac{8}{\cancel{3}} = 32,$$

the number of horses.

433. We may simplify the actual work by noticing that, as in simple proportion,

In any proportion the product of the means equals the product of the extremes.

Hence, in the above example, after reasoning out the form of the proportion, we have simply to write

$$4 \times 3 \times x = 6 \times 8 \times 8.$$

$$x = \frac{\overset{2}{\cancel{6}} \times \overset{2}{\cancel{8}} \times 8}{\cancel{4} \times \cancel{3}} = 32.$$

WRITTEN EXERCISE

1. If 4 bbl. of flour last 6 persons 12 mo., how many barrels will last 15 persons 18 mo.?

2. If 75 hens eat 16 bu. of feed in 12 wk., how many bushels will 30 hens eat in 5 wk.?

3. If a farm 130 rd. by 80 rd. costs $1750, how much will one 118 rd. by 50 rd. cost at the same rate?

4. If the total cost of feeding 75 hens for 7 wk. is $10.50, what will it cost to feed 40 hens for 9 wk.?

5. If 4 five-cent loaves of bread last 6 persons 2 da., how many ten-cent loaves will last 5 persons a week?

6. If $800 yields $324 interest in 6 yr. 9 mo., how much interest will $500 yield in 4 yr. 5 mo. at the same rate?

7. In a sewing guild 18 girls make 40 garments in 4 hr. At this rate, how many garments can 30 girls make in 3 hr.?

8. If the value of the eggs laid by 75 hens in 12 wk. is $67.50, what is the value of the eggs laid by a third as many hens in 5 wk.?

9. If 5 boys, working 8 hr. a day, can build a cabin in the woods in 12 da., how many days will it take 15 boys working 2 hr. less a day?

10. If 5 girls in a sewing class make 20 aprons in 3 da., how many girls will it take to make 32 aprons in 4 da.? to make 16 aprons in 6 da.?

11. A stone wall is to be constructed around a rectangular piece of land 2000 ft. by 1075 ft. Ten men, working 8 hr. a day, build 890 ft. in 3 da. The number of men is then diminished 50%, and the remainder work 9 hr. a day. How long will it take to complete the wall?

12. If 25 children use 10 boxes of crayons, 200 in a box, in 4 mo., how long will 6 boxes, 100 in a box, last 40 children?

13. If 2 pumps, working 10 hr. a day for 4 da., can raise 120 T. of water, how many tons can 3 pumps raise in 5 da. of 8 hr. each?

14. If 5 men, working 14 da., 6 hr. a day, can excavate a cellar, how long will it take if there are 7 more men, and they work 1 hr. longer each day?

15. If it will take 5 boys 14 da., working 6 hr. a day, to clear a stony field for a ball ground, how long will it take 12 boys, working 8 hr. a day?

16. Two men rented a pasture for $83. The first puts in 6 horses for 4 mo., the second puts in 5 horses for $3\frac{1}{2}$ mo. What is the share to be paid by each?

17. If it takes 6 boys 15 da., working 4 hr. a day, to build a playhouse, how many boys should be able to do the work in 12 da., working 3 hr. a day?

18. If 9 girls in a manual training class made 12 baskets in 4 da., working 45 min. a day, how many baskets can 15 girls make in 6 da., working an hour a day?

19. If a railroad charges $9 for transporting 3 T. of goods 360 mi., how much should be charged for transporting $7\frac{1}{2}$ T. of goods 280 mi. at the same rate?

20. If 5 girls, earning money for Christmas gifts, dressed 14 dolls in 4 da., working 6 hr. a day, how many dolls could 8 girls dress in 3 da. of 5 hr. each, at the same rate?

21. Of 2 boys raising eggs for the market, one has 40 hens and the other 75. If the 40 hens lay 1440 eggs in 12 wk., how many will the 75 hens lay in half that time, at the same rate?

Farm Problem

WRITTEN EXERCISE

Estimate the answers, putting the estimates in writing for comparison with the correct answers. Some estimates should be exact.

1. A meadow is $19\frac{1}{2}$ rd. long and $9\frac{1}{4}$ rd. wide. What is its area in acres?

2. A square bin has a capacity of 2352 cu. ft. The bin is $5\frac{1}{3}$ ft. deep. What is its length? (Given that $441 = 21 \times 21$.)

3. If 1 qt. of corn will plant 230 hills, what will the corn cost for 2760 hills @ 25¢ a quart?

4. If 1 qt. of beans will plant 150 hills, what will the beans cost for 1950 hills @ 35¢ a quart?

5. What is the cost of seed for 7500 celery plants, allowing $\frac{1}{2}$ oz. to 1000 plants, @ 22¢ an ounce?

6. What is the cost of seed for 3500 cauliflower plants, allowing 1 oz. to 1000 plants, @ \$2.30 an ounce?

7. Find the cost of oats required to sow 15 acres, allowing $2\frac{1}{4}$ bu. to the acre, the oats being worth 90¢ a bushel.

8. Find the cost of beans required to plant 5 acres, allowing 12 qt. to the acre, the beans being worth \$1.50 a peck.

9. Find the cost of peas required to plant 5 acres, allowing $1\frac{1}{2}$ bu. to the acre, the peas being worth \$3.50 a bushel.

10. Find the cost of corn required to plant 20 acres, allowing 8 qt. to the acre, the corn being worth \$2 a bushel.

11. Find the cost of potatoes required to plant 12 acres, allowing 10 bu. to the acre, the potatoes being worth \$1.15 a bushel.

12. If a tree sparrow eats an ounce of weed seed every 4 days, how many pounds will 500 such birds eat in May?

13. If a tree sparrow eats in a day enough seeds to produce 75 weeds, a dozen such birds will destroy in a week enough seeds to produce how many weeds?

14. If a woodpecker eats on an average 1690 insect pests in a day, how many will 250 woodpeckers eat during June?

15. If a woodpecker eats on an average 10% less insect pests in May than in June, how many will 50 woodpeckers eat during May? (See Ex. 14.)

16. The king bird eats 89% animal food. Out of 11 lb. 8 oz. of food eaten by these birds, how many ounces are vegetable food?

17. Of the food of bobolinks, 63% is insects. How much insect food is consumed by them while eating 7 lb. $6\frac{2}{5}$ oz. of other (vegetable) food?

18. Of the food of blackbirds, 57% is weed seed. When these birds have consumed $14\frac{1}{3}$ lb. of other food, how many pounds of weed seed have they eaten?

19. Out of 5 lb. of food eaten by orioles, $12\frac{4}{5}$ oz. consist of vegetable food, the remainder being insects injurious to plant life. What per cent of the food is injurious to plants?

20. It is estimated that the offspring of a pair of English sparrows would, were there no deaths, amount in ten years to 275 million. Estimating 75% of the food of these birds to be grain, and allowing $\frac{1}{2}$ oz. of food to each bird daily, how many pounds of grain would this offspring consume in a day? in 365 da.?

21. If a heaped barrel of potatoes contains 3 bu., how many such barrels will contain 567 bu.? How many bushels in 297 such barrels?

22. The 2,568,391,056 bu. of Indian corn produced in this country in one year was 26% more than the number of bushels of wheat. How much wheat was produced?

23. The average price of hay in Kansas recently was only \$4.06 a ton, which was $23\frac{1}{3}$% as much as in Massachusetts. What was the price in Massachusetts, and why was there this great difference?

24. When the hay crop of the country is 60 million tons, worth \$7.50 a ton, what is the value of the whole crop? If the crop of New York, the chief hay-producing state, is worth 10% of this amount, what is its value?

25. In one of the states 2.2 million acres produced 74.8 million bushels of oats in one year, worth \$18,700,000. What was the average yield per acre, the average price per bushel, and the average value of the crop per acre?

26. When the annual products of the 155,000 farms in Minnesota were worth \$170,500,000, what was the average per farm?

27. Of these farms, if 6,500,000 acres produced 97,500,000 bu. of wheat, what was the average yield per acre? What was the total yield worth at 60¢ a bushel?

28. Some of this grain went to Duluth, where an elevator distributed 20,000 bu. of wheat to a ship in one hour. Allowing 62 lb. to the bushel, how many tons did it distribute in a minute?

29. If one elevator spout carries $8602\frac{4}{5}$ bu. of wheat in an hour, how long will it take a 6-spout elevator to unload a train of 40 cars, averaging 20 tons to a car, allowing 62 lb. to the bushel, no allowance being made for moving the cars?

ORAL EXERCISE

1. At \$26 a ton, what is the cost of 200 lb. of bone meal? of 50 lb.?

2. At \$26 a ton, what is the cost of 100 lb. of cotton-seed meal? of 500 lb.?

3. At \$12.50 a ton, what is the cost of 100 lb. of acid phosphate? of 400 lb.?

4. At \$1.35 per 100 lb., what is the cost of a ton of bone meal? of 300 lb.?

5. At \$48 a ton, what does 1 lb. of nitrate of soda cost? What is the cost of 100 lb.?

6. At \$125 a ton, what is the cost of 200 lb. of sulphate of copper? of 100 lb.?

7. At \$2.50 per 100 lb., what is the cost of a ton of nitrate of soda? of 250 lb.?

8. At \$2.20 per 100 lb., what is the cost of a ton of muriate of potash? of 500 lb.?

9. At \$2.60 per 100 lb., what is the cost of a ton of sulphate of potash? of 500 lb.?

10. If nitrate of soda contains 15% nitrogen, what is the weight of nitrogen in a ton?

11. At \$3.30 per 100 lb., what is the cost of a ton of sulphate of ammonia? of 600 lb.?

12. If acid phosphate contains 14% phosphoric acid, what is the weight of phosphoric acid in a ton?

13. A certain fertilizer for tobacco land is made of 900 lb. cotton-seed meal @ \$1.30 per 100 lb., 100 lb. nitrate of soda @ \$50 per ton, 250 lb. sulphate of potash @ \$2.50 per 100 lb., 750 lb. acid phosphate @ 60¢ per 100 lb. What is the cost of each ingredient?

WRITTEN EXERCISE

1. To lessen potato scab the seed potatoes are soaked in a solution of $\frac{1}{2}$ pt. of formalin to 15 gal. of water. What is the per cent of formalin in the mixture? What is the per cent of water?

2. Bordeaux mixture, for spraying, contains $1\frac{1}{4}\%$ by weight of copper sulphate, $1\frac{1}{4}\%$ of unslacked lime, and the rest water. How many pounds of each in a barrel containing 250 lb. of the mixture?

3. Half of a peach tree was sprayed with Bordeaux mixture and the other half was not. From the sprayed half 284.8 lb. of fruit were taken, and from the other half 95% less. How many pounds were taken from the tree?

4. If only the fiber (lint) is taken from the land, a cotton crop removes from the soil only 8.6% as much plant food as a wheat crop. The wheat crop takes 32.4 lb. a year from each acre. What is the weight of plant food that a cotton crop takes each year from a 50-acre field?

5. It is estimated that a chickadee will destroy 30 egg-laying cankerworm moths during each of the 25 days that these moths, which are wingless, crawl up orchard trees. This would result in the destruction of how many eggs by a single chickadee in an orchard, if each moth would have laid 185 eggs?

6. A farmer by placing a burlap band around an apple tree caught 96 larvæ of the coddling moth in a week. Suppose each of these had emerged from its cocoon as a moth, had deposited an egg on each of 294 apples, each egg hatching into an apple-tree worm and destroying the apple, how many apples would have been saved by destroying these 96 larvæ?

7. If bone meal is 4% nitrogen, and nitrate of soda is 15% nitrogen, how many pounds of nitrogen in a ton of each? in 1500 lb.?

8. In a certain fertilizer (plant food) for land on which corn is grown, $43\frac{3}{4}\%$ is acid phosphate, $47\frac{1}{2}\%$ is cotton-seed meal, and the rest is kainit. How many pounds of each to a ton?

9. In a certain fertilizer for land on which cotton is grown, $53\frac{3}{4}\%$ is acid phosphate and the rest is 32 parts fish scrap to 5 parts muriate of potash. How many pounds of each to a ton?

10. A farmer used the following fertilizer for an acre of grapes: 300 lb. acid phosphate @ \$11.50 a ton, the same weight of bone meal @ \$26.50 a ton, and the same weight of muriate of potash @ \$42.50 a ton. What did the fertilizer cost?

11. A Virginia tobacco grower used for his land a fertilizer composed of 35% cotton-seed meal, 5% nitrate of soda, and the rest of sulphate of potash and acid phosphate in the ratio of 1 to 3. How many pounds of each in 750 lb.? in 1 T.? in $1\frac{1}{2}$ T.?

12. A farmer mixed a ton of fertilizer for his corn land, using half cotton-seed meal @ \$24.60 a ton and the rest acid phosphate and muriate of potash in the ratio of 19 to 1. The acid phosphate cost \$11.50 a ton, and the muriate of potash \$42.50 a ton. What did the fertilizer cost?

13. A cotton planter mixed a ton of fertilizer for his land, using half acid phosphate @ \$11.50 a ton and the rest cotton-seed meal and muriate of potash in the ratio of 37 to 3. The cotton-seed meal cost \$24.60 a ton and the muriate of potash \$42.50 a ton. What did the fertilizer cost?

14. A market gardener used the following fertilizer for 3 acres of vegetables: 400 lb. nitrate of soda @ $48 a ton, 900 lb. cotton-seed meal @ $24.60 a ton, 900 lb. acid phosphate @ $11.70 a ton, 900 lb. bone meal @ $26.10 a ton, and 33⅓% less muriate of potash than bone meal @ $2.15 per 100 lb. What did the fertilizer cost per acre?

15. A gardener used 700 lb. of fertilizer per acre, composed of 1 part nitrate of soda to 2 parts each of acid phosphate, bone meal, and muriate of potash. The nitrate of soda cost $48 a ton, the acid phosphate $11.50 a ton, the bone meal $2.15 per 100 lb., and the muriate of potash $42.50 a ton. What did the fertilizer cost for 5 acres?

16. Of an Iowa farm of 160 acres, 10% was swampy. Before this part was properly drained it yielded some hay, averaging in value $3.25 to the acre. After being drained and tiled it yielded grain to the value of $12 to the acre. What was the per cent of increase in productiveness of this part of the farm after it was drained?

17. By a proper selection of seed corn for three years, a farmer was finally able to grow 80 bu. of corn on one acre. On an adjoining acre with similar soil where seed was taken at random, the yield was 33¾ bu. If he had followed the first plan instead of the second for all of his 52 acres, what would have been the per cent of gain?

18. On a piece of average ground a farmer counted 22 heads of smutty wheat out of a total of 176 heads. If the yield was 16 bu. of good wheat to the acre, and the tract contained 42 acres, what was the farmer's loss in yield? By treating a certain area with formalin he killed all the smut spores, and by selecting his seed from these healthy plants he afterwards grew all of his wheat free from disease. What was now the yield per acre?

19. A cow gives 875 lb. of milk in a certain month. The milk tests 4.3% butter fat. How many pounds of butter fat does she produce in this month?

20. If the butter fat is sufficient for $\frac{1}{6}$ more than its weight of butter, how many pounds of butter could be made from the milk mentioned in Ex. 19?

21. If the butter mentioned in Ex. 20 is sold at 24¢ a pound, what is the value of the butter produced in this month from the milk of this cow?

22. A man has a dairy that produces milk averaging 3.75% butter fat during a certain month. The butter fat weighs 787½ lb. How many pounds of milk are produced?

23. The weight of the solid matter in the milk of a certain dairy during a summer was 12½% of the total weight, and the butter fat was 25% of the solids. The butter fat was what per cent of the weight of the milk?

24. The weight of the solid matter in the milk of a certain dairy during June was 13.2% of the total weight, and the butter fat was $\frac{1}{4}$ of the solids. How many pounds of butter fat to a ton of milk?

25. A farmer bought 24 cattle and sold them a year and a half later for $1200. He estimated his net gain at $10 a head, 20% of which was due to extra care in feeding. What would have been the average selling price per head if he had not taken this extra care?

26. A farmer has two cows, one supplying 1000 lb. of milk in a certain month, testing 3% butter fat, and the other 800 lb., testing 4% butter fat. If the butter fat is sufficient for 116⅔% of its weight in butter, and butter is worth 23¢ a pound, which cow pays the farmer the more for that month, the feed costing the same? How much more?

27. The books of a creamery show the following record for seven of its patrons, A, B, C, D, E, F, G, for six days. The figures represent the pounds of cream delivered each day, and the per cent of butter fat in the cream, as shown by tests.

	A	B	C	D	E	F	G
Feb. 2.	27	21	41	18	24	59	78
" 3.	26	28	42	19	34	67	81
" 4.	23	30	45	16	38	72	90
" 5.	25	32	47	20	39	76	91
" 6.	28	35	50	21	38	72	89
" 7.	23	39	52	20	39	75	92
Per cent of butter fat	29%	34%	27%	30%	26%	35%	32%

How many pounds of cream were supplied by each patron? Of this amount how many pounds were butter fat? If this butter fat was sufficient for $116\frac{2}{3}\%$ of its weight in butter, and the patrons received 24¢ per pound of butter, what did each receive for the six days?

28. One day's report of a creamery operating five stations shows the pounds of milk and cream received, as follows:

	Sta. 1	*Sta. 2*	*Sta. 3*	*Sta. 4*	*Sta. 5*
Milk	1032	1152	864	720	1008
Cream	500	480	384	288	360

a. If $16\frac{2}{3}\%$ of the milk is cream, how many pounds of cream has each station?

b. If 25% of the cream is butter fat, how many pounds of butter fat has each?

c. If the butter fat is sufficient for $116\frac{2}{3}\%$ of its weight in butter, how many pounds of butter does each produce?

d. What is the butter of each worth at 23¢ a pound?

e. Show that the total number of pounds of butter is 819.

29. A certain creamery uses 6400 lb. of milk in a week. The skim milk amounts to 80% of the whole milk, and tests $\frac{3}{5}$% butter fat. How many pounds of butter fat are lost in the skim milk? If this would suffice for $16\frac{2}{3}$% more than its weight in butter, how much butter is lost?

30. A cow gives 850 lb. of milk in a certain month. After the butter fat is extracted the milk weighs 816 lb. What per cent of butter fat did the milk yield? How many pounds? How many pounds of butter could be made from it, the butter weighing $116\frac{2}{3}$% as much as the fat? What would this butter be worth at 25¢ a pound?

31. An Illinois farmer made a fertilizer for his cornfields, using 95 parts acid phosphate, 100 parts cotton-seed meal, and 5 parts muriate of potash. He then decided to add enough acid phosphate to 100 lb. of the mixture to make it 50% of the total. How much did he add?

If he adds x lb., then 50% of $(100 + x)$ lb. $= 47\frac{1}{2}$ lb. $+ x$ lb. Hence $2\frac{1}{2} = \frac{1}{2}x$, and $x = 5$.

32. A Louisiana planter has some fertilizer for his cotton fields, consisting of $62\frac{1}{2}$% acid phosphate, 30% dried blood, $7\frac{1}{2}$% muriate of potash. He finds that his land needs more acid phosphate, and decides to add enough of it to the mixture so that it shall be 70% of the total. How many pounds of acid phosphate must he add to a ton of the fertilizer?

33. A North Carolina farmer finds that a fertilizer that he has been using is too rich in acid phosphate. The formula used is 25% dried blood, 5% nitrate of soda, 20% sulphate of potash, and the rest acid phosphate. How many pounds less of acid phosphate must he use to every 400 lb. of sulphate of potash, so that the acid phosphate shall be 45% of the mixture?

CHAPTER IV

I. BUSINESS APPLICATIONS

GOING INTO BUSINESS

434. How a boy may go into business. If a boy on leaving school goes into some trade, he will start with very low wages. He may be employed to go on errands, to wrap up goods, or to deliver parcels. He will soon show whether he has any elements of success. If he shirks work, indulges in too much talk, watches the clock for closing time, and wastes what money he receives, he will either be "out of a job" or be left in the lowest kind of place.

But if he works his best, exerts himself to be helpful in every way, is always a gentleman, and takes an interest in improving his work, he will soon begin to work his way up. If he saves money, shows himself a good manager, earns a reputation for absolute truth, and has a circle of customers who know they can always rely upon him, he will probably succeed in business for himself. These are the qualities that make great merchants and manufacturers. The world is waiting for such men.

435. How a girl may go into business. The girl who has to support herself may begin as cash girl or clerk in a store and work up to the position of buyer in some department. If she is in a factory, a position as overseer is among the possibilities. If she is a stenographer, good positions are always awaiting capable women, often demanding considerable knowledge of the business with which they are connected. If she marries, some knowledge of business is equally as important, for many homes are made unhappy because of lack of this knowledge.

BANK ACCOUNTS

436. A bank account essential. One thing essential to any one who hopes to succeed is a bank account. Any one may "open an account," as it is called, as soon as he begins to save even a small amount. In most parts of the country small sums are usually deposited in savings banks.

437. The savings banks. To deposit money a person simply goes to the bank, says that he wishes to open an account, and leaves his money with the officer in charge. The officer gives him a book in which the amount is written, and the depositor writes his name in a book of signatures, for identification. When he wishes to draw out any money, he takes his book to the bank, states how much he wishes, signs a receipt (in some parts of the country he makes out a check), has the amount entered in his book, and the money is paid to him if it does not exceed his deposits, — his "balance," as they say. If he forms the habit of depositing money when he has any to spare, and never drawing except in an emergency, he will be surprised to see how it accumulates.

WRITTEN EXERCISE

1. What does 25 ct. saved a day amount to in a year?

2. How much will 15 ct. a day, 310 days a year, amount to in 10 years, not counting interest?

3. If a boy, beginning when he is 16, saves 25 ct. a day for 300 days a year, and deposits it in a savings bank, how much will he have when he is 21, not counting interest?

4. If a father gives his daughter $1 when she is 1 yr. old, $2 on her next birthday, and so on until she is 21, depositing it for her in a savings bank, how much will she have when she is 21, not counting interest?

5. A man saves on an average 25¢ a day the first year he works on a certain farm, and 120% as much the second year, depositing it in a savings bank. If he works 306 days each year, how much does he save in the two years?

6. A farmer's savings-bank deposits average $25 a month during a certain year, and 97% as much during the next year. The year following he deposited 110% as much as he did the second year. How much did he deposit during the three years?

7. If a man saved $1 a week during the year in which he was 21, and increased his savings each year by 10% on the amount of the preceding year, and worked 50 weeks a year, how much would he save during the year in which he was 29 years old?

8. A merchant saves $375 the first year he is in business. The second year he increases his savings one third. The third year they are only 85% as much as the second year. The fourth year they increase 30%. How much does he save in the four years?

9. A man works on a salary of $15 a week for 50 weeks in a year. His expenses are $15 a month for house rent, 60% as much for clothing, 300% as much for food as for clothing, and 20% as much for other necessary expenses as for food. How much of his salary can he reserve for the savings bank?

10. A clerk in a store had a salary of $12 a week two years ago, and a commission of 2% on the goods sold by him. That year he worked 48 weeks and sold $2350 worth of goods. Last year his salary was increased 25%, his commissions remaining the same. He worked 49 weeks and sold $2750 worth of goods. How much was his income increased?

438. How savings banks pay interest. Savings banks usually pay interest every six months. This is added to the principal, and the amount draws interest.

439. Compound interest. When the interest is added to the principal as it becomes due, and the amount draws interest, the owner is said to receive *compound interest.*

Compound interest is no longer allowed on notes. But if any one collects his interest when due, and at once reinvests it, of course he practically has the advantage of compound interest. The method of finding compound interest is substantially the same as that used in simple interest.

440. Illustrative problem. Required the amount of $2000 invested in a savings bank at 4% annually, the interest compounded semiannually, for 2 years.

The simple interest for the same length of time is $160, $4.87 less than the compound interest.

$2000	= first principal
.02	
40	= first interest
2000	
2040	= amt. after 6 mo.
.02	
40.80	= int. second 6 mo.
2040	
2080.80	= amt. after 1 yr.
.02	
41.62	= int. third 6 mo.
2080.80	
2122.42	= amt. after 1½ yr.
.02	
42.45	= int. fourth 6 mo.
2122.42	
$2164.87	= amt. after 2 yr.

ORAL EXERCISE

Find the amount at compound interest:

1. $1000, 2 yr., 4%.
2. $1000, 2 yr., 5%.

In Ex. 1 we have $40 interest each year, plus 4% of $40.

3. $2000, 2 yr., 4%.
4. $2000, 2 yr., 5%.
5. $3000, 2 yr., 3%.
6. $3000, 2 yr., 2%.
7. $5000, 2 yr., 4%.
8. $5000, 2 yr., 3%.
9. $8000, 2 yr., 5%.
10. $10,000, 2 yr., 4%.

WRITTEN EXERCISE

Find the amount at simple interest, and at interest compounded annually:

1. \$2500, 4 yr., 5%.
2. \$4000, 3 yr., 6%.
3. \$3500, 3 yr., 4%.
4. \$2250, 4 yr., 3%.
5. \$2750, 4 yr., 3%.
6. \$1750, 3 yr., $3\frac{1}{2}$%.
7. \$625.50, 4 yr., $3\frac{1}{2}$%.
8. \$10,000, 6 yr., 4%.
9. \$425.50, 4 yr., $4\frac{1}{2}$%.
10. \$275.50, 3 yr., $4\frac{1}{2}$%.

Find the amount of principal and interest, the interest being compounded semiannually:

11. \$200, 3 yr., 4%.
12. \$300, 2 yr., 4%.
13. \$650, 2 yr., 6%.
14. \$700, 3 yr., 3%.
15. \$800, 3 yr., 4%.
16. \$500, 2 yr., 4%.
17. \$1000, 2 yr., $4\frac{1}{2}$%.
18. \$1000, 3 yr., 4%.
19. \$2000, 2 yr., $3\frac{1}{2}$%.
20. \$3000, 2 yr., $2\frac{1}{2}$%.

21. Which brings the more interest in 2 yr., on \$1250, 4% in a savings bank, compounded semiannually (the money being deposited at the beginning of the year), or $4\frac{1}{2}$% simple interest?

22. If a man on January 1 deposited \$2000 in a savings bank, and left it for 2 yr., at 4%, the interest being compounded semiannually, how much less interest would he receive than by loaning it at 5% simple interest?

23. If a man deposits in a savings bank \$200 January 1, \$300 February 1, \$100 May 1, \$400 August 1, and \$350 November 1, and the rules of the bank allow 4% interest on all of these sums, compounding it on July 1 and January 1, how much will the man have to his credit on the January 1 following these deposits?

441. Banks of deposit. When a man has money enough ahead to pay his bills by checks, he will need to have an account with an ordinary bank, sometimes called a *bank of deposit.*

Such banks do not pay interest on small accounts, the deposit being a matter of convenience and safety. If a man wishes to open an account, he sometimes has to give references, for banks do not wish to do business with unreliable people.

442. Deposit slips. When a man deposits money he makes out a *deposit slip*, as here shown, and leaves it at the bank.

SECOND NATIONAL BANK

BOSTON, MASS.

Deposited for the account of

.....

Date..... .....19....

Bills and small coin . .		
Gold		
Silver		
Check on.....Bank		
" " ..... "		
Total		

ORAL EXERCISE

Tell the sum of each of the following lists of deposits:

1.	2.	3.	4.
$15.50	$12.75	$23.80	$12.00
3.75	2.25	12.70	13.75

WRITTEN EXERCISE

Make out deposit slips for the following deposits, naming some bank in your town:

1. Bills, $254; silver, $40; checks on First National Bank, $87.50; Traders Bank, $127.50.

2. Bills, etc., $423.57; gold, $275; silver, $135.75; check on Cotton Exchange Bank, $342.60.

3. Bills, etc., $135.50; checks on Chemical National Bank of New York, $325; Third National, $63.73.

4. Bills, etc., $1726.45; gold, $125; silver, $100; checks on Traders Bank, $335.50; Second National, $175.40.

5. Bills, etc., $262.75; checks on Garfield National, $96.50, $200; Jefferson and Lee National, $325, $46.50.

6. Gold, $50; checks on First National, $27.62, $41.75, $32.80; Lincoln Trust Company, $37.42, $21.85.

7. Bills, etc., $146.73; silver, $275; checks on Miners National Bank, $43.50; Rocky Mountain National, $250.

8. Bills, $145; silver, $350; gold, $480; checks on Merchants National Bank, $255; Farmers Trust Company, $162.50; Second National Bank, $275.50.

9. A merchant deposited $175.80 in cash to-day, and a check for 25% of a debt of $176 due him, and a check in payment for $18\frac{3}{4}$ yd. of velvet at $2.16 a yard less $33\frac{1}{3}\%$ on account of a bargain sale. Make out a deposit slip.

10. A jeweler received cash for the following: 3 doz. forks @ $16.50; $4\frac{1}{2}$ doz. teaspoons @ $15; a watch @ $32.75; and 2 clocks @ $6.75. He also received checks on the First National Bank for the following: $2\frac{1}{2}$ doz. table spoons @ $19.50; 3 doz. dessert spoons @ $17.75; and 9 nut crackers @ $9 a dozen. Make out a deposit slip.

443. Check books. A *check book* is given the depositor. Each page has one or more *checks* and *stubs*.

Stub	*Check*
No. *275*	No. *275* CHICAGO, ILL., ...[*Date*]....19....
Date.....	FIRST NATIONAL BANK OF CHICAGO
To *J. H. Smith*	Pay to the order of
For *Bal. acct.*	*John H. Smith* 15.$\frac{50}{}$
Amt. $15.$\frac{50}{}$	*Fifteen and* $\frac{50}{100}$ ~~~~~~~~ Dollars
	Robert J. Brown

444. Payee. The person to whom a check is payable is called the *payee*. A check is usually made payable to:

1. "Self," in which case the drawer alone can collect it.
2. The order of the payee, when the payee must indorse it.
3. The payee or bearer, when any one can collect it.
4. "Cash," in which case any one can collect it.

ORAL EXERCISE

Given the following deposits and checks, tell the balance:

	Deposits	*Checks*		*Deposits*	*Checks*		*Deposits*	*Checks*
1.	$25	$15	2.	$37	$19	3.	$48	$20
	70	27		41	32		27	15
	32	42		20	15		13	17
4.	$62	$37	5.	$75	$62	6.	$50	$65
	48	41		25	17		40	30
	17	20		32	40		75	50

WRITTEN EXERCISE

1. If your deposits in a bank have been \$14.75, \$32, \$15, \$25, \$50, \$75, \$1.75, and you have drawn checks for \$9.50, \$18.75, \$30, what is your balance?

2. A man earning \$15 a week deposits \$12 every Saturday, and on each Monday gives a check for \$6 for his board. What will be his balance in 6 months (26 weeks)?

3. If your deposits in a bank have been \$32.75, \$49.63, \$28, \$14.90, \$25, \$10, \$30, \$42.75, \$71.25, and you have drawn checks for \$5.75, \$13.50, \$32, \$4.50, \$36.50, \$52.80, what is your balance?

4. A farmer having \$275.60 in the bank deposits during the next six months \$23.50, \$17.42, \$75, \$32.40, and the proceeds of the sale of 7 cows @ \$37.50 less 5%. He gives checks for \$17.92, \$8.50, \$72.80, \$9.50, \$6.75, \$13.90, and \$26.50. What is his balance?

5. A merchant's deposits this week have been \$127.42, \$135, \$72.50, \$265.75, \$327.40, \$182.50, and he has drawn checks for \$236.50, \$15.75, \$27.90, \$241.60, and \$29.50. He has also paid by check a bill for \$176.70 less 10%, and another for \$75.25 less 10%, 3%. At the beginning of the week he had \$475.80 in the bank. What is his balance at the end of the week?

6. A merchant's deposits this week have been \$261.50, \$392.75, \$62.40, \$112.30, \$98.76, \$115, and he has drawn checks for \$42.80, \$174.95, \$162.30, \$81.50, \$15, \$27.40, and \$37.50. He has also given a check for \$125 plus 6 months' interest at 5%, and one in payment of a bill for \$248.50 less 8%, 5%. At the beginning of the week he had \$692.80 in the bank. What is his balance at the end of the week?

445. Borrowing money. If the time comes when a man wishes to start in business, and he has saved some money and has an unquestionable reputation for honesty, he will be able to borrow more money for buying a business or purchasing goods. Suppose he needs $1000. A bank will be found that has heard enough about his standing to lend him the money if he has a good *indorser.*

446. Indorser. One who writes his name across the back of a note is called an *indorser.* The indorser is responsible for the note if the maker does not pay it.

If Richard Roberts will indorse for John Brown, Mr. Brown makes out a note like this:

BOSTON, MASS., *May 15,* 1906

Two months after date, *without grace,* I promise to pay to the order of *Richard Roberts* $*1000.*^{00}

One thousand ..... Dollars

at **The First National Bank, Boston, Mass.**

Value received

Residence *7030 Beacon St.* *John Brown*

Due *July 15*

It is not a very direct way of borrowing from the bank, but it is a common one. The original idea was that Mr. Brown borrowed of Mr. Roberts, and Mr. Roberts sold the note to the bank, thus getting the money to lend Mr. Brown. But now it means simply that Mr. Roberts agrees to pay if Mr. Brown does not. Banks often take notes payable directly to the cashier.

Some states allow three "days of grace," that is, two months means two months + three days; but the custom is rapidly going out of use. Teachers should use them where it is the custom.

447. Bank discount. The note described on page 411 does not mention interest. This is because the borrower, Mr. Brown, pays the interest to the bank in advance.

Interest paid on a note in advance is called *bank discount.* The note is then said to be *discounted.*

448. Illustrative problem. What is the discount on the $1000 note on page 411 at 6% a year?

The discount on $1000 for 2 mo. @ 6% a year is $10.

The rate of discount is understood to be by the year, like the rate of interest, unless the contrary is stated.

449. Proceeds. The balance after deducting the discount from the face of a note is called the *proceeds.*

Thus, the proceeds on the above note are $1000 − $10 = $990.

450. Usual time for bank notes. Notes are usually made for 30, 60, or 90 days, and are then renewed if desired.

In states where days of grace are allowed, teachers should see that they are included.

ORAL EXERCISE

State the discount on the following:

1. $50, 2 mo., 6%.
2. $75, 2 mo., 6%.
3. $125, 2 mo., 6%.
4. $650, 2 mo., 6%.
5. $100, 1 mo., 6%.
6. $250, 1 mo., 6%.
7. $400, 3 mo., 6%.
8. $600, 3 mo., 6%.
9. $600, 2 mo., 5%.
10. $1200, 3 mo., 5%.

State the proceeds on the following notes, discounted as specified:

11. $100, 2 mo., 6%.
12. $300, 2 mo., 6%.
13. $600, 1 mo., 5%.
14. $600, 3 mo., 5%.
15. $1000, 3 mo., 6%.
16. $1000, 3 mo., 5%.

WRITTEN EXERCISE

Find the discount and proceeds on the following:

1. \$875, 30 days, 6%.
2. \$975, 90 days, 5%.
3. \$425, 30 days, 6%.
4. \$2568, 60 days, 5%.
5. \$1250, 60 days, 6%.
6. \$1500, 90 days, 5%.
7. \$2565, 90 days, 5%.
8. \$3250, 90 days, $5\frac{1}{2}$%.
9. \$4250, 30 days, $4\frac{1}{2}$%.
10. \$325.50, 30 days, 6%.
11. \$427.50, 60 days, 5%.
12. \$237.50, 30 days, 5%.

13. Make out a 60-day note for \$750, dated to-day, payable to John Doe's order at some bank in your town or city. Discount it at 5%.

14. Make out a 30-day note for \$675, dated to-day, payable to Richard Roe's order at some bank of which you know. Discount it at 6%.

15. Make out a 90-day note for \$1250, dated to-day, payable to James Jameson's order at some bank of which you know. Discount it at 6%.

16. A man's bank account shows deposits, \$37.50, \$75, \$82.50, \$100, \$50, \$300, \$40, \$125. \$500; drawn by checks, \$13.75, \$62, \$5, \$125.50, \$82.75. He needs \$2000 to start him in business, and wishes to keep about \$500 in the bank. How much must he borrow, to the nearest \$100?

17. If he makes out a note for this amount for 90 days at 6%, how much discount must he pay? What are the proceeds? What are they for 60 days?

18. A merchant has to pay a debt of \$1275 less 10%. He has in the bank \$672.80, and has \$127.50 in cash in his safe. He wishes to leave about \$300 in the bank and about \$65 in his safe. How much must he borrow, to the nearest \$50? Discount the note for 30 days at 6%.

451. Discounting notes. If a dealer buys some goods for the fall trade, but does not wish to pay for them until after the holidays, he may buy them on credit, giving his note. The manufacturer may need the money at once, in which case he will indorse the note and sell it to a bank for the sum less the discount.

452. Illustrative problem. If you give a manufacturer your note dated September 1, due on January 1, for $500, with interest at 5%, and he, needing the money, discounts it at the bank on September 16 at 6%, what are the proceeds?

Face of the note	=	$500.
Interest 4 mo., 5%	=	8.33
Amount at maturity	=	$508.33
Discount 107 da., 6%	=	9.07
Proceeds,		$499.26

Banks usually compute the discount period in days. In some parts of the country both the first and last days are included in the discount period, thus making it 108 days in the above example. Teachers should be guided by the local custom.

ORAL EXERCISE

State the following:

1. 5% of $70.
2. 5% of $75.
3. 5% of $65.
4. 4% of $55.
5. 4% of $35.
6. 4% of $95.
7. 6% of $75.
8. 6% of $85.
9. 6% of $65.
10. 3% of $125.
11. 3% of $225.
12. 3% of $365.
13. 2% of $42.50.
14. 2% of $63.50.
15. 2% of $27.50.
16. 4% of $20.25.
17. 4% of $15.25.
18. 4% of $41.25.
19. 5% of $15.20.
20. 5% of $20.40.
21. 5% of $30.60.
22. 6% of $25.50.

WRITTEN EXERCISE

1. A note for \$375, at 5%, has a payment of \$18.75 indorsed annually at the close of each year from its date, for 3 years. How much is due at the end of the fourth year?

2. A note for \$500, at 6%, is dated January 1, 1906, and has the following partial payments indorsed: July 2, 1906, \$100; January 1, 1907, \$100. How much is due July 1, 1907?

3. A note for \$300, at 6%, is dated April 1, 1906, and has the following partial payments indorsed: September 1, 1906, \$50; February 16, 1907, \$75; January 1, 1908, \$100. How much is due July 1, 1908?

4. A note for \$1800, at 5%, is dated April 16, 1906, and has the following partial payments indorsed: July 2, 1906, \$500; October 1, 1906, \$250; February 1, 1907, \$100; May 1, 1907, \$375. How much is due September 5, 1907?

5. A note for \$750, at 4%, is dated June 15, 1906, and has the following partial payments indorsed: March 6, 1907, \$200; September 6, 1907, \$150; January 17, 1908, \$250. How much is due on the day of settlement, May 14, 1908?

6. A note for \$600, at 5%, is dated July 2, 1906, and has the following partial payments indorsed: January 2, 1907, \$50; February 11, 1907, \$75; July 8, 1907, \$200; September 10, 1907, \$80; November 2, 1907, \$50. How much is due January 2, 1908?

7. A note for \$1200, at 4%, is dated May 1, 1906, and has the following partial payments indorsed: July 2, 1906, by labor, \$5; July 9, 1906, by labor, \$2; September 1, 1906, \$93; May 1, 1907, \$450; November 1, 1907, \$500. How much is due January 1, 1908?

8. A note for \$750, at 6%, is dated February 2, 1907, and has the following partial payments indorsed: June 2, 1907, \$150; July 17, 1907, \$75; February 2, 1908, \$100. How much is due August 2, 1908?

9. A note for \$575, at 6%, is dated January 15, 1906, and has the following partial payments indorsed: May 7, 1906, \$30; August 13, 1906, \$50; December 18, 1906, \$75. How much is due January 30, 1907?

10. A note for \$625, at 5%, is dated November 13, 1906, and has the following partial payments indorsed: February 14, 1907, \$35; April 17, 1907, \$75.50; October 23, 1907, \$50; March 19, 1908, \$135. How much is due September 23, 1908?

11. A note for \$175, at 6%, is dated December 9, 1907, and has the following partial payments indorsed: February 4, 1908, \$25; May 27, 1908, \$63.50; September 2, 1908, \$15; February 25, 1909, \$20. How much is due April 22, 1909?

12. A note for \$675.50, at 5%, is dated August 5, 1907, and has the following partial payments indorsed: January 7, 1908, \$50; August 5, 1908, \$50; March 18, 1909, \$35; July 15, 1909, \$75; November 11, 1909, \$130. How much is due January 7, 1910?

13. A man owes another \$750 for a village building lot, and he gives his note at 6%, due on demand, with the understanding that he should pay part of it by labor. The note is dated April 15, 1908, and the following partial payments are indorsed: July 22, 1908, 4 days' labor @ \$3; September 16, 1908, 3½ days' labor @ \$3; January 21, 1909, 4½ hours' labor @ 50¢; March 11, 1909, 11½ days' labor @ \$3; July 15, 1909, 37 days' labor @ \$3; September 16, 1909, cash, \$350. How much is due January 1, 1910?

455. Merchants' rule. Where a note or account runs a year or less, and partial payments have been made, business men often compute the balance due by means of a rule sometimes called *The Merchants' Rule*, as follows:

1. *Find the amount of the principal at the time of settlement.*

2. *Find the amount of each payment from the time it was made until the time of settlement.*

3. *From the amount of the principal subtract the amount of the payments.*

The rule, not being as fair for longer periods as the United States Rule, is not legal. But for short periods it is easier and gives nearly the same result.

456. Illustrative problem. A note for $100 made January 1 has the following payments indorsed: March 1, $10; April 1, $25. Find the balance July 1, allowing 6% interest.

Amount of $100 for 6 mo.		$103
" " $10 " 4 "	$10.20	
" " $25 " 3 "	25.38	
		35.58
Balance,		$67.42

WRITTEN EXERCISE

Notes for the following amounts have indorsed the payments indicated. Find the balance at the date of settlement, using the Merchants' rule, and allowing 6% interest:

1. $875, April 1. Indorsements: $300, May 1; $125, June 15; $275, July 25. Settled November 20.

2. $250, February 1. Indorsements: $30, March 1; $70, May 15; $50, July 10. Settled September 15.

3. $750, March 1. Indorsements: May 10, $100; July 12, $75; September 4, $40. Settled October 15.

TRADE DISCOUNT

457. Advantage of trade discount. Dealers wish to buy their goods at as great a discount as possible. When they pay cash for them they get the best price, because a special discount is usually allowed for cash.

458. Several discounts. We have already met cases in this series of arithmetics in which more than one discount was allowed. Sometimes more than two are allowed, new discount lists being sent to retail dealers as the cost of production changes.

ORAL EXERCISE

1. What is the cost of goods listed at \$300, 5% off?
2. What is the cost of goods listed at \$600, at $\frac{1}{5}$ off?
3. What is the cost of goods listed at \$250, 10% off?
4. If on a bill of goods amounting to \$250 a discount of 20%, 10% is allowed, what is the net price?

\$250 − \$50 = \$200; \$200 − \$20 = \$180.

State the cost of goods listed as follows, less the discount:

5. \$250, 10%, 4%.
6. \$200, 5%, 1%.
7. \$300, 10%, 1%.
8. \$250, 20%, 4%.
9. \$100, 10%, 10%.
10. \$500, 20%, 6%.
11. \$200, 15%, 10%.
12. \$400, 25%, 10%.
13. \$1250, 20%, 3%.
14. \$660, $33\frac{1}{3}$%, 5%.
15. \$400, $12\frac{1}{2}$%, 2%.
16. \$1000, 15%, 2%.
17. \$300, $33\frac{1}{3}$%, 10%.
18. \$600, $33\frac{1}{3}$%, 10%.
19. \$600, $16\frac{2}{3}$%, 10%.
20. \$500, 20%, $12\frac{1}{2}$%.
21. \$1000, 10%, 10%.
22. \$540, $16\frac{2}{3}$%, 10%.

459. Sample price list. The following is a price list of certain school supplies, with the discounts allowed to schools and dealers where the prices are not net:

Rulers,	$0.35	per	doz.,	net
Composition books,	4.50	"	gross,	less 5%
Thumb tacks,	0.40	"	100,	" 40%
Drawing pencils,	4.71	"	gross,	" 20%
Drawing paper 9 × 12,	1.10	"	package,	less $\frac{1}{10}$
Pens,	0.61	"	gross,	less $\frac{1}{4}$, 10%
Tubes of paste,	4.05	"	"	" 10%, 5%
Penholders,	3.20	"	"	" 12%, 5%
Drawing compasses,	1.65	"	doz.,	" 10%, 5%

WRITTEN EXERCISE

1. How much must your school pay for $\frac{1}{2}$ gross of penholders? for $\frac{1}{4}$ gross? for a gross?

2. The school wishes to buy 5 packages of drawing paper and 300 thumb tacks. What will they cost?

3. What will 8 gross of pens and $\frac{1}{2}$ gross of composition books cost? 2 gross of pens and 1 doz. rulers?

4. There are 40 pupils in a class, and each needs drawing compasses and a ruler. What will they cost the school? Suppose there were 60 pupils?

5. If a dealer sells pens at a cent apiece, what does he gain per gross? If he sells penholders at 2 ct. each, what does he gain per gross?

6. A dealer buys a gross of rulers and a gross of drawing pencils. He sells both at 5 ct. each. What does he gain on the lot? Suppose he buys 2 gross of each?

7. If a dealer buys a gross of tubes of paste for mounting pictures, and sells the tubes at 5 ct. each, what does he gain on the purchase? How much on 3 gross?

460. Ordering goods. The following is a model order:

WOOD & ROBERTS
BOOKSELLERS

MEMPHIS, TENN., Feb. 15, 19 07

Messrs. Ginn & Company, Publishers,
378 Wabash Ave., Chicago, Ill.

Dear Sirs: Please send at once, by express,
75 Smith's Practical Arithmetics,
60 " Primary Arithmetics.

Yours truly,
WOOD & ROBERTS

461. Model bill. The following is the bill that Ginn & Company would send in reply:

Messrs. Wood & Roberts,
Memphis, Tenn. Feb. 16, 19 07

Bought of GINN & COMPANY
Educational Publishers
378 WABASH AVE., CHICAGO

Terms of this Invoice: *Net Cash*

75	Smith's Practical Arithmetics,	$.65	$48.75
60	" Primary Arithmetics,	.35	21.00
			$69.75
		1/6	11.62
			$58.13

It is customary to state per cents like $16\frac{2}{3}\%$, $12\frac{1}{2}\%$, 20%, and 25% in the common-fraction form, as above.

WRITTEN EXERCISE

Write both orders and bills for the following goods bought of yourself, by the person named or by some one you know:

1. Bought 2 doz. tennis rackets @ \$21; 8 doz. balls @ \$3; $\frac{3}{4}$ doz. tennis nets @ \$19.20. Discount 12%, 5%, 2%.

2. Henry James, Des Moines, Iowa: $\frac{1}{2}$ doz. rugs @ \$84; 750 yd. carpet @ 82¢; 3 doz. hassocks @ \$6. Discount $\frac{1}{5}$.

3. Bought 12 doz. tumblers @ \$1.05; 35 doz. dinner plates @ \$2.50; $1\frac{1}{2}$ doz. sugar bowls @ \$9.60. Discount $\frac{1}{6}$.

4. Jones & Co., 3497 Wabash Ave., Chicago: 2 bbl. P. R. molasses @ \$12; 200 lb. Rio tapioca @ 6 ct.; 150 lb. Mocha coffee @ 20 ct. Discount 6%.

5. Bought 5 doz. bottles of ink @ \$3; 300 lb. paper @ 30 ct.; 2 doz. bottles of mucilage @ \$4; 6 gross pencils @ \$2.75. Discount 3%, 2%, 1%.

6. George Lloyd, Lincoln, Neb.: 18 brass bedsteads, No. 142, @ \$17.25; 15 woven wire mattresses, No. 16, @ \$3.45; 3 sideboards, No. 196, @ \$17.25. Discount $\frac{1}{6}$.

7. Newton & Co., 2831 Spring St., New York: 21 Ohio steers, 1226 lb. (average), @ \$5.10 (per 100 lb.); 20 do. (ditto = the same), 1247 lb., @ \$5.10; 20 do., 1112 lb., @ \$4.95. Discount $\frac{1}{10}$.

8. Sherman & Culver, New York: 20 Illinois steers, 1157 lb. (see Ex. 7), @ \$4.60; 28 do., 996 lb., @ \$4.12$\frac{1}{2}$; 20 Kentucky steers, 1230 lb., @ \$4.80; 7 Indiana do., 1078 lb., @ \$4.70. Discount $\frac{1}{10}$.

9. R. B. Homer, Buffalo: 75 180-lb. bags of Western potatoes @ \$1.48; 50 150-lb. do. @ \$1.28; 125 168-lb. do. @ \$1.42; 1620 lb. Jersey potatoes in bulk @ \$1.47 per 180 lb. Discount $\frac{1}{6}$.

SIMPLE ACCOUNTS

462. The debit column. In his account a man charges his income to himself, and places it in the left column. He is debtor (*Dr.*) to himself for this column.

463. The credit column. His expenditures he credits to his account, and places them in the column marked *Cr.*

19.....		Dr.		Cr.	
Jan. 7	Cash on hand	7	45		
" 7	Carfare .10, pencil .05				15
" 8	Drawing book .15, ink .10				25
" 9	Geography			1	12
" 12	Paper .10, ruler .05				15
" 14	Balance			5	78
		7	45	7	45

WRITTEN EXERCISE

1. Make out the following account for a day. Cash on hand, $174.30; Receipts, mdse., $12.50, $6.75, $0.42, $17.30, $9.50, $42.75; Expenses, Perry & Co. bill, $75.82.

2. Make out the following account for a week. Cash on hand, $21.30; Receipts, from apples sold, $7.50; Eggs, $3.25; Poultry, $6.90. Disbursements, Horseshoeing, $0.75; Repairs to wagon, $1.20; Shingles, $4.20; Seed, $3.50.

3. Make out the following account for a week. Cash on hand, $1.75; Receipts, weekly allowance, $1. Expenses, Writing paper, 10 ct.; Pencils, 8 ct.; Ink, 10 ct.; Compasses, 10 ct.; Arithmetic, 90 ct.; Penholder, 3 ct.; Eraser, 5 ct.

Pupils should be required to make out imaginary accounts.

Make out the following accounts for last week, inserting the proper year, month, and day of the month:

4. Dr., Monday, Cash on hand, $1.05; Allowance for house expenses, $6; Thursday, Sewing for Mrs. Graham, $1. Cr., Tuesday, Groceries, $1.35; Meat, 85¢; Wednesday, Meat, $1.08; Thursday, Groceries, $2.30; Saturday, Meat, $1.12.

5. Dr., Monday, Cash on hand, $17.50; Eggs sold, $4.20; Tuesday, Corn sold, $26.50; Poultry sold, $6.75; Thursday, 3 T. 750 lb. hay sold @ $8. Cr., Tuesday, 4 days' wages, John Cobb, @ $2.50; Wednesday, Grocery bill, $7.28; Friday, Repairs, $1.60.

6. Dr., Monday, Cash on hand, $26.70; Saturday, Wages, $15. Cr., Monday, Allowance for house, $7.50; Carfare, 10¢; Tuesday, Carfare, 10¢; Wednesday, Rent, $20; Carfare, 10¢; Thursday, Carfare, 10¢; Friday, Shoes, $3; Saturday, Suit for Rob, $6.40.

7. Dr., Monday, Cash on hand, $9.75; Allowance for house expenses, $10; Thursday, Sale of old suit, $1.25. Cr., Tuesday, Meat bill, 95¢; Carfare, 15¢; Grocer, $1.68; Wednesday, Meat bill, $1.30; Grocer, $2.15; Thursday, Laundry, 60¢; Gas bill, $1.80; Friday, Ice, $1.60; Grocer, $3.60; Meat bill, $1.42; Broom and ironing board, $1.10.

8. Dr., Monday, Cash on hand, $278.50; Sales, $72.80; Tuesday, Sales, $98.75; Wednesday, Sales, $126.40; Sale of old show case, $6.50; Thursday, Sales, $82.75; Friday, Sales, $110.62; Saturday, Sales, $96.30. Cr., Monday, Gorham bill, $168.40 less 10%; Tuesday, Whiting bill, $68.90 net; Barton bill, $137.50 less 6%; Saturday, Frank's wages, $12.50; Morgan's wages, $15; Hulbert's wages, $14.

PARTNERSHIP

464. Partitive proportion. If partners invest equal sums and contribute equally in work, the profits are divided equally. Otherwise partitive proportion (page 381) is employed.

465. Partners usually have an annual settlement. Money withdrawn by any partner during the year is charged to him with interest at the settlement.

466. Illustrative problem. Brown, Edgcomb, and Thomas form a partnership on February 1, Brown putting in \$2500, Edgcomb \$4000, and Thomas \$3500. What is the share of each in \$6000 profits on the following February 1?

1. Since the total capital is \$10,000, and Brown put in \$2500,
   Brown's share is $\frac{2500}{10000}$, or $\frac{5}{20}$,
   Edgcomb's " " $\frac{4000}{10000}$, " $\frac{8}{20}$,
   Thomas's " " $\frac{3500}{10000}$, " $\frac{7}{20}$.
2. Therefore Brown receives $\frac{5}{20}$ of \$6000, or \$1500
   Edgcomb " $\frac{8}{20}$ " " " \$2400
   Thomas " $\frac{7}{20}$ " " " \$2100
   The total being 1 \$6000

ORAL EXERCISE

Separate into two parts having the given ratio:

1. \$150, 1 : 2.
2. \$500, 2 : 3.
3. \$400, 3 : 1.
4. \$500, 1 : 3.
5. \$250, 2 : 3.
6. \$350, 3 : 4.
7. \$180, 4 : 5.
8. \$330, 4 : 7.
9. \$560, 3 : 5.
10. \$660, 8 : 3.
11. 140 ft., 5 : 9.
12. 132 yd., 7 : 5.
13. 600 ft., 8 : 7.
14. 450 yd., 7 : 8.
15. 420 rd., 10 : 11.
16. 260 mi., 11 : 15.

WRITTEN EXERCISE

1. X, Y, and Z invest $345, $625, and $730, respectively. They make $153. What is the share of each?

2. A, B, and C invest $2200, $3350, and $1650, respectively. They make $1440. What is the share of each?

3. Roberts, Jacobs, and Jameson invest $2700, $3250, and $2050, respectively. They make $2400. What is the share of each?

4. A, B, and C pay $195 irrigation taxes for their farms. A has 250 acres, B 180 acres, C 220 acres. What is the share of each?

5. X, Y, and Z pay $1085 for some water power. X uses 30 horse power, Y 45 horse power, Z 80 horse power. What should each pay?

6. Three men rent a summer cottage. The first occupies it 5 weeks, the second 4 weeks, and the third 3 weeks. At $264 for the season, what should each pay?

7. Day and McFarlane pasture some cattle in a field, Day putting in 62 head for 7 weeks, and McFarlane 48 head for 5 weeks. The bill for pasturage being $101.10, how much should each pay?

8. Two contractors for rock excavating in a tunnel run 16 and 21 drills respectively, getting compressed air from the same engine. If the expense of furnishing the compressed air is $555, what is the share of each?

9. Two farmers whose lands join have a windmill and tank in common. They pay for the annual repairs according to the greatest number of head of cattle kept by each at any time during the year. If they have 37 and 48 head respectively, and the repairs amount to $25.50, what is the share of each?

10. Messrs. Brown and Jones form a partnership, Brown furnishing \$4200 and Jones furnishing \$3300. After a year Brown puts in \$500 more. At the end of 2 years they sell out for \$9300. How much should each receive?

11. Ayres and Ives go into partnership, Ayres putting in \$3500 and Ives \$6500. Ives is to give all of his time to the business and take out \$2000 before any division of profits. If they make \$2500 this year, to how much of it is each entitled?

12. Glover and Staughton go into partnership, Staughton putting in \$7000 and Glover \$9000. Glover doing no work, it is agreed that Staughton shall take \$2000 a year from the profits before a division. The profits last year were \$6400. What was the share of each?

13. Webb, Bull, and Smith form a partnership, Webb putting in \$8000, Bull \$3000, and Smith \$5000. It is agreed that Bull shall contribute no work, but that Webb shall receive \$1500 and Smith \$900 before the rest of the profits are divided. The year's profits were \$7200. How much did each receive?

14. Messrs. Davids, Glover, and James buy a house for \$9500. Davids puts in \$3200, and Glover and James put in the rest, Glover putting in half as much as James. They rent the house this year for 10% of its cost, and pay \$15 for insurance, \$40 for taxes, and \$40 for repairs. What is the share of each in the net receipts?

15. Wood, Wallace, and Harris own a spring, and pay for repairs to the spring house and piping according to the amount of water used by each, as shown by their water meters. If the respective amounts used during a given period are 34,000 gal., 26,500 gal., and 42,500 gal., what is the share of each in repairs amounting to \$24.72?

EXCHANGE

467. Paying bills at a distance. If a man owes money to some one living in another place, he may send it by a registered letter. He will be more likely, however, to pay his bill by a check, a money order, or a draft.

468. Exchange. The payment of money by means of checks, money orders, or drafts is called *exchange.*

469. Paying by check. If a man sends his check for the amount, the one to whom he owes the money will deposit it in the bank where he keeps his account. This bank will send it to the debtor's bank for collection, and will probably charge a small amount for the trouble.

470. Paying by money order. A money order may be purchased at the post office, or from an express company, or it may, at considerable expense, be telegraphed. Postal or express orders may be sent to the creditor, who can then obtain the money at his post or express office. The extra cost of postal money orders is as follows:

For sums not exceeding \$2.50, 3 cents; above this, not exceeding \$5, 5 cents; above this, not exceeding \$10, 8 cents; above this, not exceeding \$20, 10 cents; above this, not exceeding \$30, 12 cents; above this, not exceeding \$40, 15 cents; above this, not exceeding \$50, 18 cents; above this, not exceeding \$60, 20 cents; above this, not exceeding \$75, 25 cents; above this, not exceeding \$100, 30 cents.

ORAL EXERCISE

Referring to the above list, state the cost of money orders for:

1. \$31.50.	2. \$52.75.	3. \$92.30.	4. \$16.30.
5. \$19.90.	6. \$86.50.	7. \$69.95.	8. \$90.50.

471. Paying by bank draft. One of the most common methods of paying a debt in another place, particularly debts of large size, is by means of the bank draft.

No. 48104

MERCHANTS NATIONAL BANK OF AUSTIN

AUSTIN, TEXAS, July 6, 1907

Pay to the order of ..... John Roberts ..... $78.$\frac{75}{}$

Seventy-eight and $\frac{75}{100}$ ~~~~~~~~~~~~~~~~~~~~ Dollars

To The Chemical National Bank,
New York City

A. B. Smith
Cashier

A draft is therefore the same as a check, except that it is made by the cashier of some bank and is drawn on another bank.

Banks usually charge a slight premium on the face of the draft. Thus, a $250 draft at 0.1% premium would cost $250 + 0.1% of $250 = $250.25.

If John Roberts, who purchased the above draft, owed Robert Jones the money, he would indorse it, thus:

Pay to the order of Robert Jones
John Roberts

It might also have been made payable directly to the order of Robert Jones in the first place, but this is not the custom.

Drafts on large money centers are usually cashed for customers of the bank without any discount.

472. Illustrative problem. What would the above draft cost at 0.1% premium?

0.1% of $78.75 = $0.08, to the nearest cent.

The bank, however, would probably charge 10¢ to make a convenient amount. Therefore it would cost $78.85.

WRITTEN EXERCISE

1. What will a draft for \$300 cost at $\frac{1}{5}\%$ premium?

2. What will a draft for \$3200 cost at 0.1% premium? for \$2500 at $\frac{1}{20}\%$ premium?

3. What will a postal money order for \$37.50 cost? for \$62.75? for \$14.30? for \$86.50? for \$75.40?

4. What will a New Orleans merchant pay for a draft on Chicago for \$2750 at 40¢ premium per \$1000?

5. When the government charges 30¢ per \$100 for a money order, what per cent premium does it charge?

6. A draft cost a merchant \$2752.75, including 0.1% premium. What was the face of the draft? What was the premium? Write the draft.

7. A draft cost a merchant \$3751.50, including the premium of 40¢ per \$1000. What was the face of the draft? What was the premium?

8. If a man owes to different jobbers from whom he buys goods \$250, \$150, \$100, and \$350, what will drafts for these amounts cost at 0.1% premium?

9. Which is cheaper for you, if you owe \$75 for some goods, to send a money order, or a draft for which you have to pay 15¢ premium? How much cheaper?

10. It costs 8¢, besides postage, to register a letter. If it is not delivered the government will pay the loss, up to \$25. If you owe \$18, is it cheaper for you to send it by registered letter or by money order? How much cheaper?

11. If you owe \$100 to a manufacturer at a distance, from whom you have bought goods, how will you make the payment? Tell why you will make it in that way, and how much it will probably cost. (The premium on drafts may be taken at the common rate of 0.1%.)

473. Commercial drafts. A creditor sometimes *draws* directly on a debtor, the draft being of this form:

DAYTON, OHIO, *Feb. 15*, 19*10*

At sight pay to the order of

The First National Bank of Dayton $*735.80*

Seven hundred thirty-five and $\frac{80}{100}$ Dollars

To *The National Cash Register Co.*

James X. Wye, *Per J. K. S.*

Cleveland, Ohio

474. Parties to a draft. Here the National Cash Register Company is the *drawer;* Wye is the *drawee;* The First National Bank is the *payee.*

475. The Register Company deposits this draft with the First National Bank. The bank sends it to some bank in Cleveland. The Cleveland bank sends a messenger to Mr. Wye for the money, and, having collected it, sends the money (or its equivalent) to the Dayton bank. The Dayton bank then notifies the company that it is paid, and the amount, less some slight commission, the *proceeds* of the draft, is added to the company's account.

WRITTEN EXERCISE

1. Brown & Co. draw on J. H. Brownson for $750. The banks charge 0.1% for collection. What are the proceeds?

2. The Electric Company draws on Mr. X for $550. The banks charge 0.1% for collection. What are the proceeds?

3. The Arithmetic Publishing Company draws on R. T. Jewett for $150. The banks charge 0.2% for collection. What are the proceeds? Write the draft.

4. Robertson Bros. draw on J. P. Shipley for $37.50. The charges for collection are 10¢. This is what per cent of the face?

5. M. D. St. John collects a $150 debt through the bank, the proceeds being $149.70. What is the bank's rate for collecting this debt?

6. S. L. James of Des Moines draws on L. D. Richards of Cedar Rapids, Iowa, for $75. The bank charges $\frac{1}{5}\%$ for collecting. What are the proceeds?

7. M. T. Snell of Cleveland is the drawee of a $200 draft; The Farmers Trust Co. of Detroit is the payee; The World Soap Co. is the drawer. Write the draft.

8. A. B. Stanley owes M. S. Stanton for 2 doz. suits @ $113.50; $1\frac{1}{2}$ doz. suits @ $130; 2 doz. overcoats @ $139; $7\frac{1}{2}$ doz. pairs trousers @ $40. Stanton draws on Stanley for the money, and the bank charges him 50¢ for collecting. What rate is this?

9. Suppose William Bentley of Winnipeg, Manitoba, owes you $250, and you wish to draw upon him for this amount. Write out a draft, payable to the order of some bank near your home. If the bank charges $\frac{1}{10}\%$ for collecting, what are the proceeds?

10. R. H. Dudley owes Cayley & Co. for 4 dressers @ $14, and 6 washstands @ $3.50, all less 15%, and a bedroom set at $34.55 net. They draw on him for the amount, the bank charging 10¢ for collecting. What is the net amount received by Cayley & Co.?

11. R. J. Doane of Montreal owes A. D. Kane of Pittsburg for 10 steel girders @ $50, and tells the latter to draw upon him for the amount. Mr. Kane keeps his account at the Iron Exchange Bank. Make out a draft for Mr. Kane. What is the bank's charge for collecting, at $0.1\frac{1}{2}\%$?

476. The rates of exchange. A money order is always sold at a slight advance over its face, and usually a bank draft costs more than its face. In each case the variation from the face is called the *rate of exchange.*

477. Premium. If the rate of exchange is added to the face, exchange is *at a premium,* as we have already seen.

478. Par. If there is no rate of exchange, exchange is *at par.*

479. Discount. If the rate of exchange is subtracted from the face, exchange is *at a discount.*

480. For small sums, say for $500 or less, New York, Chicago, or Philadelphia exchange usually sells at a premium of about 0.1%. This is to pay the bank for its trouble and for the expense of shipping the money when its balance at these cities gets low. Banks usually buy such drafts at their face value, thus making no charge for cashing them.

481. But on large sums the rate of exchange varies. If the Chicago banks owe the New York banks $2,000,000, they must send that amount by express, an expensive proceeding. If a man in Chicago at that time wished to buy a draft on New York for $30,000, they would charge him more than usual because they would have to express that much more to New York. But if a man in New York wished to buy a draft on Chicago, he might buy it for less than $30,000 because the bank would get its money at once and the risk and expense of transmitting it would be saved.

482. The premium or discount is usually quoted as a certain per cent of the face of the draft, but sometimes as so much on $1000. In the latter case the quotation of $\frac{1}{4}\%$ premium is the same as that of $2.50 premium.

The explanation of the *Clearing House*, a place where bank officials of a city meet daily to exchange drafts and checks, and to pay balances due one another, is too technical for most classes, and if given at all should be explained verbally by the teacher.

WRITTEN EXERCISE

Find the cost of the following drafts:

1. \$3756.70, at par.
2. \$3500, 0.1% discount.
3. \$750, 0.2% premium.
4. \$6750, $\frac{1}{20}$% premium.
5. \$2450, 0.1% premium.
6. \$17,500, 0.2% discount.

7. What is the cost of a draft on San Francisco for \$5200 at $\frac{3}{4}$% discount? Write the draft.

8. What must be paid in Memphis for a draft on Chicago for \$3400, exchange being at $\frac{1}{4}$% premium?

9. A draft for \$4800 was bought for \$4794. Was exchange at a premium or a discount? What was the rate?

10. When a Boston draft for \$35,000 can be bought in New Orleans for \$34,930, is exchange at a premium, at par, or at a discount? What is the rate?

11. If J. R. Glover draws on J. B. Thornton for \$250, and the banks charge 0.1% for collection, what are the net proceeds that Glover will receive? Write the draft.

12. My agent in Toronto sells a house for me for \$2500. He charges 2% commission, and the bank charges \$2.50 premium for a draft for any amount between \$2000 and \$2500. What sum does he remit to me?

13. A telegraphic money order costs twice the rate for a ten-word message, plus 1% premium on the face. A ten-word message from Kansas City to Albany costs 50 ct. What will a telegraphic money order for \$375 cost?

14. Mr. Edgcomb of Denver owes Mr. Nourse \$3243.24 in that city. Mr. Nourse has gone to New York on business and asks that the money be sent, less the cost of exchange. Exchange being at 0.1% premium, what is the face of the draft?

483. Foreign exchange. If a man buys foreign goods, he often has to send money abroad.

484. Table of English money:

12 pence (d.) = 1 shilling (s.) = \$0.243 +.
20 shillings = 1 pound (£) = \$4.8665.

We commonly think of the pound as about \$5, the shilling as about 25¢, and the penny as about 2¢. Canada uses the same table of money as the United States.

485. Table of French money:

100 centimes (c.) = 1 franc (fr.) = \$0.193.

We commonly think of the franc as 20¢.

This system is used in several European countries.

486. Table of German money:

100 pfennigs (pf.) = 1 mark (M.) = \$0.238.

We commonly think of the mark as 25¢, and 4 pf. as 1¢.

ORAL EXERCISE

Taking the above approximate values, state about how much the following sums represent in our money:

1. £7.	2. £6 5s.	3. £9 4s.	4. £8 10s.
5. £50.	6. £10 16s.	7. £12 2s.	8. £15 12s.
9. £150.	10. £50 4s.	11. £60 7s.	12. £90 9s.
13. 50 fr.	14. 75 fr.	15. 125 fr.	16. 300 fr.
17. 800 fr.	18. 1200 fr.	19. 80 M.	20. 100 M.
21. 240 M.	22. 640 M.	23. 840 M.	24. 1200 M.
25. £8 4s. 6d.	26. £7 2s. 4d.	27. £5 7s. 9d.	
28. 50 fr. 50 c.	29. 75 fr. 25 c.	30. 80 fr. 20 c.	
31. 8 M. 20 pf.	32. 10 M. 40 pf.	33. 80 M. 4 pf.	

34. Express \$40 approximately as German money; as English money; as French money.

WRITTEN EXERCISE

Express as pence:

1. £3. 2. £2 4s. 3. £3 5s. 6d.
4. 3s. 8d. 5. 18s. 3d. 6. £5 1s. 2d.

Express as shillings and decimals:

7. 8s. 8d. 8. 9s. 3d. 9. £3 6s. 9d.

Express as pounds and decimals:

10. £2 2s. 11. £3 4s. 12. £6 4s. 6d.

Express as centimes:

13. 25 fr. 14. 37 fr. 15. 35 fr. 30 c.

Express as francs and decimals:

16. 275 c. 17. 475 c. 18. 1275 c.

Express as marks and decimals:

19. 350 pf. 20. 480 pf. 21. 9275 pf.

Express as pfennigs:

22. 175 M. 23. 200 M. 24. 10 M. 75 pf.

Taking £1 *as equal to* \$4.87, *express in our money:*

25. £75. 26. £68. 27. £16 10s.

Taking £1 *as equal to* \$4.87, *express in English money:*

28. \$38.96. 29. \$24.35. 30. \$43.83.

Taking 1 M. *as equal to* 23.8¢, *express in our money:*

31. 75 M. 32. 125 M. 33. 3750 M.

Taking 1 M. *as equal to* 23.8¢, *express in German money:*

34. \$9.52. 35. \$14.28. 36. \$71.40.

Taking 1 fr. *as equal to* 19.3¢, *express in our money:*

37. 85 fr. 38. 230 fr. 39. 750 fr.

487. Bills of exchange. Foreign drafts are also called *bills of exchange.*

488. Rate of exchange. The rate of foreign exchange varies continually, depending on the demand.

Thus, when English exchange is quoted at 4.90 (that is, a draft for £1 costs $4.90) it is above par, for 4.8665 is par.

489. Foreign quotations. English exchange is quoted at dollars to the pound, as 4.92.

French exchange is quoted either at francs to the dollar, 5.14 meaning that $1 will buy a draft for 5.14 fr., or at cents to the franc, 19.7 meaning that a draft for 1 fr. costs 19.7 ct.

German exchange is quoted at cents to the 4 marks, 97 meaning that a draft for 4 M. costs 97 ct., or at cents to the mark, 24.1 meaning that a draft for 1 M. costs 24.1 ct.

490. At present much of the foreign exchange for small sums is carried on by post-office or express money orders.

491. Newspaper quotations. Exchange is often quoted thus:

	Demand	*60 Days*	*90 Days*
Sterling	4.83	4.79	4.77
Francs	5.20	5.21	5.23
Marks	$94\frac{5}{8}$	$94\frac{3}{8}$	$93\frac{7}{8}$

This means that a £1 draft on England (sterling) will cost $4.83 if payable on demand (at sight), $4.79 if payable 60 days after sight, or $4.77 90 days after sight. The quotations for the following examples may be taken as above or from a newspaper.

ORAL EXERCISE

State the cost of demand drafts for:

1. £10.	2. £5.	3. £2.	4. £1000.
5. 5.20 fr.	6. 52 fr.	7. 104 fr.	8. 5200 fr.
9. 40 M.	10. 400 M.	11. 4000 M.	12. 40,000 M.

492. Illustrative problems. 1. What will a 60-day draft for £40 cost?

1. £1 costs \$4.79.
2. £40 cost 40 times \$4.79 = \$191.60.

2. What will a demand draft for 75 M. cost?

1. 4 M. cost \$0.94⅝, 1 M. costs ¼ of \$0.94⅝.
2. 75 M. cost 75 times ¼ of \$0.94⅝ = \$17.74.

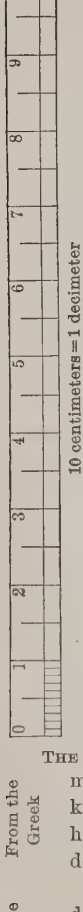
3. What will a 90-day draft for 125 fr. cost?

1. 5.23 fr. cost \$1, 1 fr. costs $\frac{\$1}{5.23}$.
2. 125 fr. cost $\frac{\$125}{5.23}$ = \$23.90.

WRITTEN EXERCISE

1. Find the cost of a 60-day draft for £90 10s.
2. Find the cost of a demand draft for 305 fr. 50 c.
3. Find the cost of a 90-day draft for 750 M.; 620 M.
4. How large a sterling 90-day draft will \$71.55 buy?
5. How large a demand draft on Paris will \$125 buy?
6. How large a 60-day draft on Leipzig will \$76 buy?
7. A merchant buys Scotch tweeds in London to the amount of £127 10s. What will a 60-day draft cost?
8. A city library buys books in Paris to the amount of 578 fr. 31 c. How much will a draft for this amount cost when exchange is quoted at 5.21?
9. I have bought 400 M. worth of books in Leipzig. Which is better for me, to buy a demand draft or an express money order @ 24¢ to the mark?
10. I have bought a painting in Florence for 670 lire (lērā, francs), and 7 lire for packing. If exchange on Florence is 5.18, how much will a draft for this amount cost?

METRIC SYSTEM



493. Where used. If a man buys foreign goods, except from England, the measures will probably be on a more convenient system than ours, one invented in France about 1800, and now used in a large part of the civilized world. It is easily learned, it is much more simple than our system, and we need it in all scientific work and in our newspaper reading. It is called the *Metric System.*

494. Meter. The unit of length is the *meter.* It is nearly 39.37 in. long, or nearly one ten-millionth of the distance from the equator to the pole.

495. Liter. The unit of capacity is the *liter* (lētēr), a cube $\frac{1}{10}$ of a meter on an edge. It is nearly 1 qt.

496. Gram. The unit of weight is the *gram.* It is practically the weight of a cube of water $\frac{1}{100}$ of a meter on an edge. It equals nearly 15.4 grains.

497. The prefixes. The tables are easily learned when the prefixes are known.

Just as 1 mill = $\frac{1}{1000}$ of a dollar,
so 1 millimeter = 0.001 of a meter.
Just as 1 cent = $\frac{1}{100}$ of a dollar,
so 1 centimeter = 0.01 of a meter.
Just as decimal means tenths,
so 1 decimeter = 0.1 of a meter.

	THE PREFIX	MEANS	AS IN	WHICH MEANS	
From the Greek	myria-	10,000	myriameter	10,000	meters.
	kilo-	1000	kilogram	1000	grams.
	hekto-	100	hektoliter	100	liters.
	deka-	10	dekameter	10	meters.
		1		1	
From the Latin	deci-	0.1	decimeter	0.1	of a meter.
	centi-	0.01	centigram	0.01	of a gram.
	milli-	0.001	millimeter	0.001	of a meter.

498. Table of length:

A myriameter	=	10,000	meters.
A kilometer (km.)	=	1000	"
A hektometer	=	100	"
A dekameter	=	10	"
Meter (m.)			
A decimeter (dm.)	=	0.1	of a meter.
A centimeter (cm.)	=	0.01	"
A millimeter (mm.)	=	0.001	"

In the tables the most important names are in heavy type.

499. Approximate values. The meter is about 39.37 in., $3\frac{1}{4}$ ft., or a little over a yard; the kilometer is about 0.6 of a mile.

500. Abbreviations. The abbreviations in this book are recommended by various scientific associations. Some, however, use Km., Dm., dm., for kilometer, dekameter, and decimeter.

ORAL EXERCISE

Express as meters and decimals:

1. 1 km.	2. 65 km.	3. 225 dm.
4. 375 dm.	5. 700 dm.	6. 750 km.
7. 3275 cm.	8. 4550 cm.	9. 6500 cm.
10. 120,000 mm.	11. 216,500 mm.	12. 100,575 mm.

Express approximately as meters (1 m. = $3\frac{1}{4}$ ft. = 39 in.):

13. 13 ft.	14. 39 ft.	15. 65 ft.
16. 3.9 in.	17. 78 in.	18. 390 in.
19. 325 ft.	20. 650 ft.	21. 0.39 in.
22. 9 ft. 9 in.	23. 975 ft.	24. 3 ft. 3 in.

Express approximately as feet, inches, or miles:

25. 8 m.	26. 24 m.	27. $\frac{1}{6}$ km.	28. $\frac{2}{3}$ km.
29. 100 km.	30. 300 km.	31. 60 km.	32. 0.5 m.
33. 30.5 km.	34. 20.5 km.	35. 100 m.	36. 1200 km.

WRITTEN EXERCISE

Express as meters and decimals:

1. 275.3 mm. 2. 476.4 cm. 3. 293.8 dm.

Express as kilometers and decimals:

4. 4862 m. 5. 12,758 cm. 6. 628,341 mm.
7. 47 mi. 8. 26.5 mi. 9. 10,560 ft.

Express as miles:

10. 751 km. 11. 286 km. 12. 34.9 km.
13. 5280 m. 14. 2976 m. 15. 14,781 dm.

Express as feet, taking $3\frac{1}{4}$ ft. $= 1$ m.:

16. 17 m. 17. 64 m. 18. 108 m.
19. 6894 cm. 20. 2986 cm. 21. 81,296 mm.

Express as inches, taking 39.37 in. $= 1$ m.:

22. 47 m. 23. 324 cm. 24. 4680 mm.
25. 34.5 m. 26. 2.83 cm. 27. 3000 mm.

28. Express the diameter of a 7-cm. gun in inches.

29. A certain hill is 203 m. high. Express this in feet.

30. A certain tower in Paris is 37.5 m. high. Express this in feet.

31. The distance from Dieppe (Dē-ep′) to Paris is 209 km. Express this in miles.

32. The distance between two places in Germany is 178 km. Express this in miles.

33. The distance from Paris to Cologne is 306 mi. What is the railway fare at 12 centimes per kilometer?

34. The distance from Paris to Brussels is 326 km., and it takes 7 hours to make the trip by railway. What is the average number of miles an hour?

501. Table of square measure:

A square myriameter	=	100,000,000	square meters.
" kilometer (km^2.)	=	1,000,000	" "
" hektometer	=	10,000	" "
" dekameter	=	100	" "
Square meter (m^2.)			
A square decimeter (dm^2.)	=	0.01	of a square meter.
" centimeter (cm^2.)	=	0.0001	" "
" millimeter (mm^2.)	=	0.000001	" "

The abbreviation sq. m. is often used for m^2.

502. Land measure. The square dekameter is also called an **are** (ăr); and since there are 100 square dekameters in 1 hm^2., a square hektometer is called a **hektare** (ha). The hektare equals 2.47 acres = nearly $2\frac{1}{2}$ acres.

ORAL EXERCISE

Express as square meters:

1. 2 km^2.
2. 30,000 cm^2.
3. 5 square dekameters.
4. 1000 dm^2.
5. 100,000 cm^2.
6. 2 square hektometers.

Express as square dekameters:

7. 5000 m^2.
8. 2 km^2.
9. 5 square hektometers.

Express as hektares:

10. 10,000 m^2.
11. 1 km^2.
12. 5 square hektometers.

Express as hektares, taking 1 ha. = $2\frac{1}{2}$ A.:

13. 5 A.
14. 25 A.
15. 100 A.
16. 50 A.

WRITTEN EXERCISE

Express as square centimeters:

1. 750 km^2.
2. 37 m^2.
3. 4296 mm^2.
4. 6.25 m^2.
5. France has an area of 322,335 $km.^2$ How many acres?

503. Table of cubic measure:

A cubic myriameter	=	10^{12} cubic meters.	
" kilometer	=	10^{9} " "	
" hektometer	=	1,000,000 " "	
" dekameter	=	1000 " "	
Cubic meter (m^3.)			
A cubic decimeter (dm^3.)	=	0.001	of a cubic meter.
" centimeter (cm^3.)	=	0.000001	" "
" millimeter (mm^3.)	=	0.000000001	" "

504. The stere. The cubic meter is also called a **stere** (stēr, st.), a unit used in measuring wood.

ORAL EXERCISE

Express as cubic meters:

1. 17 st. 2. 5000 dm^3. 3. 2,000,000 cm^3.
4. 1000 dm^3. 5. 2 cubic dekameters.

Express as cubic decimeters:

6. 7 m^3. 7. 8 st. 8. 5000 cm^3.
9. 19 m^3. 10. 26 st. 11. 10,000 cm^3.

12. How many centimeters in a meter? square centimeters in a square meter? cubic centimeters in a cubic meter? cubic decimeters in a cubic dekameter?

13. Estimate the length, width, and height of your schoolroom, in meters. How many cubic meters in the room?

WRITTEN EXERCISE

Express as cubic meters:

1. 19.75 cubic dekameters. 2. 427,653.84 mm^3.
3. 37.5 st. + 98.9 st. + 764 st. + 27.43 st. + 196.8 st. + 37 st.
4. 0.000001 cubic hektometer. 5. 0.0000001 km^3.
6. $34\frac{2}{6}\frac{1}{4}$ cu. ft. (calling $3\frac{1}{4}$ ft. equal to 1 m.).

505. Table of weight:

A **metric ton** (t.)	= 1,000,000	grams.
A quintal (q.)	= 100,000	"
A myriagram	= 10,000	"
A **kilogram** (kg.)	= 1000	"
A hektogram	= 100	"
A dekagram	= 10	"
Gram (g.)		
A decigram	= 0.1	of a gram.
A centigram (cg.)	= 0.01	"
A milligram (mg.)	= 0.001	"

The metric ton is nearly the weight of 1 m^3. of water at its greatest density; the kilogram, of 1 dm^3.; and the gram, of 1 cm^3.

506. Approximate values. A kilogram is about $2\frac{1}{5}$ lb. A 5-ct. piece weighs 5 g. A metric ton is nearly 2204.6 lb.

ORAL EXERCISE

Express as grams:

1. 147 cg. 2. 3400 mg. 3. 5200 cg.
4. 348 kg. 5. 2950 kg. 6. 1728 kg.

Express as centigrams:

7. 121 g. 8. 3 kg. 9. 19 kg.
10. 20 mg. 11. 135 mg. 12. 6800 mg.

Express as milligrams:

13. 3.75 g. 14. 4.25 g. 15. 55.2 cg.

Express as kilos (kilograms):

16. 17 t. 17. 3.5 t. 18. 4.25 t.
19. 18,000 cg. 20. 15,000 mg. 21. 2460 g.

Express as pounds:

22. 25 kg. 23. 30 kg. 24. 50 kg.
25. 500 kg. 26. 100 kg. 27. 125 kg.

WRITTEN EXERCISE

Express as kilos (kilograms):

1. 374 lb.	2. 352 oz.	3. 3275 g.
4. 15,428 lb.	5. 48.4 lb.	6. 7275 g.
7. 1 T. 204 lb.	8. 300,000 cg.	9. 173.8 lb.

10. What is the weight of 35 dm^3. of water in pounds?

11. How many 5-ct. pieces will it take to weigh a kilo?

12. Express a gram as a fraction of an ounce; of a pound.

13. In traveling on the continent of Europe, 25 kg. of baggage is usually transported free. How many pounds?

14. What is the weight, in metric tons, of water in a tank 2.5 m. by 3 m. by 1.5 m.? 3.4 m. by 6 m. by 4.25 m.?

15. What is the weight, in metric tons, of water in a tank 5.2 m. by 3.4 m. by $1\frac{3}{4}$ m.? 2.8 m. by 4 m. by 0.75 m.?

16. Steel being 7.8 times as heavy as water, what is the weight of a bar of steel 7.8 cm. wide, 3.1 cm. thick, and 1.5 m. long?

The number 7.8 is called the *specific gravity* of steel. From Exs. 18 and 19 the specific gravity of lead is 11.3, and of gold 19.3.

17. The specific gravity of granite being 2.7, what is the weight of a block of granite 1.2 m. by 0.75 m. by 0.25 m.?

18. Lead being 11.3 times as heavy as water, what is the weight of a bar of lead 3.3 cm. square on the end, and 25.8 cm. long?

19. Gold being 19.3 times as heavy as water, how many grams does a cube of gold 3 cm. on an edge weigh? Express the result also as a fraction of a kilogram.

20. A man traveling abroad steps on a penny-in-the-slot weighing machine and finds he weighs 75 kg. How many pounds does he weigh?

507. Table of capacity:

A hektoliter (hl.)	=	100 liters.
A dekaliter	=	10 "
Liter (l.)		
A deciliter	=	0.1 of a liter.
A centiliter	=	0.01 "
A milliliter (ml.)	=	0.001 "

508. Approximate values. A liter is the same as 1 dm^3. Hence 1 l. of water weighs 1 kg. A liter is practically the same as our quart.

ORAL EXERCISE

Express as liters:

1. 5000 ml.	2. 3 hl.	3. 17 hl.
4. 2500 ml.	5. 7.5 hl.	6. 2.25 hl.
7. 5 gal.	8. 8 pt.	9. $12\frac{1}{2}$ gal.

Express as hektoliters:

10. 250 l.	11. 375 l.	12. 125.5 l.
13. 2575 l.	14. 3500 l.	15. 625.3 l.
16. 300 qt.	17. 2600 qt.	18. 50 gal.

Express as quarts:

19. 175 l.	20. 16.25 l.	21. 4 hl.
22. 20.75 l.	23. 12.50 hl.	24. 0.8 hl.

Express in kilograms the weight of the following amounts of water:

25. 25.5 l.	26. 18.75 l.	27. 16 hl.
28. 17.50 l.	29. 26.50 hl.	30. 0.01 hl.

31. How many decimeters in a meter? How many cubic decimeters in a cubic meter? How many liters in a cubic meter? A cubic meter of water weighs how many kilos?

WRITTEN EXERCISE

1. Milk weighing 1.032 times as much as water, how many kilos does 7 l. weigh?

2. Alcohol being 0.83 as heavy as water, find the weight of 7 l. Express the result in kilos; in pounds.

3. The Eiffel Tower in Paris is 300 m. high. Express this as inches; as feet; as a fraction of 1 km.

4. Gold being 19.3 times as heavy as water, how much does 17.5 cm^3. weigh? Answer in grams; in grains.

5. Cork being one fourth as heavy as water, how much does 9 dm^3. of cork weigh? Answer in kilos; in pounds.

6. The specific gravity of silver (see page 446, Ex. 16) being 10.5, what is the weight of a piece of silver 12.5 cm. by 7.2 cm. by 3.1 cm.?

7. The specific gravity of petroleum being 0.7, what is the weight of 7.34 dm^3. of petroleum?

8. The specific gravity of alcohol being 0.79, what is the weight of alcohol in a tank 5 dm. by 3 dm. by 2.5 dm.?

9. The specific gravity of copper being 8.9, what is the weight of a piece of copper 1 m. by 2 dm. by 9 cm.?

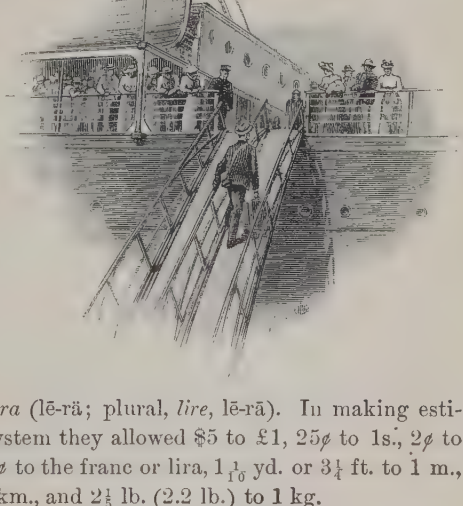
10. One piece of land is 20 m. long and 30 m. wide, and another is 20 yd. long and 30 yd. wide. Find the area of the first in hektares and the second in acres.

11. Samples of merchandise may be sent abroad by mail if they do not weigh more than 350 g. Express this as a fraction of a pound; as ounces; as a fraction of 1 kg.

12. Find in kilos the weight of water in a tank 3 m. by 2 m. by 5 m.; also the weight in pounds of water in a tank 3 ft. by 2 ft. by 5 ft. (1 cu. ft. of water weighs 62.5 lb. Notice the simplicity of the metric system.)

Mr. Field's Trip to Europe

Mr. and Mrs. Field took a vacation trip to Europe, landing at Liverpool, traveling through England, France, Germany, Switzerland, and Italy, and sailing home from Naples. After leaving England they found the Metric System used by every one, and of course they did not meet our system of money until they landed in New York again. They found that Switzerland and France used the same system of money, and that Italy had the same values, but called the franc a *lira* (lē-rä; plural, *lire*, lē-rā). In making estimates in our own system they allowed \$5 to £1, 25¢ to 1s., 2¢ to 1d., 25¢ to 1 M., 20¢ to the franc or lira, $1\frac{1}{10}$ yd. or $3\frac{1}{4}$ ft. to 1 m., $\frac{3}{5}$ mi. (0.6 mi.) to 1 km., and $2\frac{1}{2}$ lb. (2.2 lb.) to 1 kg.



ORAL EXERCISE

1. At \$90 a steamer ticket each way, less 10% on the return portion of the ticket, what did both tickets cost?

2. On the voyage to Liverpool he gave £3 5s. for stewards' fees. How many dollars did he give?

3. He landed in Liverpool at 2 P.M., and at once telegraphed his brother in San Francisco and his sister in New York. Allowing an hour for delay, at what time did the messages reach their respective destinations?

4. He paid 21s. 9d. a ticket from Liverpool to London. How much did he pay for the two tickets? How many dollars?

5. He paid the cab driver in London 2s. 6d. to take them to their hotel. How much is this in our money?

6. He paid 12s. a day for each, at one of the smaller hotels. They remained there a week. How much did it cost the two? How much in our money?

7. He paid 33s. each for two tickets to Paris. How much did he pay for both? How much in our money?

8. At Paris he paid the cab driver 2 fr. 50 c. to take them to their hotel. How much is this in our money?

9. He paid 14 fr. a day for each, at a hotel near the Arch of Triumph, and they spent 11 days in Paris. How much was the hotel bill? How much in our money?

10. The Arch of Triumph is 50 m. high. How many feet high? It is at the end of a beautiful avenue 2.2 km. long. Express this distance in miles.

11. Not far from his hotel is the Eiffel Tower, 300 m. high. Express this height in feet.

12. The distance from Paris to Cologne is 510 km. Express this distance in miles.

13. He bought two tickets at 44 fr. each. How much did they cost? How much in our money?

14. They reached Cologne at 5 : 30 P.M. (Mid-European time, 15° E.). What time (standard) was it then in England? in Boston? in Chicago? in Portland, Oregon?

15. They spent two days in Cologne, paying 12 M. each per day. How much was the hotel bill? How much in our money?

16. At Cologne Mrs. Field did some shopping, spending 150 M. How much is this in our money?

17. Among her purchases was some silk for a gown. She bought 18 m. How many yards?

18. From Cologne they went up the Rhine to Mainz, a distance of 150 km. How many miles?

19. The tickets from Cologne to Mainz were 15 M. 50 pf. each. What did the two cost? How many dollars?

20. They spent two days at Mainz, paying 14 M. each per day. How much was the hotel bill? How many dollars?

21. They then went to Lucerne, a distance of 400 km. How many miles?

22. The tickets to Lucerne cost 32 M. apiece. How much did the two cost? How many dollars?

23. They spent two weeks in Switzerland, their expenses averaging 22 fr. 50 c. apiece per day. How much were they for two? How many dollars? How many dollars for the two weeks?

24. Their tickets from Lucerne right through to Naples, including Florence and Rome, cost 100 fr. apiece. How much for the two tickets? How many dollars?

25. They spent three weeks in Italy, their expenses averaging 20 fr. apiece per day. How much were they for two? How many dollars?

26. Their other purchases and extras in England amounted to £8, in France to 125 fr., in Germany to 120 M., in Switzerland to 60 fr., and in Italy to 80 lire. Express each in dollars.

WRITTEN EXERCISE

Taking the expenses as stated in the Oral Exercise, find the cost of the trip taken by Mr. and Mrs. Field.

TAXES

509. What are taxes? Whatever a man's occupation, one of his expenses will be *taxes*, money paid for the support of the village, town, city, county, or state.

510. Expenses of our government. The expenses of the United States government vary from year to year, but they average about $1,300,000 a day, or $473,500,000 a year. Some items of our income and expenditures are as follows, varying from year to year:

Income:

Customs (duties on imported goods)	$275,000,000
Internal revenue (tobacco, etc.)	275,000,000
Sale of public lands	7,500,000
Miscellaneous	36,000,000

Expenditures:

War Department	$112,000,000
Navy Department	88,000,000
Pensions	138,000,000
Indians	10,000,000
Salaries, diplomatic service, etc.	150,000,000

WRITTEN EXERCISE

1. When our income is $575,000,000 and our customs receipts are $242,000,000, these receipts are what per cent of the income?

2. Taking our expenditures as $488,000,000 a year, what per cent of this goes to the War Department, as above stated? to the Navy Department? to pensions?

3. The colleges and universities of the country cost $23,850,000 a year, and are not supported by the government. This is what per cent of the $488,000,000 in Ex. 2?

4. Our income from public lands is what per cent of that from our customs?

In this and Exs. 5–8, use the items as given on page 452.

5. Our public lands receipts are what per cent less than the income from internal revenue?

6. Our income from internal revenue is what per cent greater than that from our public lands?

7. What per cent of our internal revenue would be necessary to pay the Navy Department expenditures?

8. What per cent of our customs receipts would be necessary to pay the War Department expenditures?

9. In a certain year the government paid $69,210,000 for carrying the mails, and 33% as much for salaries to postmasters. How much did it pay for both of these items?

10. The income of the Post-Office Department through the New York and Brooklyn offices in a certain year amounted to $16,206,000. This was what per cent of the total income ($146,000,000) of the department that year?

11. In a certain year the government received $135,810,015 from taxes on spirits, 33% as much from taxes on tobacco, and 10% more on fermented liquors than on tobacco. What were its total receipts from these three sources?

12. In a certain year the internal revenue receipts of the country amounted to $233,000,000. The largest amount paid by any state was 23%, paid by Illinois. What was this amount? What did the rest of the country pay?

13. In a certain year the government received $143,820,000 from the sale of postage stamps and money orders, which was 94% of the expenses of the Post-Office Department. How much deficiency did Congress have to vote to the Post-Office Department that year?

511. Our post-office system. We do not often think of our post-offices as places where we all pay taxes (when we buy postage stamps). The department is nearly supported by such receipts. In the following miscellaneous problems the annual income may be taken as $150,000,000, and the number of pieces of mail matter handled as 9 billion, but the figures vary from year to year.

WRITTEN EXERCISE

1. The average number of mistakes reported against clerks who handle the mails is only 5 out of 57,500 pieces. What is the per cent of errors?

2. If the post offices issue 46 million domestic money orders a year, amounting to $358,800,000, what is the average amount of each money order?

3. If the receipts of the New York post office amount to $13,650,000 a year, this is what per cent of our total annual postal income as stated above?

4. How much does it cost to register a letter or parcel? If the post offices transmit 22,831,400 registered letters and parcels annually, what is the income from this source?

5. If we have 75,000 post offices, what is the average annual income for each? If the sum of the salaries of the postmasters is $22,200,000, what is their average salary?

6. If the number of postal routes is 35,000, averaging 14.5 miles each, what is their total length? If the average number of annual trips over each route is 936, what is the total distance traveled?

7. If the contractors who carry the mail receive over the stage mail routes 6.58¢ per mile traveled, how much will a contractor receive a year who travels 25 miles daily, except 52 Sundays?

512. Our army and navy. It is necessary for us to keep a navy and a small army of sufficient strength to protect us from foreign attack. It is an expense and therefore is considered under taxes, with certain review problems.

WRITTEN EXERCISE

1. Two of our naval cruisers cost together \$6,810,000, one costing $11\frac{79}{161}\%$ more than the other. What did each cost?

2. Gun metal is composed of 11 parts of copper to 2 parts of tin. How much tin must be added to 2607 lb. of copper to make gun metal?

3. The bronze trimmings of the guns are composed of 17 parts of copper to 4 parts of tin. How much copper must be added to 108 lb. of tin to make bronze?

4. A certain battle ship carries 2000 short tons of coal. How many pounds is this? If it is $12\frac{33}{38}\frac{2}{9}\%$ of the loaded weight of the ship, what is this weight?

5. The speed of a 19-knot battle ship is $20\frac{5}{6}\%$ less than that of a fast mail boat. How long would it take the mail boat to overtake it, giving the battle ship 120 knots the start?

6. The battle ships *Connecticut, Kansas, Louisiana, Minnesota,* and *Vermont* cost \$4,200,000 each. The amount paid for these five ships would send how many men through an agricultural or trade school, at \$1400 each?

7. If our government pays for the War and Navy Departments \$207,000,000 a year, this is how many times the \$9,000,000 annually paid by our people to educate their children in colleges and universities? The tuition is what per cent of the war and navy expenses?

8. If our swiftest torpedo boat makes 31.395 knots an hour, which is $36\frac{1}{2}\%$ faster than our best cruiser, what is the speed of the latter?

9. The 7-in. guns are protected by 6.9-in. steel armor, which is $68\frac{32}{101}\%$ as thick as that protecting the 12-in. guns. How thick is the latter?

10. A 12-in. gun weighs 64 T. 719 lb. and fires a 1000-lb. projectile with 487 lb. of powder. How many pounds does it weigh when loaded? (Always use the 2000-lb. ton unless otherwise directed.)

11. The cruiser *Minneapolis* is $21\frac{1}{15}\%$ faster than the 19-knot battle ship *Georgia*. At these rates, if the battle ship has 288 knots the start, how long will it take the cruiser to overtake it in a race?

12. One of our battle ships has four 12-in. guns, or 20% as many as it has 7-in. guns. The rest of its guns are 3-in. The 12-in. and 7-in. guns together are $54\frac{6}{11}\%$ of the total number. How many 7-in. and 3-in. guns are there?

13. A shot from a certain 12-in. gun exerts enough force at the muzzle to lift a weight of 46,240 tons to the height of 1 ft. This is 70% more force than that exerted by a certain 10-in. gun. Find the force exerted by the latter.

14. A 16-in. gun fires a projectile over 5 ft. long, weighing 1 T. 400 lb., with 576 lb. of smokeless powder. At \$1 a pound for such powder, and 5¢ a pound for the projectile, how much does it cost the government to fire such a gun once? to fire it a dozen times?

15. One ship fired a signal to another, which was answered as soon as it was heard. The first ship heard the answering gun $18\frac{1}{4}$ sec. after the first gun was fired. What was the distance between the ships, sound traveling (at the temperature then observed) 1142 ft. per second?

513. Tariff. The United States collects a large part of its income by a tax on goods brought into the country. This income is called *customs revenue, tariff,* or *duty.*

514. Customhouse. Customs revenue is collected at *customhouses.* These are situated at *ports of entry.*

515. Classes of goods. Goods imported may be:

1. On the *free list,* and not subject to duty, as raw silk.

2. Subject to *ad valorem* (on the value) duty, a certain per cent on the value at the place of purchase, as clocks, on which the duty is 40% *ad valorem.*

3. Subject to *specific* duty, a certain amount per bushel, etc., as potatoes, on which the duty is 25¢ a bushel.

4. Subject to both *ad valorem* and *specific* duty, as velvet carpets, the duty being 60¢ per square yard plus 40%.

ORAL EXERCISE

State the duty on the following:

1. 750 T. of hay, duty $4 per ton.
2. $275 worth of goods, duty 10%.
3. $484 worth of goods, duty 25%.
4. $800 worth of goods, duty 40%.
5. $325 worth of goods, duty 30%.
6. $700 worth of goods, duty 60%.
7. $500 worth of goods, duty 35%.
8. $2000 worth of goods, duty 45%.
9. $88.40 worth of goods, duty $12\frac{1}{2}$%.
10. $60.30 worth of goods, duty $16\frac{2}{3}$%.
11. 600 lb. of goods, duty 22¢ per pound.
12. 1100 lb. of goods, duty 44¢ per pound.
13. 6000 sq. ft. of plate glass, duty 8¢ per square foot.

WRITTEN EXERCISE

1. What is the duty on $4350 worth of bronzes at 45%? Is this specific or *ad valorem?*

2. What is the duty on 3500 bu. of barley at 30 ct. per bushel? Is this specific or *ad valorem?*

3. What is the duty on 15 sets of Thackeray's works at £7 a set, allowing $4.87 to the pound, the duty being 25%?

4. What is the duty on 2 doz. sets of Scott's works at £4 10s. a set, allowing $4.87 to the pound, the duty being 25%?

5. What is the duty on 2000 yd. of tapestry Brussels, 27 in. wide, invoiced at $1.50 a yard, at 28 ct. per square yard and 40% *ad valorem?*

6. What is the duty on $3500 worth of ready-made clothing at 50% *ad valorem*, and $1800 worth of silk dresses at 60% *ad valorem?*

7. A man bought a painting in Rome, paying 750 lire (francs) for it. Allowing 19.3 ct. to the lira, what did it cost in our money, and what was the duty at 25%? What was the total cost, including the duty?

8. A jeweler bought an invoice of Swiss watches, paying 9875 fr. for them in Geneva. Allowing 19.3 ct. to the franc, what was the cost in our money, and what was the duty at 40%? What was the total cost, including the duty?

9. A lady traveling in Europe bought $650 worth of jewelry (duty 60%), $250 worth of silk dresses (duty 60%), $75 worth of engravings (duty 25%), and a 10×12 Oriental rug costing $700 (duty 10 ct. per square foot and 40%). What duty did she have to pay, if $100 worth of goods (jewelry in this case) was admitted duty free?

516. Table of customs duties. The following is a table of certain dutiable articles:

Books in English	25%	Barley, 48 lb. to the bu.	30¢ per bu.
Bronzes	45%	Blankets	22¢ per lb. + 30%
Cheese	6¢ per lb.	Flannel cloth	22¢ per lb. + 30%
Cotton handkerchiefs	45%	Fruits, preserved	1¢ per lb. + 35%
Diamonds, cut and set	60%	Glass, plate	8¢ per sq. ft.
Hay	$4 per ton	Knit woolen	44¢ per lb. + 50%
Paintings	20%	Matches	8¢ per gross
Potatoes	25¢ per bu.	Perfumery	60¢ per lb. + 45%
Watches	40%	Soap, toilet	15¢ per lb.

WRITTEN EXERCISE

Find the duty on the following:

1. A $1500 diamond necklace.
2. An oil painting worth $1250.
3. A bronze statue costing $2500.
4. A car load of barley weighing 5 tons.
5. A shipment of cheese weighing 650 lb.
6. A dozen Swiss watches worth $45 each.
7. An English encyclopedia valued at $150.
8. A case of matches containing 1000 gross.
9. A 350-lb. bale of flannel cloth worth $400.
10. A shipment of toilet soap weighing 500 lb.
11. A 150-lb. box of knit woolen goods valued at $425.
12. A hundredweight of preserved fruits valued at $40.
13. Three hundred pieces of plate glass, each $16'' \times 24''$.
14. Fifty tons of hay and a car load of 250 bu. of potatoes.
15. A bale of blankets weighing 225 lb. and worth $300.
16. A hundred dozen cotton handkerchiefs @ 60¢ a dozen.

517. State and local taxes. State and local taxes are usually a certain per cent levied on the property of business concerns, on land, on money, and on other property.

518. Assessors. The property to be taxed is valued by officers called *assessors.*

519. Assessed valuation. The value placed upon property for taxation is called the *assessed valuation.*

520. Rate of taxation. Upon the assessed valuation a certain *rate of taxation* is fixed.

The words *rate of taxation* are often used to designate the *number of mills* of tax on each dollar of valuation.

Thus, a tax of $5\frac{1}{4}$ mills means $5\frac{1}{4}$ mills on a dollar.

Male citizens over 21 yr. of age pay a *poll* (head) *tax* in parts of the country.

This is a fixed sum, usually about $1.

521. Illustrative problem. For example, if a village with an assessed valuation of $3,200,000 must raise $16,800, what is the rate of taxation?

$16,800 \div 3,200,000 = \$0.005\frac{1}{4}$.

$$3200000\overline{)\$16800}\quad\$0.005\tfrac{1}{4}$$
$$160000$$
$$8000$$
$$\tfrac{8}{32} = \tfrac{1}{4}$$

ORAL EXERCISE

State the tax on the following amounts:

1. $1000 @ 5 mills.
2. $1000 @ 4 mills.
3. $2000 @ 6 mills.
4. $5000 @ 5 mills.
5. $5000 @ 4 mills.
6. $7500 @ 4 mills.
7. $7000 @ 5 mills.
8. $1000 @ $4\frac{3}{4}$ mills.
9. $7500 @ 6 mills.
10. $2000 @ $5\frac{1}{2}$ mills.
11. $1000 @ $3\frac{1}{2}$ mills.
12. $10,000 @ $5\frac{3}{4}$ mills.

WRITTEN EXERCISE

1. What is the tax on \$6500 at $6\frac{1}{4}$ mills on a dollar?

2. The rate of taxation being $1\frac{1}{2}\%$, how much must a company pay whose property is assessed at \$75,000?

3. A town has an assessed valuation of \$4,500,000, and it has to raise \$24,750 by taxation. What is the rate?

4. The rate of taxation being $7\frac{1}{2}$ mills, how much tax must a man pay on \$8500, together with \$1 poll tax?

5. A town has an assessed valuation of \$720,000, and it wishes to levy a tax of \$5400 for highways and schools. What must be the rate?

6. What is the total tax of a man whose property is assessed at \$8400, the rate being 1 mill for state purposes, 3 mills for county purposes, $\frac{1}{2}$ mill for the town, and 2 mills for school purposes, together with \$1 poll tax?

Given the following assessed valuations and tax levies, find the rates of taxation:

7. \$4,200,000, \$16,800.
8. \$2,900,000, \$8700.
9. \$3,700,000, \$20,350.
10. \$7,875,000, \$23,625.
11. \$9,250,000, \$41,625.
12. \$14,500,000, \$87,000.

Given the following tax levies and rates of taxation, find the assessed valuation:

13. \$13,750, 5 mills.
14. \$21,000, 6 mills.
15. \$57,750, 7 mills.
16. \$73,000, 8 mills.

Given the following assessed valuations and rates, find the tax levies:

17. \$4,230,000, $4\frac{1}{2}$ mills.
18. \$6,350,000, $7\frac{1}{2}$ mills.
19. \$7,250,000, $5\frac{1}{2}$ mills.
20. \$12,780,000, $6\frac{1}{2}$ mills.

522. Tax table. Collectors make out a *tax table* like this:

TAX TABLE. RATE $5\frac{1}{2}$ MILLS ON $1

	0	1	2	3	4	5	6	7	8	9
0	0000	0055	0110	0165	0220	0275	0330	0385	0440	0495
1	0550	0605	0660	0715	0770	0825	0880	0935	0990	1045
2	1100	1155	1210	1265	1320	1375	1430	1485	1540	1595
3	1650	1705	1760	1815	1870	1925	1980	2035	2090	2145
4	2200	2255	2310	2365	2420	2475	2530	2585	2640	2695
5	2750	2805	2860	2915	2970	3025	3080	3135	3190	3245
6	3300	3355	3410	3465	3520	3575	3630	3685	3740	3795
7	3850	3905	3960	4015	4070	4125	4180	4235	4290	4345
8	4400	4455	4510	4565	4620	4675	4730	4785	4840	4895
9	4950	5005	5060	5115	5170	5225	5280	5335	5390	5445

Here the first figure of the number of dollars assessed is given at the left, and the second one at the top. Thus, at $5\frac{1}{2}$ mills on $1, the tax on $100 is $0.55; on $250, 1.37\frac{1}{2}$.

523. Collector's commission. The law allows the collector a commission on the tax collected, usually about 1%.

524. Illustrative problem. What must a man pay on $8250, at $5\frac{1}{2}$ mills on $1, the collector's commission being 1%?

By the table, the tax on $8200 is $45.10
" " " 50 " 0.28 (really $27\frac{1}{2}$¢)
" " " $8250 " $45.38
Add collector's 1% on the *tax* .46 (really 45.38¢)
$45.84

WRITTEN EXERCISE

Find the tax at $5\frac{1}{2}$ mills, as in § 523:

1. $6500. 2. $8750. 3. $4875. 4. $7675.
5. $12,000. 6. $16,500. 7. $24,400. 8. $23,500.
9. $31,100. 10. $23,250. 11. $18,875. 12. $45,250.

FIRE INSURANCE

525. Insurance. An agreement to compensate any one for some specified loss is called *insurance.*

526. Policy. The written agreement of an insurance company to pay a certain amount in case of loss is called a *policy.* The insurance companies are often called *underwriters.*

527. Face of policy. The amount specified to be paid in case of loss is called the *face of the policy.*

528. Premium. The cost of insurance is called the *premium.* The *rate of premium* is sometimes stated as a certain sum for each \$100, and sometimes as a certain rate per cent.

Houses are usually insured for three or five years, business property is usually insured for one.

529. Illustrative problem. What is the premium for insuring a store against loss by fire, for \$4000 at \$1.20 a year?

1. Since the rate is \$1.20 on \$100, it is \$0.012 on \$1.
2. 4000 times \$0.012 equals \$48, the premium.

ORAL EXERCISE

1. What is the premium on a \$650 policy at 2%?
2. What is the premium on a \$3000 policy at $1\frac{1}{2}$%?
3. A farmer insured his growing crop of wheat for \$800 at 4%. What was the premium?
4. A building which cost \$24,000 is insured for $\frac{5}{8}$ of its cost. What is the face of the policy?
5. A man paid \$20 for insuring his house, the rate being 1%. What was the face of the policy?
6. A schoolhouse is insured at 1%, the premium being \$50. The face of the policy is $\frac{5}{8}$ of the value of the building. Required the value.

State the premiums on the following policies at the rates specified:

7. \$1000, $1\frac{1}{4}\%$.
8. \$725, 4%.
9. \$2000, $1\frac{3}{4}\%$.
10. \$825, 4%.
11. \$2000, $2\frac{1}{2}\%$.
12. \$500, $2\frac{1}{2}\%$.
13. \$3000, $1\frac{1}{2}\%$.
14. \$700, $2\frac{1}{2}\%$.
15. \$4000, $2\frac{3}{4}\%$.
16. \$900, $1\frac{1}{2}\%$.
17. \$2500, $1\frac{1}{5}\%$.
18. \$650, $1\frac{1}{2}\%$.
19. \$3500, 1.2%.
20. \$1250, 4%.
21. \$4250, \$2.
22. \$6250, \$2.
23. \$1750, \$2.
24. \$3750, \$2.
25. \$12,500, \$1.
26. \$12,000, \$3.
27. \$25,000, \$3.
28. \$15,000, \$2.
29. \$10,000, \$1.75.
30. \$50,000, \$1.50.

State the faces of the policies, given the following premiums and rates:

31. \$70, 2%.
32. \$90, 2%.
33. \$17, $\frac{1}{2}\%$.
34. \$50, $2\frac{1}{2}\%$.
35. \$25, $2\frac{1}{3}\%$.
36. \$13.75, 1%.
37. \$17.50, 1%.
38. \$18.50, $\frac{1}{2}\%$.

State the rates, given the following faces of policies and premiums:

39. \$1500, \$15.
40. \$2000, \$10.
41. \$2500, \$50.
42. \$3500, \$35.
43. \$5000, \$25.
44. \$7500, \$150.
45. \$6000, \$60.
46. \$6500, \$130.

47. A building worth \$8000 is insured for $\frac{3}{4}$ of its value at 1%. What is the premium?

WRITTEN EXERCISE

1. At \$0.95, what is the premium on a \$2500 policy?

2. At \$1.10, what is the premium on a \$2800 policy?

3. At \$1.15, what is the premium on a \$3750 policy?

4. A building worth \$12,000 is insured for $\frac{2}{3}$ of its value at 2%. What is the premium?

5. If you insure the contents of your store for \$6000, what is the premium, at \$1.25?

6. The premium for insuring some property at \$1.50 is \$52.50. What is the face of the policy?

7. A man insured his factory, valued at \$135,000, for $\frac{4}{5}$ of its value, at \$1.90. What was the premium?

8. A dealer insured his stock of goods, valued at \$14,000, for $\frac{7}{8}$ of its value, at \$1.65. What was the premium?

9. If a 3-year policy for \$1750 costs \$27, what is the rate of premium for the 3 years? What is the rate per year?

10. If a 3-year policy for \$3000 costs \$36, what is the rate of premium for the 3 years? What is the rate per year?

11. A merchant insured his stock for $\frac{3}{4}$ of its value, at $1\frac{3}{4}$%. The premium was \$131.25. What was the value of the stock?

12. A manufacturer insured his factory for $\frac{5}{8}$ of its value, at \$2. The premium was \$210. What was the face of the policy? the value of the factory?

13. A factory worth \$33,000 is insured for $\frac{5}{6}$ of its value, at 2%. In case of total loss how much would the owner lose, including his premium paid?

14. A man insured his library for its full value, \$2500, at \$1.25. What was the premium? If a fire destroyed $\frac{3}{5}$ of the library, how much could he recover?

15. Which is the cheaper, a 3-year policy on \$3500 at $1\frac{1}{2}\%$, or an annual policy renewed for the same period, at 60¢ a year? How much cheaper?

16. If Mr. Wood insures his stock of goods, invoiced at \$28,000, for 80% of its value, at $1\frac{1}{4}\%$, and the property is entirely destroyed by fire, what is his loss, including the premium paid?

Include the premium in Exs. 17, 18.

17. A dealer insured his stock of goods, valued at \$21,800, for $\frac{7}{8}$ of its value, at 1.3%. The property was entirely destroyed by fire. What was his loss?

18. If Mr. Baldwin insures his stock of goods, invoiced at \$37,500, for 80% of its value, at 95¢, and the property is entirely destroyed by fire, what is his loss?

19. Suppose a company willing to take a risk covering the value of a stock of goods and the premium paid, and the face of the policy is \$25,500, the rate being 2%, what is the value of the stock of goods?

20. A dealer pays \$21 premium for insuring a shipment of grain at $1\frac{3}{4}\%$. What is the face of the policy? When this grain is placed in the elevator it is again insured for the same amount at $\frac{1}{4}\%$. What is the premium?

21. A manufacturer of especially inflammable goods finds that the insurance rate is 6% a year. Rather than pay such a rate, he could afford to have his stock burn once in how many years, not counting interest?

22. If Roberts & Eastman insure their business block for \$12,000, at \$1.15; the first-floor contents for \$8000, at \$1.25 (on account of the difficulty of removing goods); and the contents of the other floors for \$3000, at \$1.40, what is the total amount of premiums paid?

MARINE INSURANCE

530. Marine insurance. Insurance against loss by navigation is called *marine insurance.*

The value placed on the goods is the same as the face of the policy unless the contrary is stated.

WRITTEN EXERCISE

1. A man paid \$36 for insuring some goods from Philadelphia to Marseilles, the rate being $\frac{1}{2}\%$, less 20% of the premium. What was the face of the policy?

2. A new ocean-going tug running between Norfolk, Va., and Eastport, Me., is insured at 5% a year, the premium being \$2245. What is the face of the policy?

3. A man pays \$24,793.35 for a cargo of hides in Galveston, which included the premium at $\frac{1}{2}\%$ net for shipping the same to New York. What was the amount of premium?

4. What is the premium on a shipment of \$10,200 worth of boots and shoes by steamer from New York to Manila, the rate being $1\frac{1}{4}\%$, less 20% (see Ex. 1)? How much is it by sailing vessel, the rate being 2%, less 20%?

5. What is the premium on an under-deck shipment of \$9000 worth of lumber from Georgia to New York, the rate being $\frac{3}{4}\%$, less 20%? What is it for a shipment made on deck, the rate being $2\frac{1}{4}\%$, less 20%? How much premium is saved by taking the under-deck shipment?

6. What is the premium on a shipment of \$8460 worth of typewriters by passenger steamer from New York to Liverpool, the rate being 0.2% net? How much by a tramp steamer, the rate being $\frac{3}{8}\%$, less 20%? How much by a sailing vessel, the rate being $1\frac{1}{4}\%$, less 20%?

LIFE INSURANCE

531. Why a man should insure his life. If a man is careful as to the future and considers the fact that in later years there will probably be one or more persons dependent upon him, he will wish to insure his life. The earlier this is done the lower will be the premium and the sooner he will cease to make payments if he takes a 10-, 15-, or 20-payment policy.

Premiums are stated at so much on $1000. They vary with the age, being smaller for young persons.

The best plan is to take a policy in a company of unquestionable standing, and never forfeit it by nonpayment of premiums.

ORAL EXERCISE

State the premiums on the following policies, the premium on $1000 being given:

1. $15,000, $26.
2. $3500, $25.
3. $2000, $21.30.
4. $4500, $30.
5. $3000, $22.10.
6. $5500, $30.
7. $4000, $22.50.
8. $12,000, $25.
9. $5000, $28.40.
10. $2500, $22.20.
11. $10,000, $27.65.
12. $20,000, $31.20.

State the faces of the policies, given the following premiums and the premiums on $1000:

13. $131, $26.20.
14. $175, $25.
15. $54.20, $27.10.
16. $49, $24.50.
17. $96.60, $32.20.
18. $63.90, $21.30.

State the premiums on $1000, given the following policies and premiums:

19. $3000, $84.
20. $4000, $27.

532. Kinds of policies. Four of the leading kinds of policies are:

1. *Ordinary life*, the insured agreeing to pay a certain premium, usually annually, for life.

The rate is always given as the cost of $1000 worth of insurance. That is, the rate $25.50 means that the annual premium on a policy for $1000 is $25.50. Some insurance companies allow *dividends* each year, thus reducing the premium slightly.

2. *Limited life*, the premiums being payable for some fixed number of years, as twenty, the policy then being called *paid up* (no more premiums being due), but the face not being paid until the death of the insured.

Naturally the premiums are higher on this form.

3. *Endowment*, the premiums being paid for some fixed number of years, as ten, fifteen, or twenty, at the end of which time the face of the policy will be paid to the insured.

Since this may be paid during the lifetime of the insured, the premiums are also higher than on an ordinary life policy.

4. *Term insurance*, the premiums being paid for a specified length of time and the face of the policy being payable if the insured dies within the term of insurance.

Thus, a person may insure his life or his health for thirty days, as in certain forms of accident insurance. He may also insure his life for a certain number of years only. The premium on this form of policy is low, since the company may not have to pay the face at all.

There are various other forms of policy, but most of them are modifications of the above types. Some are arranged so that the face will not be paid on the death of the insured, but a certain amount will be paid annually (an annuity) during the lifetime of the one for whose benefit the policy was written.

WRITTEN EXERCISE

1. What is the premium on a $5000 policy at $27.39 on $1000?

2. What annual premium must a man pay on a 10-year endowment policy for $4000 at $102.60 per thousand? What would he pay in the 10 years?

3. If a young man takes out a $5000 20-payment policy at $27.39 per thousand, how much will he have paid when the policy matures (that is, after 20 payments)?

4. What is the premium on a $5000 20-payment policy of such a kind that the rate is $34.20 on a thousand? What are the total premiums for the 20 years?

5. What is the premium on a $7500 20-payment policy of such a kind that the rate is $32.50 on a thousand? What are the total premiums for the 20 years?

6. If a man takes out a $2500 policy in one company at $26.40 per thousand, and a $3500 policy in another at $23.50 per thousand, what are his annual premiums?

7. A young man takes out a $10,000 policy on the 25-payment plan, at $26.36 per thousand. What is the annual premium? How much are the premiums for the 25 years?

8. A man takes out a $5000 policy, paying $26.40 a thousand. He dies just before the eighth annual payment is due. How much does his estate receive above what he paid to the company?

9. A young man took out a $5000 20-payment policy 20 years ago, paying $136.95 a year. How much has he paid in the 20 years? Estimating that the insurance company has had the use of all this amount for the equivalent of 10 years, at 4%, what is the total amount received by the company?

10. A man pays \$61.50 annually on a \$2500 policy. What is the rate of premium per thousand dollars?

11. At the age of 21 the annual premium on \$1000 of term insurance for 10 years is \$11, and the dividends decrease each premium, after the first, 15% of this sum. What is the total net cost of \$10,000 of such insurance for 10 years?

12. At the age of 21 the annual premium on an ordinary life policy is \$19.12 per thousand. What would be the total amount of premiums paid on \$5000 in 20 years? What would be the net amount if the company allowed \$198 per thousand in dividends during this period?

13. How much more would the policy of Ex. 12 cost if taken out at the age of 32, the premium then being \$25.09, and the dividends for 20 years being \$237 per thousand?

14. At the age of 21 the annual premium on a 20-payment limited life policy is \$28.98, and the dividends allowed by a certain company amount to \$222 per thousand in 20 years. What is the net cost of \$3000 of such insurance before it is paid up?

Of course the insured also loses the interest on the premiums paid, but this item is not required in the example.

15. How much more would the policy of Ex. 14 cost if taken out at the age of 50, the premium then being \$54.65, and the total amount of the dividends for 20 years being \$625 per thousand?

16. At the age of 21 the annual premium on a 20-year endowment policy is \$48.48, and the dividends amount to \$353 per thousand during this period. What is the difference between the amount paid the company (less dividends) and the amount received in return on a \$5000 policy?

II. BUSINESS ARITHMETIC COMPLETED. MENSURATION

CORPORATIONS

533. Corporation. The laws of our various states permit a number of persons who wish to go into business together to organize as one body, called a *corporation.*

534. Capital. The members contribute the money to start the business. This is called the *capital.*

535. Shares of stock. The capital is divided into *shares,* usually of $100 each, although sometimes less. Each person who owns one or more shares is a *stockholder,* and receives a *certificate of stock* stating how many shares he owns.

For example, if the capital is $100,000, and the shares are $100 each, there are 1000 shares. If a man has 50 shares, he owns $5000 worth of stock, or 5% of the whole corporation.

536. Directors and officers. The stockholders elect a few of their number to have general direction of the company. These are called *directors,* and they elect the officers.

537. Dividends. The earnings of the company, after paying the expenses and providing for a bank account sufficient for probable needs, are divided into *dividends,* the directors deciding on the *rate of dividend.* These dividends are sent to the stockholders, by means of checks.

For example, if a company with a capital of $100,000 earns $6000 beyond all expenses, it may declare a 6% dividend. Then a man who owns $5000 worth of stock will receive a check for $300.

ORAL EXERCISE

State the dividends to be paid on the following:

1. $3000, 5%. 2. $7000, 4%. 3. $12,000, $5\frac{1}{2}$%.
4. $2500, 3%. 5. $8000, $1\frac{1}{4}$%. 6. $20,000, $2\frac{1}{2}$%.

WRITTEN EXERCISE

1. How many shares in a company with \$1,500,000 capital? (In these exercises let one share be \$100.)

2. A company divides \$50,000 in dividends, and stockholders receive \$4 per share. What is the capital?

3. A company with \$3,000,000 capital declares a 5% dividend. What does the holder of 100 shares receive?

4. A company with \$500,000 capital divides \$45,000 in dividends. What does the holder of 30 shares receive?

5. How much does the holder of 30 shares of a certain railway stock receive when a $4\frac{1}{2}\%$ dividend is declared?

6. A man receives from the treasurer of a certain company \$25 every three months, as dividends on his 20 shares of stock. What is the rate of quarterly dividends?

7. A company with a capital of 50 million dollars declares a dividend of $2\frac{3}{4}\%$ every six months. How much money does it distribute among its stockholders annually?

8. A company with \$250,000 capital declares four dividends a year, each of $1\frac{1}{4}\%$. What are the total annual dividends? What is the annual income of a man who owns 50 shares?

9. A company with \$350,000 capital declares two dividends a year. A stockholder who owns 20 shares receives an annual income of \$110. What is the rate of the semi-annual dividends?

10. A company with a capital of \$250,000 has earned \$15,000 this year above all expenses. It decides to save \$2500 of this for emergencies, and to divide the rest in dividends. What is the rate?

538. Above and below par. If stock is paying a high rate of dividend, that is, more than can be received from good ordinary investments, people will be so anxious to buy it that they will pay more than \$100 for a \$100 share. It is then said to be *above par.* If a \$100 share can be bought for just \$100, the dividends are about on a par with other investments, and the stock is said to be *at par.* If the dividends are low, say 1% or 2%, people will not be anxious to buy the stock, and it will sell *below par.*

539. Buying stock. Stocks are usually bought and sold through a *broker,* generally at a *stock exchange,* a kind of auction room for such business.

540. Brokerage. The broker charges *brokerage* or *commission,* usually $\frac{1}{8}\%$ *of the par value,* making this charge both for buying and for selling.

541. Meaning of stock quotations. A quotation of $118\frac{3}{4}$ means that \$100 worth of stock will cost \$118.75 besides $\$\frac{1}{8}$ (or $12\frac{1}{2}$ ct.) brokerage. The buyer then pays $\$118.87\frac{1}{2}$ per share, while the seller, who must also pay his broker, receives $\$118.75 - \$0.12\frac{1}{2} = \$118.62\frac{1}{2}$ per share.

In stock quotations fractions are always expressed in halves, fourths, or eighths. Fractions of a share cannot be bought on the stock exchange. The standard amount bought is 100 shares, although smaller lots are often purchased.

542. Illustrative problems. 1. What is the cost of 10 shares of stock quoted at $137\frac{1}{8}$, brokerage as usual?

One share costs $\$137\frac{1}{8} + \$\frac{1}{8}$ brokerage, or $\$137\frac{1}{4}$.
10 shares cost 10 times $\$137\frac{1}{4}$, or \$1372.50.

2. What is the amount received from the sale of 100 shares of stock quoted at $96\frac{5}{8}$, allowing the usual brokerage?

One share brings $\$96\frac{5}{8} - \$\frac{1}{8}$ brokerage, or $\$96\frac{1}{2}$.
100 shares bring 100 times $\$96\frac{1}{2}$, or \$9650.

ORAL EXERCISE

State the cost of 10 shares of stock quoted as follows, adding the brokerage in each case:

1. $94\frac{7}{8}$.	2. $73\frac{7}{8}$.	3. $69\frac{7}{8}$.	4. $84\frac{3}{8}$.
5. $114\frac{7}{8}$.	6. $63\frac{3}{8}$.	7. $47\frac{3}{8}$.	8. $82\frac{5}{8}$.
9. $106\frac{3}{8}$.	10. $127\frac{3}{8}$.	11. $75\frac{5}{8}$.	12. $109\frac{7}{8}$.

State the amount received from the sale of 10 shares of stock quoted as follows, allowing for the brokerage:

13. $87\frac{1}{8}$.	14. $69\frac{1}{8}$.	15. $54\frac{1}{8}$.	16. $96\frac{5}{8}$.
17. $134\frac{1}{8}$.	18. $68\frac{5}{8}$.	19. $72\frac{5}{8}$.	20. $107\frac{1}{8}$.

State the cost of the following, including brokerage:

21. 100 shares @ $47\frac{7}{8}$.	22. 100 shares @ $34\frac{7}{8}$.
23. 50 shares @ $119\frac{7}{8}$.	24. 50 shares @ $123\frac{7}{8}$.

WRITTEN EXERCISE

Find the cost of the following, including brokerage:

1. 75 shares @ $127\frac{1}{4}$.	2. 25 shares @ $136\frac{1}{2}$.
3. 60 shares @ $141\frac{3}{4}$.	4. 125 shares @ $62\frac{5}{8}$.
5. 150 shares @ $109\frac{1}{8}$.	6. 250 shares @ $172\frac{3}{8}$.

Find the amount received from the sale of the following:

7. 80 shares @ $134\frac{1}{4}$.	8. 500 shares @ 64.
9. 175 shares @ 169.	10. 250 shares @ 178.
11. 140 shares @ $147\frac{3}{4}$.	12. 120 shares @ $142\frac{1}{2}$.

13. A man paid, including brokerage, $2047.50 for 30 shares of stock. At what price was it quoted?

14. A man received for some stock, less the brokerage, $5195, when it was quoted at 130. How much did he sell?

543. Newspaper quotations. Daily newspapers give the stock quotations. The following examples may be solved by using the newspaper quotations, or the following:

At., Top. & S. F.	$85\frac{7}{8}$	N. Y. Central	$138\frac{1}{8}$
Balt. & Ohio	$95\frac{1}{8}$	Penn. R.R.	$139\frac{1}{2}$
Can. Pac.	$130\frac{1}{4}$	Southern Pac.	$62\frac{3}{4}$
Ill. Cent.	$152\frac{1}{8}$	Union Pac.	$113\frac{1}{4}$
Louisv. & Nash.	$132\frac{3}{8}$	Western Un. Tel.	93
Missouri Pac.	$101\frac{5}{8}$	Wisconsin Cent.	20

In the following examples add $\frac{1}{8}\%$ to the quotation if you are buying, and subtract $\frac{1}{8}\%$ if you are selling, to pay the broker.

544. Illustrative problem. If a man buys 10 shares of Union Pacific when quoted at $113\frac{1}{4}$ and sells it when quoted at $115\frac{3}{4}$, how much does he gain?

1. He buys it at $113\frac{1}{4} + \frac{1}{8}$ (brokerage), or $113\frac{3}{8}$.
2. He sells it at $115\frac{3}{4} - \frac{1}{8}$ (brokerage), or $115\frac{5}{8}$.
3. Therefore he gains $2\frac{1}{4}$, or \$2.25 a share.
4. On 10 shares he gains 10 times \$2.25, or \$22.50.

$$\begin{array}{r} \$115\frac{5}{8} \\ 113\frac{3}{8} \\ \hline \$2\frac{1}{4} \\ 10 \\ \hline \$22.50 \end{array}$$

WRITTEN EXERCISE

1. A man buys 50 shares of Atchison, Topeka & Santa Fé stock as quoted, and sells at $89\frac{1}{8}$. What is his gain?

2. A man bought 50 shares of Wisconsin Central when it was quoted at $25\frac{7}{8}$, and sold it at the above quotation. How much did he lose?

3. If we buy 20 shares of Baltimore & Ohio at 90, and 100 shares of Canadian Pacific at 131, and sell at the above quotations, what do we gain or lose?

4. Find the cost of 50 shares of Pennsylvania R.R., 100 shares of Southern Pacific, 20 shares of Union Pacific, and 10 shares of Western Union Telegraph Co.

Find the gain or loss on buying 50 shares of the following stocks as quoted on the preceding page, and selling at the prices here given:

5. Ill. Cent. 153.
6. Ill. Cent. $149\frac{7}{8}$.
7. Can. Pac. 125.
8. Union Pac. 115.
9. Can. Pac. $130\frac{1}{2}$.
10. Penn. R.R. $137\frac{1}{8}$.
11. Southern Pac. $63\frac{1}{4}$.
12. Union Pac. $114\frac{1}{2}$.
13. N. Y. Central $136\frac{1}{4}$.
14. Penn. R.R. $139\frac{3}{4}$.
15. At., Top. & S.F. $86\frac{1}{2}$.
16. Balt. & Ohio $97\frac{1}{4}$.
17. Louisv. & Nash. 134.
18. Missouri Pac. $100\frac{1}{8}$.
19. Western Un. Tel. $87\frac{1}{2}$.
20. Wisconsin Cent. $18\frac{1}{4}$.

21. A man bought 40 shares of stock and sold it when quoted at $127\frac{1}{8}$. He gained \$80 on the deal. What was the quoted price when he bought it?

22. A man bought 150 shares of New York Central stock when quoted as on page 476. He sold it so as to gain \$600. What was the quoted price when he sold it?

23. If a dealer buys 100 shares of Illinois Central, 50 shares of Louisville & Nashville, and 25 shares of Missouri Pacific at the quotations on page 476, what does it all cost?

24. A man bought some Illinois Central stock when quoted as on page 476. He sold it when quoted at $150\frac{3}{8}$, and lost \$500 by so doing. How many shares did he buy?

25. A man bought some Canadian Pacific stock when quoted as on page 476. He sold it when quoted at $132\frac{1}{2}$, and made \$400 by so doing. How many shares did he buy?

26. A man buys 100 shares of New York Central as quoted on page 476, holds it a year, receives 5% in dividends (5% on the par value), and sells it at $141\frac{5}{8}$. Money being worth 4% a year, what does he gain?

545. Bonds. When corporations wish to borrow any considerable amount of money they issue *bonds*.

A written or printed promise to pay a certain sum at a specified time, signed by the maker and bearing his seal, is called a *bond*.

546. Bonds secured by a mortgage. Bonds are secured by a *mortgage*, an agreement that the owners of the bonds may sell the property if the bonds and the interest are not paid.

547. Difference between stocks and bonds. Bonds differ from stocks in this way: the stockholders of a railway are the owners; the bondholders are the ones to whom the road owes money. Bonds bear a fixed rate of interest, but the income of stocks depends on the earnings of the company after the interest on the bonds and the running expenses have been paid.

548. Computing dividends. Stock dividends and bond incomes are always computed on the par value.

549. Illustrative problems. 1. How much is the income on \$5000 worth of 4% bonds?

4% of \$5000 = \$200.

2. What is the income on 50 shares of railway stock at 5%?

1. 50 shares have a par value of \$5000.
2. 5% of \$5000 = \$250.

3. If I buy a 5% bond at $124\frac{7}{8}$, what is the rate of income on my investment?

1. I pay $\$124\frac{7}{8} + \$\frac{1}{8}$ (brokerage) = \$125 for a \$100 bond.
2. My income is 5% of \$100, or \$5.
3. Since $x\%$ of \$125 = \$5,

therefore $x\% = \$5 \div \$125 = .04$.

4. Hence my income is 4% on my *investment* (but 5% on the *par value*), not considering the date of maturity).

WRITTEN EXERCISE

1. What will 75 shares of stock cost when quoted at $96\frac{1}{8}$? at $102\frac{5}{8}$? at $68\frac{1}{2}$? at $99\frac{7}{8}$? (Remember the $\frac{1}{8}\%$.)

2. I buy a 6% bond at $149\frac{7}{8}$. What is the rate of income on the money invested? (Remember the $\frac{1}{8}\%$.)

3. Which gives the better income, 5% stock bought when quoted at $139\frac{7}{8}$ or $3\frac{1}{2}\%$ bonds bought when quoted at $99\frac{7}{8}$?

$5 ÷ $140, compared with $3.50 ÷ $100.

4. Which gives the better rate of income, a 6% bond at 120 or a 5% promissory note, leaving out the question of brokerage in this example?

$6 ÷ $120, compared with $5 on $100.

5. A man buys some stock that regularly pays 7% dividends, when quoted at $199\frac{7}{8}$. What is the rate of income on the money invested?

6. When United States 4% bonds are quoted at $116\frac{7}{8}$, what rate of income does a purchaser receive on his investment, not considering the question of the date of maturity of the bonds?

7. If you had some money to invest, which would you prefer, a stock that regularly pays 8%, quoted at $159\frac{7}{8}$, a 5% bond at $109\frac{7}{8}$, or a $5\frac{1}{2}\%$ promissory note, the security being equally good?

Compare $8 return on $160 invested, $5 return on $110 invested, and $5.50 return on $100 invested. The $8 on $160 gives an income of $\frac{8}{160}$ or 5%. What per cent do the others give?

8. If you had some money to invest, which would you prefer, a stock that regularly pays 7%, quoted at $149\frac{7}{8}$, a 4% bond at $89\frac{7}{8}$, or a $4\frac{1}{2}\%$ promissory note, the security being equally good?

Find which pays the better per cent, and how much, not considering the brokerage, in Exs. 9–21:

9. A 5% bond at 121, or a 4% bond at 97.

10. A 5% bond at 115, or a 4% bond at 92.

11. A 5% bond at 130, or a 6% bond at 156.

12. A 3% bond at 102, or a 5% bond at 170.

13. A 6% bond at 130, or a 5% bond at 108.

14. A 6% bond at 140, or a 5% bond at 117.

15. A 3% bond at 92, or a $3\frac{1}{2}\%$ bond at 107.

16. A $4\frac{1}{2}\%$ bond at par, or a 5% bond at 111.

17. A 5% bond at 20 above par, or a 4% bond at 4 below par.

This means a 5% bond at 120, or a 4% bond at 96.

18. A 6% bond at 30 above par, or a 4% bond at 14 below par.

19. A 5% bond at 10 above par, or a 4% bond at 12 below par.

20. A stock paying regularly 7%, at 140, or one paying regularly 5%, at 95.

21. A stock paying regularly 6%, at 137, or one paying regularly 5%, at 114.

22. An investor buys 100 shares of stock at $110\frac{1}{4}$, holds the stock 2 years, receives 8 quarterly dividends of $1\frac{1}{2}\%$, and sells it at $113\frac{3}{8}$. Is this better than to have invested his money for the same time at 5%? State the two incomes.

23. A man buys 100 shares of stock when quoted at $74\frac{3}{8}$, holds the stock two years, receives 8% in dividends, and then sells it at 72. Compare his gain or loss with that resulting from investing his money in a savings bank so that it earns 3% compound interest.

BUYING PRODUCE

550. Large dealings in produce. A man's work may lead him into large dealings in the products of the soil. These products are bought and sold in large quantities in great auction houses, known by various names, such as the Board of Trade, the Produce Exchange, and the Cotton Exchange.

Teachers should not require these business customs to be memorized but should seek to make the transactions seem real and legitimate.

551. Buying grain. As a rule, not less than 1000 bu. of grain are sold at a time on the Board of Trade at Chicago, the great center for such transactions. The broker charges $\frac{1}{8}¢$ per bushel for buying, and the same for selling. The quotations are by the number of cents to the bushel and always vary by multiples of $\frac{1}{16}¢$ per bushel.

552. Buying pork. As a rule, not less than 250 bbl. of pork are sold on the exchanges. The broker's commissions are $2\frac{1}{2}¢$ per barrel for buying, and the same for selling. The quotations are by the number of dollars to the barrel and always vary by multiples of $2\frac{1}{2}¢$ per barrel.

In all of the following examples remember the broker's commissions as stated above.

WRITTEN EXERCISE

1. What is the cost of 3000 bu. of wheat quoted at $91\frac{7}{8}¢$?

2. If a man buys 5000 bu. of wheat at $89\frac{3}{16}¢$ and sells it at $91\frac{1}{8}¢$, how much does he gain?

3. If a dealer buys 6000 bu. of corn at $47\frac{3}{16}¢$ and sells it at $46\frac{7}{8}¢$, how much does he lose?

4. If a firm of produce dealers buys 750 bbl. of pork at $\$12.72\frac{1}{2}$, and sells it at \$13.20, what is the gain?

553. Buying lard. This is one of the leading products of the central part of our country. As a rule, not less than 250 tierces are sold on the exchanges. A tierce is 340 lb. The broker charges $2\frac{1}{2}¢$ per tierce. The quotations are in dollars per tierce.

554. Buying cotton. This is the great product of the South. As a rule, not less than 100 bales are sold on the exchanges. A bale is considered as 500 lb. The broker charges $5 per 100 bales. The quotations are in cents per pound, and vary by hundredths of a cent.

555. Buying coffee. This is one of the chief imported products dealt in on the exchanges. As a rule, not less than 250 bags are sold. A bag is considered as 130 lb. The broker charges $10 per 250 bags. The quotations are in cents per pound.

In all of the examples remember the broker's commissions.

WRITTEN EXERCISE

1. What will 1250 bags of coffee cost at 8.42¢?
2. What will 750 tierces of lard cost at $7.02?
3. What will 3000 bales of cotton cost at 12.30¢?
4. What will 1000 tierces of lard cost at $6.97\frac{1}{2}$?
5. What will 2500 bales of cotton cost at 11.75¢?
6. If a man buys 1500 tierces of lard at $6.87\frac{1}{2}$, and sells it at $6.82\frac{1}{2}$, how much does he lose?
7. A man buys 250 bags of coffee at 8.25¢, and sells it at 8.60¢. Does he gain or lose, and how much?
8. A cotton factory buys 500 bales at 10.61¢, 300 bales at 10.68¢, and 200 bales at 10.75¢. What is the cost?
9. A dealer buys 1200 bales of cotton at 11.50¢ and sells it at 11.61¢. Does he gain or lose, and how much?

INDUSTRIAL PROBLEMS

556. Problems concerning our various industries. These problems may be used when the industries are being studied in geography. In general they involve percentage.

WRITTEN EXERCISE

1. If when we had a population of 79,374,120 there were 661,451 persons engaged in the cloth industry, one person out of how many was engaged in this work?

2. There were 297,929 wage earners in the cotton industries of our country in a certain year, and they received $84,909,765 a year. There were also 4713 salaried clerks and officials, receiving in all $7,121,343 a year. What was the average income of each class per capita?

3. If our factories produce $297,000,000 worth of woolen goods in a certain year, and the materials cost 61%, the labor 19%, the salaries of officers 2%, and the miscellaneous expenses 6% of this sum, how much is expended for each of these items, and how much is left for profit?

4. Of all the timber cut in this country in a certain year, 21½% was white pine, 9.8% was hemlock, 4.2% was spruce, 27.8% was yellow pine, and 12.8% was oak. We cut 2,530,000 M ft. that year, B.M. How many feet of each of these woods were cut? How many of all the other woods?

5. This timber was worth, on an average, $2.18 per M, standing, and $6.28 when ready for the mill. How much did the cutting and logging add to the value of the total?

6. Out of this increase, $1.76 per M went for wages, and 94¢ for other expenses, the rest being profit. What was the total profit?

7. If it costs \$1.55 a ton to manufacture ice, and the wholesale price is \$2 a ton, what is the per cent of profit to the manufacturer?

8. The value of our leather product in a certain year was \$204,000,000, an increase of $18\frac{2}{4}\frac{6}{3}\%$ in 10 years. What was the value 10 years before?

9. The value of the shoes manufactured in this country in a certain year was \$262,700,000, an increase of $18\frac{1}{3}\%$ in 10 years. What was it 10 years before?

10. At the opening of the century the amount of money invested in this country in the production of leather was \$174,000,000, an increase of $77\frac{2}{4}\frac{7}{9}\%$ over the amount 10 years before. What was the amount then?

11. In a certain year we produced 67,890,000 pairs of boots and shoes for men, 21,110,000 for boys, 65,000,000 for women, 42,000,000 for girls and children, 17,000,000 pairs of slippers, and 6,000,000 other pairs. The men's boots and shoes were what per cent of the total?

12. Some of the leather that year went into 2,895,700 dozen pairs of gloves, of which 87% were for men. How many gloves were produced for women and children?

13. In a certain year we had 134,000 wage earners engaged in making cotton cloth, and they received \$46,900,000. Ten years before that we had 89,000 wage earners, and they received \$33,820,000. Had the wages increased or decreased per capita, and how much?

14. When New York City had a population of $3\frac{1}{2}$ millions it manufactured 411,000 tons of ice a year, while New Orleans, with a population of 300,000, manufactured 140,000 tons. The population and ice product of New Orleans are what per cents of those of New York? Why does the former manufacture relatively more?

15. In a year when our flour mills produced $589,950,000 of flour, our meat products were worth 33⅓% more. What was the value of the meat products?

16. In a year when our wool products amounted to $446,740,000, this was 75% more than the value of our boot and shoe products. What was the value of the latter?

17. In a year when our mines produced 610,815,384 lb. of copper, the total output was worth $73,297,846.08, which was 20% more per pound than it was worth five years before. What was it worth per pound then?

18. In a year when the wheat crop of the world amounted to 3,124,422,000 bu., we produced 22% of the total, and Canada produced 15% as much as we did. How many bushels did Canada and the United States each produce? How many together?

19. Suppose the grain elevators of Duluth to have a capacity of 34 million bushels. Estimating a bushel as 1¼ cu. ft., what is this capacity in cubic feet? (It would be interesting to see how many times the volume of your schoolroom this is.)

20. Of three great flour mills of Minneapolis, if one has a capacity of 28,000 bbl. a day, a second of 3¾% less, and a third of 33⅓% less than the second, and all should run at their full capacity for 300 working days of a year, how many barrels would they produce?

21. In a certain state it takes 21 lb. of milk to make 1 lb. of butter, while it takes 22 lb. in a second state, 23 lb. in a third, and 24 lb. in a fourth. The first state takes what per cent less than the second? The second what per cent more than the first? The fourth what per cent more than the third? The third what per cent less than the fourth?

557. Our sugar industry. This country produces about 500,000 tons of sugar a year, the greater part being made from sugar cane, and the rest from beets. At present the chief beet-sugar producing states are California, Michigan, Colorado, and Utah. Our cane sugar comes from Louisiana.

WRITTEN EXERCISE

1. If 2457 tons of beet sugar are worth $275,184, what is the average price per ton? What would it be if the price were decreased $2\frac{1}{2}\%$?

2. If 23,241 tons of sugar beets are worth $92,964, what is the average price per ton? What would it be if the price were increased $7\frac{1}{2}\%$?

3. If we manufacture 5325 tons of maple sugar in a year and 300,000 tons of cane sugar, the amount of maple sugar is what per cent of that of cane?

4. In a year when we had 132,441 acres planted to sugar beets, the average yield was 6 tons to the acre. What was the value of the crop at $4.10 a ton?

5. When we produced 300,000 long tons of cane sugar and 195,000 long tons of beet sugar, how many pounds of each did we produce? Each was what per cent of the sum?

6. If the world's production of beet and cane sugar in one year is 9,835,392 tons, and the cane sugar is 839,595 tons more than one third of the total, how much beet sugar is produced?

7. If we have 1970 wage earners employed in making beet sugar, and their wages are $1,091,380, and if we pay 100% more to each salaried person employed in this industry than to the wage earner, and if these salaries amount to $356,776, how many salaried persons are employed?

Problems in Iron Working

WRITTEN EXERCISE

1. What is the cost of 12′ 8″ of iron rod, $2\frac{1}{8}$ lb. to the foot, at $1\frac{3}{4}¢$ a pound?

2. What is the weight of a steel girder 26′ 6″ long, weighing $42\frac{1}{2}$ lb. to the foot?

3. The wooden pattern from which an iron casting is made weighs $6\frac{1}{4}\%$ as much as the iron. The pattern weighs $45\frac{5}{8}$ lb. How much does the casting weigh?

4. If steel rails weighing 80 lb. to the yard are used between New York and Chicago, a distance of 980 mi., how many tons will be required for a double-track road?

5. An iron tire expands $1\frac{9}{16}\%$ on being heated for shrinking on a wheel. A wooden wheel needs a tire 4′ 8″ in diameter. If the circumference is $3\frac{1}{7}$ times the diameter, how much longer will the tire be when thus heated?

6. In a certain blast furnace the casting machine turns out 20 pigs per minute, averaging in weight 110 lb. each. If this machine runs at this rate for 308 days, 16 hours a day, how many long tons of pig iron will it turn out?

7. A cellar window 3′ × 2′ is to be fitted with an iron grating. This is to be made by constructing an iron frame and then putting in cross pieces of iron rods 3″ between centers, the top and bottom ones being 3″ from the window casing. The iron frame weighs $2\frac{1}{4}$ lb. per running foot, and the rods weigh 13 oz. per running foot. What will the grating cost at $6\frac{1}{4}¢$ a pound, not allowing for corners nor for welding the joints? Draw to scale a plan of the grating.

Railway Problems

WRITTEN EXERCISE

1. If ties are 8″ wide, and are placed 18″ apart, how many ties are there to a mile?

2. We use 90 million new ties a year, averaging 8″ × 6″ × 8′ 6″. How many cubic feet of timber do we use?

3. The modern coal car has a capacity of 100,000 lb. At 35 cu. ft. to the ton (of 2000 lb.), what is the volume?

4. The Pikes Peak railway makes an ascent of 7552 ft. in a length of $8\frac{3}{4}$ mi. What is the average gradient (ratio of ascent to length)?

5. The rails are usually 30′ long and often weigh 80 lb. to the yard. If a man carries 160 lb., how many men will it take to carry a rail?

6. The standard gauge in America and England is 4′ $8\frac{1}{2}$″. Express this in meters, as used in most European countries, the meter being 39.37 in.

7. The standard American gauge (see Ex. 6) is $\frac{1}{5}$ more than the gauge of a certain mountain road. What is the gauge of the latter?

8. If the cost of maintaining the single track of a certain railway averages $846 a mile, what is the cost of maintaining its 450 mi. of double track?

9. In a year when our country had 1,189,000 men employed in the railroad business, 4% were employed as engineers. The average wages for engineers that year were $3.84 a day, which was 20% more than the average wages for conductors. There were 35,070 conductors employed that year. How much more was the total paid that year for engineers' wages than for conductors'?

558. The great ocean steamers. Some idea of the great shipping industry may be obtained by thinking that in one year 857,000 immigrants (steerage or third-cabin passengers) came to our country, besides the first- and second-cabin passengers. A large ocean steamer has a capacity of over 2,000,000 cu. ft., perhaps a thousand times the capacity of your recitation room.

WRITTEN EXERCISE

1. One of the fastest day's runs recorded for a steamer is 601 knots. How many statute miles is this?

2. A certain ocean steamer can carry 3192 persons, including the crew. The crew number 12% as many as the passengers. How many are there of each?

3. Prior to 1860 the best steamship record between New York and Queenstown was 9 da. 2 hr. When it was reduced to 5 da. 7 hr., what was the per cent of reduction?

4. To build one of the large Atlantic steamers, 1400 plates of steel were used in the hull alone. They weighed 4 long tons each. How many tons did they all weigh? How many pounds?

5. A steamer carries 350 first-cabin passengers, 48% as many in the second cabin, and 400% as many in the third cabin (steerage) as in the other two together. How many passengers does it carry?

6. A 20,000-ton boat carries 12 times as much freight as the old-style ocean steamers, makes 25% better time, and the expenses are only 4 times as much. In one year such a modern boat will do how many times the work of the old kind? At the same cost, it will carry how many times as much freight?

559. Problems in meteorology. Problems concerning rainfall, temperature, and the general state of the weather are called problems in *meteorology*. They are so related to agriculture as to have a place among industrial problems.

WRITTEN EXERCISE

1. What is the weight of an inch of rainfall upon an acre, taking the weight of 1 cu. ft. of water as 1000 oz.? What is it on a 200-acre farm? Answer in tons.

2. The lowest daily temperatures registered during a certain week in January, in Buffalo, were 7°, 3.6°, – 9° (9° below zero), 6°, – 3°, 0°, 8°. What is the average of these readings?

3. In our common Fahrenheit (F.) thermometer the freezing point of water, 32°, is the 0° in the Centigrade (C.) thermometer, and the boiling point, 212°, is the 100° in the Centigrade. Express 4° C. in Fahrenheit, and 62° F. in Centigrade. Draw each thermometer to scale.

4. A barometer registered 29.024 in. on Monday. In the next 4 days it rose 0.135 in., 0.044 in., 0.095 in., and 0.573 in. On Saturday and Sunday it fell 0.021 in. and 0.417 in. What was the Friday reading? the Sunday reading? What were the weather indications on Friday? on Sunday?

A rising barometer indicates fair weather. When the mercury falls rapidly a storm is indicated.

5. The mean (average) annual rainfall at Mobile is 62.2 in.; at Sacramento, 20.9 in.; at Denver, 14.5 in.; at Indianapolis, 43 in.; at Des Moines, 33.1 in.; at St. Paul, 27.5 in.; at St. Louis, 41.1 in.; and at Portland, Oregon, 46.8 in. Taking the weight of 1 cu. ft. of water as 62.5 lb., what is the weight of water annually falling on a square mile in each of these cities? Answer in tons.

6. If light travels 186,000 mi. per second, and it takes it $8.158\frac{1}{3}$ sec. to come from the sun to the earth, what is the distance traveled?

7. Water is composed of two gases, oxygen and hydrogen, 88.89% by weight being oxygen. What is the weight of the hydrogen in a cubic foot of water?

8. The air is composed of two gases, oxygen and nitrogen. In every cubic foot of air there are 345.6 cu. in. of oxygen. What per cent of the volume of the air is nitrogen?

9. A certain cirrus cloud is observed to be 6 mi. above the earth, or 240% higher than a certain rain cloud observed a few hours before. How high was the rain cloud?

10. At the temperature when sound travels 1120 ft. per second, what is the distance of a thunder cloud in which lightning is seen $17\frac{2}{5}$ sec. before the thunder is heard?

11. If a large drop of rain falls at the rate of 20 ft. per second, and a small one, blown by the wind, only 25% as fast, how long will it take the small one to reach the earth from a cloud $2\frac{1}{2}$ mi. high?

12. The wind pressure in a hurricane has been known to be as great as 49.2 lb. per square foot. In such a storm how many tons pressure on the side of a large office building 104 ft. long and 308 ft. high?

13. In a great storm the velocity of the wind often reaches 88 ft. per second. What is then its velocity per minute? In a hurricane it has been known to be $66\frac{2}{3}$% greater. What is then its velocity per hour?

14. When the mercury in the barometer is at 30 in., the pressure of air on every square inch of surface is 15 lb. What is the pressure on a pane of glass 2 ft. by 3 ft., when the increased air pressure forces the mercury up to 31 in.? Why does the glass not break?

POWERS AND ROOTS

560. Square numbers and square roots. If a square has a side 4 units, it has an area 16 square units. Therefore 16 is called the *square* of 4, and 4 the *square root* of 16.

561. Square roots of areas. Therefore, considering the *abstract* numbers representing the sides and area,

The side of a square is the square root of its area.

562. Writing squares and roots. The square of 4 is written 4^2; the square root of 16 is written $\sqrt{16}$.

563. Perfect squares. A number like 16 is a *perfect square*, but 10 is not a perfect square. We speak, however, of $\sqrt{10} = 3.16+$, because 3.16^2 nearly equals 10.

564. Square roots of perfect squares. Square roots of perfect squares may often be found by factoring.

$$\begin{array}{r} 3\,)\overline{441} \\ 3\,)\overline{147} \\ 7\,)\overline{49} \\ \hline 7 \end{array}$$

For example, $\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7}$

$= \sqrt{21 \times 21} = 21.$

ORAL EXERCISE

State the square roots of the numbers in Exs. 1–8:

1. 64.	2. 9.	3. 81.	4. 49.
5. 121.	6. 144.	7. 1600.	8. 4900.

What are the sides of squares whose areas are as follows?

9. 64 sq. in.	10. 49 sq. ft.	11. 1.44 sq. in.
12. 0.25 sq. ft.	13. $\frac{1}{144}$ sq. yd.	14. $\frac{1}{121}$ sq. in.

State the perimeters of squares whose areas are:

15. 1.21 sq. ft.	16. 0.49 sq. ft.	17. 169 sq. in.

WRITTEN EXERCISE

By factoring, find the square roots of the following:

1. 625.	2. 324.	3. 484.	4. 729.
5. 576.	6. 2304.	7. 1296.	8. 1089.
9. 65.61.	10. 12.25.	11. 40.96.	12. 14,641.

Find the sides of squares whose areas are as follows:

13. 4.84 sq. in.	14. 1.96 sq. ft.	15. 2.25 sq. ft.
16. 4.41 sq. in.	17. 4356 sq. in.	18. 15,625 sq. in.
19. 5929 sq. yd.	20. 10.89 sq. yd.	21. 0.1225 sq. ft.

Find the perimeters of squares whose areas are as follows:

22. 6561 sq. ft.	23. 12,100 sq. in.	24. 11,025 sq. ft.
25. 19,600 sq. ft.	26. 146.41 sq. in.	27. 20,736 sq. in.
28. 16,384 sq. in.	29. 16,900 sq. ft.	30. 129,600 sq. in.

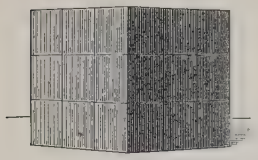
31. From the corner of a square piece of land containing 576 sq. rd. a small square lot containing 64 sq. rd. is cut out. Draw the plan of the lots and find the perimeter of each.

32. A square lot has an area of 169 sq. rd. How far is it around the lot? How far is it around a lot of four times this area? The second perimeter is how many times the first? Plot each lot to a scale.

33. A man has two adjacent building lots fronting on the street, each lot being square. The area of the two together is 89 sq. rd., that of the larger being 64 sq. rd. What is the frontage of the lots?

34. A square lot has an area of 289 sq. rd. How far is it around the lot? (Try the prime numbers between 10 and 20.) How far is it around a lot of nine times this area? The second perimeter is how many times the first?

565. Cube numbers and cube roots. If a cube has an edge 3 units, it has a volume 27 cubic units. Therefore 27 is called the *cube* of 3, and 3 the *cube root* of 27.



566. Cube roots of volumes. Therefore, considering the *abstract* numbers representing the edges and volume,

The edge of a cube is the cube root of its volume.

567. Writing cubes and roots. The cube of 3 is written 3^3; the cube root of 27 is written $\sqrt[3]{27}$.

568. Powers. Squares and cubes are called *powers*. We also have higher powers, like the fourth, fifth, and so on. Raising to powers is sometimes called *involution;* extracting roots, *evolution.*

569. Cube roots of perfect cubes. Cube roots of perfect cubes may often be found by factoring.

For example, $\sqrt[3]{216} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$

$= \sqrt[3]{(2 \times 3) \times (2 \times 3) \times (2 \times 3)}$

$= \sqrt[3]{6 \times 6 \times 6} = 6.$

$2)\overline{216}$
$2)\overline{108}$
$2)\overline{54}$
$3)\overline{27}$
$3)\overline{9}$
3

ORAL EXERCISE

1. $\sqrt[3]{8}$.
2. $\sqrt[3]{125}$.
3. $\sqrt[3]{64}$.
4. $\sqrt[3]{1000}$.
5. $\sqrt[3]{27}$.
6. $\sqrt[3]{0.027}$.
7. $\sqrt[3]{0.125}$.
8. $\sqrt[3]{0.064}$.

WRITTEN EXERCISE

1. $\sqrt[3]{1331}$.
2. $\sqrt[3]{729}$.
3. $\sqrt[3]{512}$.
4. $\sqrt[3]{1728}$.
5. $\sqrt[3]{15625}$.
6. $\sqrt[3]{2744}$.
7. $\sqrt[3]{4096}$.
8. $\sqrt[3]{5832}$.
9. What is the edge of a cube of volume 10,648 cu. in.?

570. Letters used to represent numbers. If we have two letters, like x and y, the product of their values is indicated by xy. If $x = 5$ and $y = 7$, then $xy = 5 \times 7 = 35$, $2xy = 70$, $x^2 = 25$, and $y^2 = 49$.

This is all the work with letters necessary for the understanding of square root. If not already known, a few minutes of drill upon similar work will suffice.

571. Square on the sum of two lines. If we have two lines, f and n, and construct a square on their sum, we see by this figure that there are two squares and two rectangles, f^2, n^2, and fn, fn. Therefore

280	49
1600	280
40	7

$f \quad + n$

The square of the sum of two numbers equals the square of the first, plus twice the product of the first and second, plus the square of the second.

That is, $\quad (f + n)^2 = f^2 + 2fn + n^2$.

572. Illustrative problem. What is the square of 47?

$$47^2 = (40 + 7)^2 = 40^2 + 2 \times 40 \times 7 + 7^2$$
$$= 1600 + 560 + 49 = 2209.$$

ORAL EXERCISE

Square as in § 571:

1. 15. 2. 31. 3. 52. 4. 22. 5. 25.
6. 14. 7. 41. 8. 51. 9. 61. 10. 72.
11. $a + b$. 12. $m + n$. 13. $f + n$. 14. $a + x$.
15. $x + y$. 16. $t + u$. 17. $2 + y$. 18. $4 + x$.

State the square root of:

19. $c^2 + 2cd + d^2$. 20. $30^2 + 2 \times 30 \times 7 + 7^2$.
21. $t^2 + 2tu + u^2$. 22. $20^2 + 2 \times 20 \times 6 + 6^2$.

573. How to find a square root. Find $\sqrt{289}$.

Because 289 is not so readily factored as the numbers we have considered, we take another method of finding the square root.

Imagine a square containing 289 square units (Fig. I).

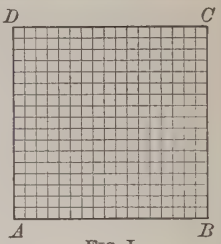


FIG. I

The greatest square of *tens* in 289 (Fig. I) is 100 (Fig. II), for $20^2 = 400$, and this is greater than 289.

Taking away 10^2, or 100, as marked off in Fig. III, we have left 189, as shown in Fig. IV. We may now lay Fig. IV lengthwise as in Fig. V.

Now we know that Fig. V has an area of 189 square units and a length a little over $10 + 10$ units (because of the little square). Therefore if we divide 189 by $10 + 10$, or 20, we shall find nearly the width PB, or the side SC of the little square $SCTQ$. Dividing, $189 \div 20 = 9$ (nearly). But this would make the total length RP (Fig. V) equal $10 + 9 + 10 = 29$, and the area would be $9 \times 29 = 261$, which is greater than 189. Therefore 9 is too large. In the same way, 8 is found to be too large, for it makes the area 224. But 7 is found to be just right, for $7 \times (10 + 7 + 10) = 7 \times 27 = 189$.

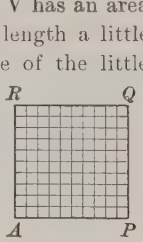


FIG. II

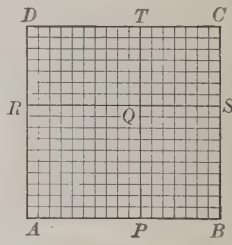


FIG. III

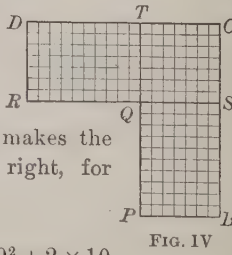


FIG. IV

Therefore $\sqrt{289} = 17$.

Again we see the truth of § 571, for $10^2 + 2 \times 10 \times 7 + 7^2 = 100 + 140 + 49 = 289$. That is, Fig. II + Fig. IV = Fig. I or Fig. III.

The check (or proof) for square root is, of course, the squaring of the result.

Here $17^2 = 289$.

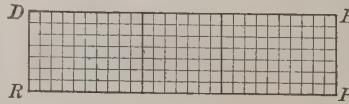


FIG. V

574. This explanation may also be given as follows:

If we let f = the *f*ound part of the root,
and n = the *n*ext figure of the root,
then $(f+n)^2 = f^2 + 2fn + n^2$. (§ 571)

Therefore if we take away f^2 (Fig. II), we shall have $2fn + n^2$ (Fig. IV, where each oblong is $f \times n$, and the small square is n^2).

If we divide this by $2f$, we shall find nearly n, as in Fig. V, where we divided by 2×10 to find 7.

Since the entire explanation of square root depends on this fact, teachers are advised to see that this is clearly understood, both from the figure and from the formula, before proceeding.

	17			
	289	contains	$f^2 + 2fn + n^2$	(Fig. I)
$f^2 =$	100			(Fig. II)
$2f =$ 20	189	"	$2fn + n^2$	(Fig. IV)
$2f + n =$ 27	189	=	" "	

The greatest square of *tens* in 289 is 100, for 20^2 is greater than 289.

This is f^2, and therefore $f = 10$, for 10 is evidently the square root of 100.

189 contains $2fn + n^2$, because f^2 has been subtracted from a number containing $f^2 + 2fn + n^2$.

Dividing this by $2f$ we approximate n, and $n = 7$. I.e., $189 \div 20 = 7$ (nearly), for 8 and 9 would be found to be too large, as on page 496.

But $2f + n$ multiplied by n equals $2fn + n^2$ (Fig. V), thus completing the square.

Therefore $289 = f^2 + 2fn + n^2 = 10^2 + 2 \times 10 \times 7 + 7^2 = 17^2$.

575. The square root of common fractions. It is easy to find the square root of common fractions. For since $(\frac{2}{3})^2 = \frac{4}{9}$, therefore $\frac{2}{3} = \sqrt{\frac{4}{9}}$. That is,

To extract the square root of a common fraction take the square root of each term.

WRITTEN EXERCISE

1. $\sqrt{3249}$.	2. $\sqrt{3721}$.	3. $\sqrt{3969}$.	4. $\sqrt{5041}$.
5. $\sqrt{1681}$.	6. $\sqrt{2209}$.	7. $\sqrt{2809}$.	8. $\sqrt{3481}$.
9. $\sqrt{4489}$.	10. $\sqrt{5329}$.	11. $\sqrt{5929}$.	12. $\sqrt{6241}$.

Find the sides of the squares, given the areas:

13. 6724 sq. ft.	14. 9409 sq. ft.	15. 7225 sq. in.
16. 7569 sq. ft.	17. 7921 sq. rd.	18. 8281 sq. in.
19. 9025 sq. ft.	20. 6889 sq. yd.	21. 9801 sq. yd.

Find the value of the following:

22. $\sqrt{\frac{144}{529}}$.	23. $\sqrt{\frac{961}{1024}}$.	24. $\sqrt{\frac{1089}{1156}}$.	25. $\sqrt{\frac{1936}{5041}}$.
26. $\sqrt{\frac{3025}{3249}}$.	27. $\sqrt{\frac{3481}{8281}}$.	28. $\sqrt{\frac{441}{9801}}$.	29. $\sqrt{\frac{289}{9409}}$.

576. Number of figures in the square root. It is easy to tell in advance the number of figures in the square root of a perfect square. For

The square of units has 2 figures or 1 figure, since

$$9^2 = 81,\ 1^2 = 1.$$

The square of a number of 2 integral places has 4 or 3 integral places, since

$$99^2 = 9801,\ 10^2 = 100.$$

The square of a number of 3 integral places has 6 or 5 integral places, and so on, since

$$999^2 = 998{,}001,\ 100^2 = 10{,}000.$$

577. *If a square number be separated into periods of two figures each, beginning at the decimal point, the number of periods will equal the number of figures in the root.*

Thus, $\sqrt{43'69'21}$ has 3 integral places;

$\sqrt{1.23'21}$ " 1 " and 2 decimal places.

578. Square root with decimals. Find $\sqrt{151.29}$.

The greatest square of 10's in 151.29 is 100. This is f^2, and therefore $f = 10$.

Then 51.29 contains $2fn + n^2$. (Why is this?)

Dividing by $2f(20)$, we find $n = 2$.

We have now found $f + n = 12$, the square being $100 + 44 = 144$.

Since 12 has been *found*, let us call this f (for *found*). Of course, this is not the same as the first number found; it is larger, because we have found more.

	12.3
	1'51.29
	100.
$2f = 20$	51.29
$2f + n = 22$	44.
$2f = 24$	7.29
$2f + n = 24.3$	7.29

7.29 contains $2fn + n^2$, because we have subtracted $f^2 = 144$.

Dividing by $2f$ (24), we find $n = 0.3$.

$2f + n$ multiplied by n equals $2fn + n^2$, the rest of the square.

579. From the examples solved we see that the following are the steps in extracting square root:

1. *Separate into periods of two figures each, beginning at the decimal point.* (See § 577.)

2. *Find the greatest square in the left-hand period and subtract it, bringing down the next period.*

That is, find f^2, the greatest square of 10's, 100's, or 1000's, etc., and subtract it, leaving as a remainder $2fn + n^2$.

3. *Divide the remainder by twice the part already found.*

That is, divide $2fn + n^2$ by $2f$ to find approximately n.

4. *Add the number thus found to this divisor, and multiply by the same number.*

That is, add n to $2f$, and multiply by n, obtaining $2fn + n^2$.

5. *Subtract this result, bring down the next period, and proceed as in 3 and 4.*

That is, having now subtracted a new f^2, proceed as before.

WRITTEN EXERCISE

Extract the square roots in Exs. 1–17:

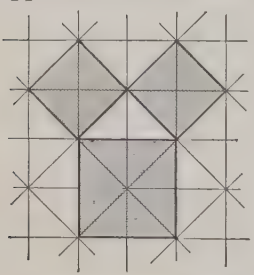
1. 12,321.	2. 54,756.	3. 110.25.
4. 8046.09.	5. 19.4481.	6. 0.2809.
7. 1176.49.	8. 82.2649.	9. 63,001.
10. 21,224,449.	11. 49,112,064.	12. 96,275,344.

In Exs. 13–18 carry the root to two decimal places only.

13. 2. 14. 5. 15. 7. 16. 8. 17. 11.

18. Find the side of a square containing 1 acre (160 sq. rd.).

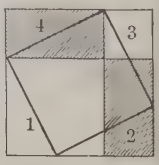
580. Hypotenuse. In a right-angled triangle the side opposite the right angle is called the *hypotenuse.*



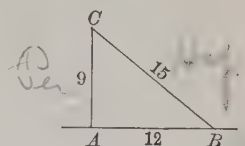
581. Square on the hypotenuse. If a floor is made up of triangular tiles like this, it is easy to mark out a right-angled triangle. In the figure it is seen that the square on the hypotenuse contains 8 small triangles, while each square on a side contains 4 such triangles. Hence

The square on the hypotenuse equals the sum of the squares on the other two sides.

582. This is true for any right-angled triangle. We see that if the 4 triangles, $1 + 2 + 3 + 4$, are taken away from this figure, there remains the square on the hypotenuse. But if we take away the 2 shaded rectangles, which equal the 4 triangles, there remain the squares on the two sides. Therefore the square on the hypotenuse must equal the sum of these two squares.



583. Illustrative problem. If $AB = 12$, and $AC = 9$, how long is BC?



Since $AB^2 + AC^2 = BC^2$,

therefore $12^2 + 9^2 = BC^2$,

or $144 + 81 = 225 = BC^2$,

and $\sqrt{225} = BC$. Therefore $BC = 15$.

WRITTEN EXERCISE

1. How long is the diagonal of a hall 51 ft. by 68 ft.?

2. How long is the diagonal of a square containing 4 sq. ft.? (Two decimals.)

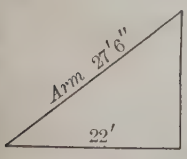
3. The two sides of a right-angled triangle are 20 in. and 30 in. Find the hypotenuse. (Two decimals.)

4. The two sides of a right-angled triangle are 57 in. and 76 in. Find the hypotenuse.

5. What is the direct distance from the cornice of a 100-ft. building to a spot 75 ft. from the foot?

6. Find the length of the hypotenuse when the sides are 321 in. and 428 in.; 40 in. and 75 in.; 72 ft. and 135 ft.

7. A telegraph pole is set perpendicular to the ground, and a wire is fastened to it 18 ft. from the ground, and then to a stake 13 ft. 6 in. from the foot of the pole, so as to hold it in place. How long is the wire?



8. A derrick for hoisting coal has its arm 27 ft. 6 in. long. It swings over an opening 22 ft. from the base of the arm. How far is the top above the opening?

Reversing the procedure in § 581, the square on either side equals the difference of what squares?

Find to two decimal places the hypotenuse of each of the right-angled triangles of which the sides are here given:

9. 35 ft., 26 ft.
10. 81 ft., 35 ft.
11. 10 rd., 13 rd.
12. $42\frac{1}{2}$ ft., $63\frac{1}{4}$ ft.
13. 4.5 in., 7.2 in.
14. 6.25 in., 7.5 in.

It is a good exercise to make up problems like those in Exs. 15–19.

15. A room is 16 ft. long, 12 ft. wide, and 9 ft. high. How far is it from an upper corner diagonally through the room to the opposite lower corner? (Two decimals.)

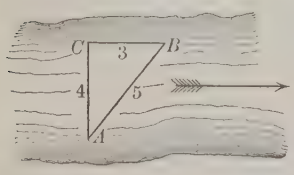
First draw the picture. Then find the hypotenuse on one wall. Then find the hypotenuse required.

16. How long is the diagonal of a cube whose volume is 8 cu. in.? (Two decimals.)

First find the edge. Then proceed as in Ex. 15.

17. A school flag pole is broken by the wind 16 ft. from the ground. The two pieces hold together, and the top of the pole touches the ground 30 ft. from the base. Find the length of the pole.

18. If I start to row directly across a stream, in the direction AC, at the rate of 4 mi. an hour, and if the stream carries me in the direction CB at the rate of 3 mi. an hour, my course will really be AB, the result of these two motions. Suppose I row at the rate of 4.5 mi. per hour, and the stream flows 6 mi., what is my rate of progress?



19. Suppose I walk across the deck of a steamer at the rate of 4 mi. per hour, while the boat moves at the rate of 8 mi. per hour, at what rate am I moving? Answer to two decimal places. (Draw a plan to scale.)

584. Helpful approximations. Because of their frequent use, it is helpful to learn the following approximations:

$\sqrt{2} = 1.414 \quad \sqrt{3} = 1.732 \quad \sqrt{5} = 2.236 \quad \sqrt{10} = 3.162$

585. Illustrative problem. What is the diagonal of a square whose side is 7 in.?

1. The square on the diagonal (hypotenuse) is 49 sq. in. + 49 sq. in. $= 2 \times 49$ sq. in.
2. Therefore the diagonal $= \sqrt{2 \times 49}$ in.
3. But $\sqrt{2 \times 49} = \sqrt{2} \times 7$
4. $= 1.414 \times 7 = 9.898.$

WRITTEN EXERCISE

1. From § 585 write out a rule for finding the diagonal of a square by multiplying its side by a certain number.

Find the diagonals of squares with sides here given:

2. 19.2 in. 3. 32.8 in. 4. 683 ft. 5. 750 rd.

6. If the diamond of a baseball field is a square 90 ft. on a side, how far is it from the first base directly across to the third?

7. A gate 3 ft. high and 6 ft. wide is to be braced by a stick fastened diagonally across it. How long is the stick?

Notice that $\sqrt{45} = \sqrt{9 \times 5} = 3 \times \sqrt{5}$.

8. How far is it from one lower corner of a room directly to the opposite upper corner, the dimensions of the floor being $12' \times 16'$ and the room being 8 ft. high?

Find the diagonals of squares with areas here given:

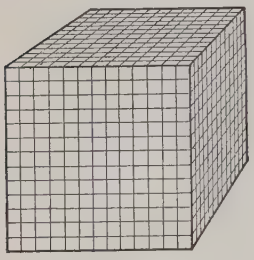
9. 81 sq. ft. 10. 256 sq. ft. 11. 6.25 sq. ft.

Find the sides of squares with diagonals here given:

12. 14.14 in. 13. 4.242 in. 14. 7.07 ft.

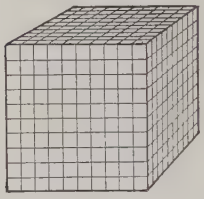
586. How to find a cube root. Find $\sqrt[3]{2197}$.

This subject may be omitted without interfering with the work that follows.



A

Because 2197 is not so readily factored as the numbers we have considered, we take another method of finding the cube root.

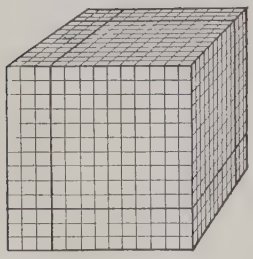


B

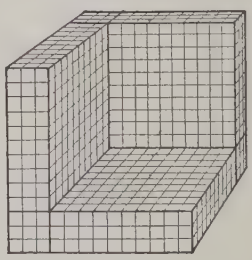
$$\begin{array}{rr} & 13 \\ & \overline{2197} \\ & 1000 \\ 300,\ 99,\ 399 & \overline{1197} \\ & 1197 \end{array}$$

Imagine a cube containing 2197 cubic units, as in A, where of course we cannot see all of the small cubes.

The greatest cube of *tens* in 2197, or A, is 1000 (B), for $20^3 = 8000$, and this is greater than 2197.



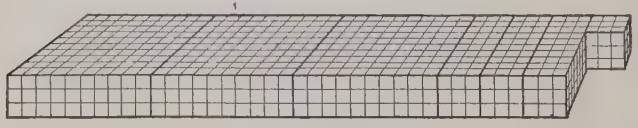
C



D

Taking away 10^3, or 1000, as marked off in C, we have left 1197, as shown in D.

We may now lay D lengthwise, as in E.



E

Now we know that the solids in E have a volume of 1197 cubic units, and that the top surface of the three larger blocks has an area of 3×10^2 units, or 300 square units. The 300 represents nearly the whole top area, the rest being that of the three oblong blocks and the small cube. Therefore if we divide 1197 by 300 (sometimes called the *trial divisor*), we shall have approximately the thickness, or the edge of the small cube. Dividing, $1197 \div 300 = 3$ (nearly).

Now if 3 is the thickness, the area of the top of each oblong piece is 3×10, and the area of the three is $3 \times 3 \times 10$, or 90. The area of the top face of the small cube is 3^2, or 9, making the area of the top of these four pieces $90 + 9$, or 99. Adding the area of the top of the three square blocks, we have $300 + 99$, or 399, as the total top area.

Multiplying 399, the number of square units in the top area, by 3, the number of units of thickness, we have 1197, the number of cubic units, which just completes the cube, D applied to B just making C or A.

587. The cube of $f + n$. From the illustrations in § 586 we see that if we call the first part of the root f and the next part n, the cube of $f + n$ (A) equals the cube of f (B) plus 3 blocks, each with a volume of f^2n, plus 3 other blocks (the oblong ones), each with a volume of fn^2, plus the small cube with a volume n^3. This shows that

$$(f + n)^3 = f^3 + 3f^2n + 3fn^2 + n^3.$$

This we may also see by multiplying $(f + n)^2$ by $f + n$, or $(10 + 3)^2$, which is $10^2 + 2 \times 10 \times 3 + 3^2$, by $10 + 3$, as here shown.

$$\begin{array}{rl}
 & 10^2 + 2 \times 10 \times 3 + 3^2 \\
 & \qquad\qquad\qquad\quad 10 + 3 \\
\hline
 & 10^2 \times 3 + 2 \times 10 \times 3^2 + 3^3 \text{ multiplying by } 3 \\
10^3 + 2 \times 10^2 \times 3 + & \qquad 10 \times 3^2 \qquad\quad \text{“} \qquad \text{“ } 10 \\
\hline
10^3 + 3 \times 10^2 \times 3 + 3 \times 10 \times 3^2 + 3^3 = & f^3 + 3f^2n + 3fn^2 + n^3
\end{array}$$

WRITTEN EXERCISE

Find the cube root of each of the following:

1. 4913.	2. 6859.	3. 9261.	4. 5832.
5. 13,824.	6. 19,683.	7. 29,791.	8. 46,656.
9. 132,651.	10. 157,464.	11. 226,981.	12. 300,763.
13. 551,368.	14. 753,571.	15. 884,736.	16. 941,192.
17. 592,704.	18. 778,688.	19. 857,375.	20. 970,299.

By the method of § 587, *find the cube of each of the following, proving the result by multiplication:*

21. 12, that is, $10 + 2$.

$$(10 + 2)^3 = 10^3 + 3 \times 10^2 \times 2 + 3 \times 10 \times 2^2 + 2^3$$
$$= 1000 + 600 + 120 + 8 = 1728.$$

22. 14.	23. 21.	24. 25.	25. 30.
26. 55.	27. 60.	28. 75.	29. 99.

588. Number of figures in the cube root. It is easy to tell in advance the number of figures in the cube root of a perfect cube. For

The cube of units has 3, 2, or 1 figure, since

$$9^3 = 729,\ 4^3 = 64,\ 1^3 = 1.$$

The cube of a number of 2 integral places has 6, 5, or 4 integral places, since

$$99^3 = 970{,}299,\ 40^3 = 64{,}000,\ 10^3 = 1000.$$

The cube of a number of 3 integral places has 9, 8, or 7 integral places, and so on. Therefore

589. *If a cube number be separated into periods of three figures each, beginning at the decimal point, the number of periods will equal the number of figures in the root.*

Thus, $\sqrt[3]{1{,}771{,}561}$ has 3 integral places;

$\sqrt[3]{26.463{,}592}$ " 1 " and 2 decimal places.

590. Cube root by the formula for $(f+n)^3$. 1. Find $\sqrt[3]{2197}$.

If we let $f =$ the *f*ound part of the root,
and $n =$ the *n*ext figure of the root,
then $(f+n)^3 = f^3 + 3f^2n + 3fn^2 + n^3$. (§ 587)

Therefore if we take away f^3 (B on page 504), we shall have $3f^2n + 3fn^2 + n^3$ (D, page 504, where each square block is f^2n, each oblong fn^2, and the small cube is n^3).

If we divide this by $3f^2$, we shall find approximately n (as in E, page 504, where we divided the number of units of volume by the number of units of area of the three squares).

				1 3	
				2,197	$\sqrt[3]{2,197} = 13$
$3f^2$	$3fn+n^2$	$3f^2+3fn+n^2$	$f^3 =$	1000	
300	99	399		1197	contains $3f^2n+3fn^2+n^3$
				1197	= " " "

The greatest cube of *tens* in 2197 is 1000, for $20^3 = 8000$, and this is greater than 2197. Therefore f^3 is 1000. Since $f^3 = 1000$, f is 10.

The remainder, 1197, contains $3f^2n + 3fn^2 + n^3$, because from a number containing $(f+n)^3$, or $f^3 + 3f^2n + 3fn^2 + n^3$, f^3 has been subtracted.

Dividing this by $3f^2$ (as seen also on page 504), we approximate n, and $n = 3$.

Now the quantity which multiplied by n equals $3f^2n + 3fn^2 + n^3$, and thus, with the f^3 already found, completes the cube of $f+n$, is $3f^2 + 3fn + n^2$. (See also page 504.)

Therefore we add to $3f^2$ the quantity that will make this, that is, $3fn + n^2$. Now $3fn + n^2 = 3 \times 10 \times 3 + 3^2 = 99$, and $300 + 99 = 399$.

Multiplying 399 by 3, the result is 1197, exactly completing the cube.

It is easily seen that 2197 must be the cube of 13, for (§ 587) $13^3 = 10^3 + 3 \times 10^2 \times 3 + 3 \times 10 \times 3^2 + 3^3 = 1000 + 900 + 270 + 9 = 1000 + 1179$, which are exactly the numbers above subtracted from 2197, leaving no remainder.

2. Find $\sqrt[3]{1{,}771{,}561}$.

$3f^2$	$3fn+n^2$	$3f^2+3fn+n^2$	$f^3=1\,000\,000$				
30,000	6400	36,400	771 561	contains	$3f^2n+3fn^2+n^3$	$f=100$	
			728 000	=	"	"	$n=20$
43,200	361	43,561	43 561	contains	"	"	$f=120$
			43 561	=	"	"	$n=1$

$\sqrt[3]{1{,}771{,}561}=121$

There cannot be any thousands in the root, for $1000^3 = 1{,}000{,}000{,}000$, and this is larger than the given cube.

The greatest cube of *hundreds* in 1,771,561 is 1,000,000, because 200^3 is larger than the given number. Therefore $f = 100$.

Then the remainder, 771,561, contains $3f^2n + 3fn^2 + n^3$, because f^3, or 1,000,000, has been subtracted from a number containing $f^3 + 3f^2n + 3fn^2 + n^3$.

Because this contains approximately $3f^2n$, if we divide by $3f^2$ (or 30,000) we shall find approximately n. We therefore find that $n = 20$.

Adding $3fn + n^2$, or $3 \times 100 \times 20 + 20^2$, to $3f^2$, we have 36,400 (as explained on page 507). This multiplied by 20 (that is, by n) equals 728,000, thus completing the cube of 120.

That 1,000,000 and 728,000 are together the cube of 120 is evident from the fact that $120^3 = 100^3 + 3 \times 100^2 \times 20 + 3 \times 100 \times 20^2 + 20^3$ (§ 587) $= 1{,}000{,}000 + 600{,}000 + 120{,}000 + 8000 = 1{,}000{,}000 + 728{,}000$.

	1 2 1
	1,771,561
	1
300	771
64	
364	728
43200	43561
361	
43561	43561

We may now consider 120 as the found part of the root, and proceed as before, finding $n = 1$. The root is therefore 121.

The annexed arrangement shows all of the necessary figures, and is the one recommended after the process is understood. It simply preserves the figures actually used from step to step, not writing the others.

In case there are more figures in the root we may subtract and proceed as before, dividing again by three times the cube of the found part.

591. The following are therefore seen to be the steps in extracting cube root:

1. *The number being already separated into three-figure periods, beginning at the decimal point, find the greatest cube in the left-hand period. This is the cube of the number represented by the first figure.*

2. *Subtract this cube, bringing down but one period.*

3. *Divide this remainder by three times the square of the found part considered as tens (the so-called trial divisor) to find the next figure.*

4. *To three times the product of the two parts add the square of the second, considered as units, and add all this to the trial divisor, thus making the complete divisor.*

5. *Multiply the complete divisor by the second, thus completing the cube of the first two.*

6. *Subtract this, bring down the next period, and proceed as before.*

The subject of cube root being now so commonly postponed until algebra is studied, a more extended explanation is not felt to be necessary.

WRITTEN EXERCISE

Extract the cube root of each of the following:

1. 2197.	2. 3375.	3. 2744.
4. 68,921.	5. 74,088.	6. 91,125.
7. 97,336.	8. 110,592.	9. 140,608.
10. 205,379.	11. 250,047.	12. 4,330,747.
13. 1,295,029.	14. 1,771,561.	15. 2,628,072.
16. 4,096,000.	17. 4,826,809.	18. 50,653,000.
19. 54,010,152.	20. 97,336,000.	21. 99,252.847.
22. 91,733.851.	23. 0.114791256.	24. 34,645,976.

MENSURATION

592. Ratio of circumference to diameter. By measuring several circles, dividing each circumference by its diameter, and taking the average of the results, the circumference will be found to be about $3\frac{1}{7}$ times the diameter.

593. Value of π. It is proved in Geometry that this ratio of circumference to diameter is more nearly 3.1416. The ratio is denoted in mathematics by the Greek letter π (pī).

594. Formula for circumference. Therefore, if c = circumference, d = diameter, and r = radius, we have

$$\frac{c}{d} = \pi,$$

whence $c = \pi d$,

or, because $d = 2r$, $c = \pi \times 2r = 2\pi r$

$= 2 \times 3\frac{1}{7} \times$ radius (nearly).

595. Illustrative problems. 1. Required the circumference when the radius is 7 in.

$$c = 2\pi r = 2 \times 3\tfrac{1}{7} \times 7 \text{ in.} = 44 \text{ in.}$$

More exactly, $2 \times 3.1416 \times 7$ in. = 43.9824 in.

2. Required the diameter when the circumference is 2827.44 in.

Since $\pi d = c$, $d = \dfrac{c}{\pi}$, or $d = \dfrac{2827.44 \text{ in.}}{3.1416} = 900$ in.

ORAL EXERCISE

Using $\pi = 3\frac{1}{7} = \frac{22}{7}$, state the circumferences of circles of diameters as follows:

1. 7 in.	2. 21 in.	3. 14 in.	4. 70 in.
5. 28 in.	6. 35 in.	7. 42 in.	8. 700 in.
9. 77 in.	10. 140 in.	11. 280 in.	12. 350 in.
13. 0.7 in.	14. 0.21 in.	15. 0.14 in.	16. 0.28 in.

WRITTEN EXERCISE

Find the circumference ($\pi = 3\frac{1}{7}$), *given the diameter:*

1. 68.2 in.	2. 48.3 ft.	3. 423 in.
4. 5.11 ft.	5. 53.9 ft.	6. 6.37 in.
7. 4.69 ft.	8. 58.1 in.	9. 6.02 ft.
10. 13 ft. 6 in.	11. 17 ft. 8 in.	12. 64.4 ft.

Find the diameter ($\pi = 3\frac{1}{7}$), *given the circumference:*

13. 132 ft.	14. $58\frac{2}{7}$ ft.	15. $97\frac{3}{7}$ ft.
16. 176 in.	17. 770 ft.	18. $96\frac{4}{5}$ ft.
19. 68.2 in.	20. 3.96 ft.	21. 0.484 ft.

Find the circumference ($\pi = 3.1416$), *given the diameter:*

22. 17 in.	23. 13 in.	24. 2.8 in.
25. 4.37 in.	26. 2.25 ft.	27. 6 ft. 2 in.

Find the diameter ($\pi = 3.1416$), *given the circumference:*

28. 53.4072 in.	29. 84.8232 in.	30. 97.3896 in.
31. 94.2478 ft.	32. 182.2228 ft.	33. 138.2304 ft.

Find the circumference ($\pi = 3\frac{1}{7}$), *given the radius:*

34. 49 in.	35. 77 in.	36. 91 in.
37. 105 in.	38. 15.4 in.	39. $1\frac{3}{4}$ in.

40. What is the diameter of a water tank whose circumference is 74.8 ft.? ($\pi = 3\frac{1}{7}$.)

41. What is the circumference of a rod whose diameter as measured by the calipers is 2.1 in.? ($\pi = 3\frac{1}{7}$.)

42. What is the circumference of a wire whose diameter as measured by the calipers is 0.63 in.? ($\pi = 3\frac{1}{7}$.)

43. What radius must be used in drawing a pattern for a wheel that shall be $91\frac{1}{7}$ in. in circumference? ($\pi = 3\frac{1}{7}$.)

596. Area of a circle. A circle can be separated into figures which are nearly triangles. The height is the radius, and the sum of the bases is the circumference. If these

were exact triangles the area would be $\frac{1}{2} \times r \times c$, or $\frac{1}{2}rc$. It is proved in Geometry that this is the true area.

597. Given the radius, to find the area. If a = area, c = circumference, and r = radius,

$a = \frac{1}{2}rc$, or, because c is the same as $2\pi r$, we may write

$$a = \tfrac{1}{2}r \times 2\pi r, \text{ or } a = \frac{r \times \cancel{2}\pi r}{\cancel{2}} = \pi r^2.$$

598. *The area of a circle is π times the square on the radius.*

599. Illustrative problem. Required the area of a circle whose radius is 5 in.

$a = \pi r^2 = \pi \times 25$ sq. in. $= 3.1416 \times 25$ sq. in. $= 78.54$ sq. in.

In the rest of the problems involving π, use the value $3\frac{1}{7}$ unless otherwise directed.

ORAL EXERCISE

State the area, given the radius as follows:

1. 7 in. 2. 70 in. 3. $\frac{1}{2}$ in. 4. $\frac{7}{2}$ in.

5. If you take a radius half as long, the area will be what part as great?

6. If you double the length of the radius, the area will be how many times as large?

600. Illustrative problem. Required the radius, the area being 50.2656 sq. in.

1. Since $a = \pi r^2$, $r^2 = \frac{a}{\pi}$, by dividing these equals by π.
2. 50.2656 sq. in. $\div$ 3.1416 = 16 sq. in.
3. Since $\sqrt{16} = 4$, the radius is 4 in.

WRITTEN EXERCISE

1. What is the area of the cross section of a water pipe that is 2 in. in diameter?

2. How many square feet in the base of a water tank that is 42 ft. in diameter?

3. A horse tethered by a rope 21 ft. long can graze over how many square feet of ground?

4. A horse tethered by a rope can graze over 3850 sq. ft. of ground. How long is the rope?

5. How long is the equator on a globe 12.67 in. in diameter? What is the area of the equator circle cut from such a globe?

6. A school flag pole has a circumference of 24.2 in. at the base. What is the diameter? the radius? the area of a cross section?

7. What is the area of the cross section of a circular iron beam whose circumference is 31.416 in.? (Use $\pi = 3.1416$.) Suppose the circumference were 53.4072 in.?

8. A boy has a ball tied to a string 1 yd. 6 in. long. As he swings it around, how long is the circumference traveled by the ball? What is the area of the circle inclosed?

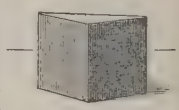
9. A tinsmith wishes to make a pattern for the bottom of a pail, the area being 154 sq. in., and to allow $\frac{1}{4}$ in. all around for soldering. What radius should he use in drawing the circle?

ORAL EXERCISE

1. A represents a cube 1 in. on an edge. What is its volume? Suppose it were 2 in. on an edge?

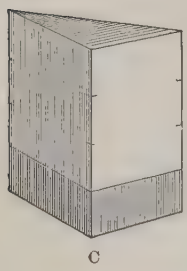
A

2. B represents half of a cube that was 1 in. on an edge. What is its volume? Suppose the original cube were 3 in. on an edge?


B

3. What is the volume of a prism 1 in. high, with a base $\frac{1}{2}$ sq. in.? 1 sq. in.? 3 sq. in.?

4. Suppose the area of the base in C is 5 sq. in., what is the volume of the lower shaded part that is 1 in. high? What is the total volume?



C

601. Volume of a prism. We see that, if b is the area of the base of a prism, h is the height, and v is the volume, $v = bh$.

Pupils should be led to read their own rules from all such formulas. Thus,

602. *The volume of a prism equals the area of the base multiplied by the height.*

This is to be understood with the meaning given in § 367.

State the volumes of prisms with bases and altitudes as follows:

5. 9 sq. in., 4 in.
6. 17 sq. in., 7 in.
7. 16 sq. in., 5 in.
8. 440 sq. in., 25 in.
9. 480 sq. in., $12\frac{1}{2}$ in.
10. 900 sq. in., 50 in.
11. 330 sq. in., $33\frac{1}{3}$ in.
12. 480 sq. in., $16\frac{2}{3}$ in.
13. 8000 sq. in., 75 in.
14. 500 sq. ft., $10\frac{1}{5}$ ft.

WRITTEN EXERCISE

Find the volumes of prisms with bases and altitudes as follows:

1. 375 sq. in., 29 in.
2. 1.28 sq. ft., 3.2 in.
3. 67.9 sq. in., 4.8 in.
4. 42.6 sq. in., 7.3 in.
5. 61.3 sq. ft., 2.91 ft.
6. 62.8 sq. in., 3.17 in.

Find the altitudes of prisms with volumes and bases as follows:

7. 243 cu. in., 29 sq. in.
8. 783 cu. in., 87 sq. in.
9. 140 cu. in., $17\frac{1}{2}$ sq. in.
10. $178\frac{1}{4}$ cu. in., $19\frac{1}{4}$ sq. in.

Find the volumes of rectangular solids as follows:

11. 42 in. by 6.8 in. by 3.5 in.
12. 2.9 in. by 3.8 in. by 6.4 in.
13. 3.91 ft. by 4.27 ft. by 6.8 ft.
14. 2.8 ft. by 3.25 ft. by 4.75 ft.
15. 6.5 ft. by 4.75 ft. by 3.25 ft.
16. 21.3 in. by 29.2 in. by 3.7 in.

17. The area of one face of a cube is 3721 sq. in. What is the volume of the cube?

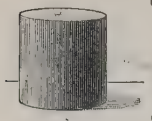
18. A prism with a square base of 127.69 sq. in. is twice as high as it is wide. Find its volume.

19. A prism has a square base, a volume of 4851 cu. in., and a height of 11 in. Required the lengths of the edges.

20. A prism has a square base, a volume of $1687\frac{1}{2}$ cu. in., and an altitude that is half its width. Find its altitude.

If the altitude equaled the width, $1687\frac{1}{2}$ would be the cube of the edge. As it is, $1687\frac{1}{2}$ is therefore half the cube of the side of the base.

603. Volume of a cylinder. The same reasoning as that employed on page 514 evidently gives the volume of a cylinder. Hence we may say that



$$v = bh.$$

604. *The volume of a cylinder equals the area of the base multiplied by the height.*

605. If the cylinder is the common one with a circular base, the area of the circle is πr^2 (§ 597). Therefore, with such a cylinder,

$$v = \pi r^2 h.$$

ORAL EXERCISE

State the volumes, given bases and altitudes as follows:

1. 0.48 sq. in., $\frac{7}{8}$ in.
2. 1.2 sq. ft., $\frac{1}{12}$ ft.
3. 300 sq. in., $33\frac{1}{3}$ in.
4. 150 sq. in., 50 in.
5. 600 sq. in., $66\frac{2}{3}$ in.
6. 80 sq. in., $12\frac{1}{2}$ in.

WRITTEN EXERCISE

1. Find the volume of a water tank 40 ft. high and 40 ft. in diameter.

2. How many cubic feet in a log 16 ft. long, the average diameter being 1 ft. 11 in.?

3. How many gallons (231 cu. in.) in a cylindrical tank 32 ft. high and 30 ft. in diameter?

4. Find the number of cubic feet in a boiler 15 ft. long and 3 ft. 6 in. in diameter, internal measure.

5. Find the number of cubic inches in a pipe 18 ft. long, 4 in. in external diameter, the metal being $\frac{1}{4}$ in. thick.

6. What is the volume of a cylinder with radius 7 in. and height 17 in.? with radius 9 in. and height 45 in.?

ORAL EXERCISE

1. If we slit the curved surface of the cylinder shown on page 516, and spread it out flat, what will it become? How may we find the area of this surface? Try it.

2. What is the area of the curved surface of a cylinder 6 in. high and 8 in. around? 9 in. high and 9 in. around?

3. How many square feet of tin are needed for a pipe 8 ft. long and 6 in. around, allowing 1 sq. ft. for overlapping?

606. Curved surface of a cylinder. We see that *the area of the curved surface of a cylinder equals the product of the height and the circumference.*

That is, if c = circumference, h = height, and a = area,

$$a = ch.$$

Or, if r = radius of the base,

$$a = 2\pi r \times h.$$

State the area of the curved surfaces of cylinders with heights and circumferences as follows:

4. 25 in., 80 in.
5. $33\frac{1}{3}$ in., 69 in.
6. 75 in., 40 in.
7. 50 in., 106 in.
8. $66\frac{2}{3}$ in., 48 in.
9. $12\frac{1}{2}$ in., 24 in.
10. $16\frac{2}{3}$ in., 36 in.
11. $33\frac{1}{3}$ in., 33 in.

12. How many square inches in the curved surface of a wire 1 in. around and 200 ft. long?

13. A tin cup is 7 in. around and 3 in. high, both measures including allowance for soldering. How many square inches of tin are needed for the curved surface?

14. A tin water pipe has a circumference of 9 in. and a length of 10 ft., both measures including allowance for soldering. How many square inches of tin are needed?

WRITTEN EXERCISE

1. How many square inches of surface on a wire 10 ft. long and $\frac{7}{8}$ in. in circumference?

2. If the height and diameter of a solid cylinder are both 7 in., what is the total area of surface?

To the cylindrical surface add the areas of the circles forming the top and bottom.

3. How many square inches of tin are needed for a cylindrical cup 4 in. high and 4 in. in diameter?

4. How many square feet in the curved surface of a water tank that is 40 ft. high and 127.3 ft. around?

5. Write a rule for finding the circumference of a circle, given the radius; also for finding the area of a circle.

6. How many square feet of surface on the outside of a smokestack 30 ft. high and 2 ft. in exterior diameter?

7. The curved surface of a granite cylindrical shaft, 3 ft. 6 in. in diameter and 22 ft. 3 in. long, is to be polished. How many square feet of surface are to be polished?

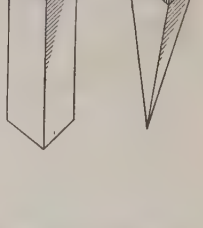
8. In a certain factory there is a room heated by 210 ft. of steam pipe, 2 in. in diameter. Required the radiating surface. (That is, the area of the curved surface which radiates the heat.)

9. A large suspension bridge has 4 cables, each 1872 ft. long and 1 ft. 2 in. in diameter. In letting the contract for painting these cables it is necessary to know their surface. Compute it.

10. In making metal bedsteads some iron rods 0.75 in. in diameter are covered with thin rolled brass. Suppose a shop needs 6000 ft. of such rods, how many square feet of rolled brass will be needed, not allowing for waste?

607. Volume of a pyramid. If we construct a hollow prism and a hollow pyramid of the same base and height, filling the former with sand, the latter can be filled three times with the same amount. Therefore

$$v = \tfrac{1}{3}\, bh.$$



608. *The volume of a pyramid equals one third the product of its base and height.*

ORAL EXERCISE

State the volumes of pyramids with bases and altitudes as follows:

1. 75 sq. ft., 7 ft.
2. 21 sq. ft., 9 ft.
3. 33 sq. ft., 6 ft.
4. 120 sq. ft., 7 ft.
5. 70 sq. in., 9 in.
6. 63 sq. ft., 12 ft.
7. 39 sq. in., 10 in.
8. 38 sq. in., 15 in.
9. 450 sq. in., 20 in.
10. 480 sq. in., 10 in.

WRITTEN EXERCISE

Find the volumes of pyramids with bases and altitudes as follows:

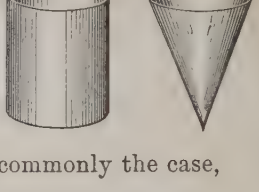
1. 129.3 sq. in., 3.72 in.
2. 702.39 sq. in., 16.8 in.
3. 202.11 sq. ft., 8.92 ft.
4. 187.02 sq. ft., 10.7 ft.

Find the bases of pyramids with volumes and altitudes as follows:

5. 89.6 cu. in., 12.8 in.
6. 123.2 cu. in., 17.6 in.
7. 178.2 cu. in., 19.8 in.
8. 140.4 cu. in., 23.4 in.

9. The Great Pyramid in Egypt is $480\frac{3}{4}$ ft. high. Its base is a square, 764 ft. on a side. What is its volume? If it weighs 168 lb. per cubic foot, what is its weight?

609. Volume of a cone. In the same way that we found the volume of a pyramid, we may find that of a cone.



The volume of a cone equals one third the product of its base and height.

610. If the base is circular, as is commonly the case,

$$v = \tfrac{1}{3}\pi r^2 h.$$

It is desirable that these solids should be constructed by the class.

611. Illustrative problem. What is the volume of a cone 5 in. high, the radius of the base being 2 in.?

1. The area of the base $= \pi r^2 = 3.1416 \times 4$ sq. in.
2. The volume $= \frac{1}{3} \times 5 \times 3.1416 \times 4$ cu. in. $= 20.944$ cu. in.

WRITTEN EXERCISE

Find the volumes of cones, given the radii and heights as in Exs. 1–4, using $\pi = 3\frac{1}{7}$ in all the exercises:

1. $r = 7.7$ in., $h = 8.2$ in.
2. $r = 4.2$ in., $h = 7.3$ in.
3. $r = 6.3$ in., $h = 9.8$ in.
4. $r = 11.9$ in., $h = 5.7$ in.

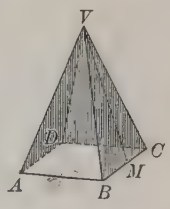
5. What is the volume of a cone whose altitude is 18 in. and the circumference of whose base is $25\frac{1}{7}$ in.?

6. A water tank is 40 ft. high and $13\frac{3}{4}$ ft. in circumference. What is its capacity in cubic feet? in gallons?

7. What is the volume of a cone whose altitude equals the diameter of the base, the circumference of the base being 44 in.?

8. The interior diameter of a water pipe is 4 in. The water flows through at the rate of 2 ft. per second. How much water flows through in a minute?

612. Regular pyramid. If the sides and angles of the base of a pyramid are respectively equal, and the vertex is exactly over the center of the base, the pyramid is said to be *regular*.



613. Slant height. The altitude of any triangle forming a side of a regular pyramid is called the *slant height* of the pyramid.

VM is the slant height of the pyramid shown.

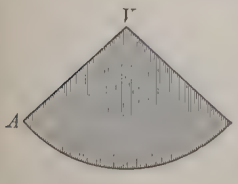
614. Surface of a regular pyramid. Since the lateral (side) surface of a regular pyramid is made up of triangles, each of which equals half the base times the altitude (page 324), therefore

The lateral area of a regular pyramid equals the perimeter of the base multiplied by half of the slant height.

For example, if AB in the above pyramid is 5 ft., and VM is 8 ft., the lateral area is $4 \times 5 \times \frac{1}{2}$ sq. ft., or 80 sq. ft.

615. Surface of a cone. If we should slit the surface of this common form of cone, often called a *right circular cone*, and flatten it out, as here shown, we should have part of a circle. In the same way that we found the area of a circle (§ 596) we find that

The lateral area of a right circular cone equals the circumference of the base multiplied by half of the slant height.



For example, if the circumference is 10 ft. and the slant height is 8 ft., the lateral area is 10×4 sq. ft., or 40 sq. ft.

In the exercises on page 522 it is understood that regular pyramids and right circular cones are given unless the contrary is stated.

WRITTEN EXERCISE

Find the lateral surfaces of regular pyramids with the following perimeters of bases and slant heights:

1. 16 in., 4 in.
2. 32 in., 17 in.
3. 48 in., 15 in.
4. 3 ft. 4 in., 7 in.
5. 2 ft. 8 in., 11 in.
6. 6 ft. 7 in., 15 in.

Also of right circular cones with the following circumferences and slant heights:

7. 17.2 in., $5\frac{3}{8}$ in.
8. 16.5 in., $4\frac{7}{8}$ in.
9. 14.9 in., 2.8 in.
10. 6 ft. 8 in., 1 yd.
11. 3 ft. 4 in., 17 in.
12. 2 ft. 7 in., 19 in.

13. A cone has a lateral area of 220 sq. in. The radius of the base is 7 in. Find the slant height.

14. A pyramid has a lateral area of 200 sq. in. The slant height is 8 in. Find the perimeter of the base.

15. A conical spire has a slant height of 68 ft. The perimeter of the base is 60 ft. What is the lateral surface?

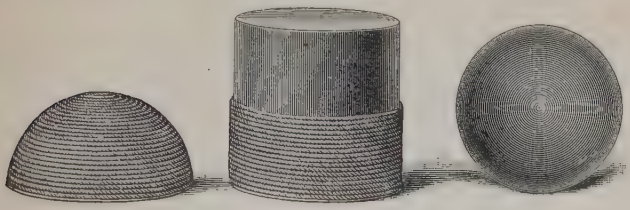
16. What is the entire area of a cone, including the base, the slant height of which is 7 ft., the diameter of the base being 7 ft.?

17. The Great Pyramid of Cheops has a square base 764 ft. on a side. The slant height is about 500 ft. About what is the lateral area of the pyramid?

18. How many square feet should be allowed for slating a steeple in the form of a regular pyramid of slant height 40 ft., the perimeter of the base being 48 ft.?

19. A dome is surmounted by a gilded cone whose slant height is 16 ft. 8 in., the radius of the base being 3 ft. 6 in. How many square feet must be allowed for gilding?

616. Surface of a sphere. If we wind a sphere with cord, and wind a cylinder whose radius equals the radius of the



sphere, and whose height equals the diameter, we find that it takes as much cord for the cylinder as for the sphere.

In the picture, for convenience, only half of each is wound.

Therefore *the surface of a sphere equals the curved surface of a cylinder of the same radius and height.*

617. Formula for area of the sphere. Since the curved surface of the cylinder is $2\pi r \times h$, where $h = 2r$, therefore

$$\text{area of sphere} = 2\pi r \times 2r = 4\pi r^2.$$

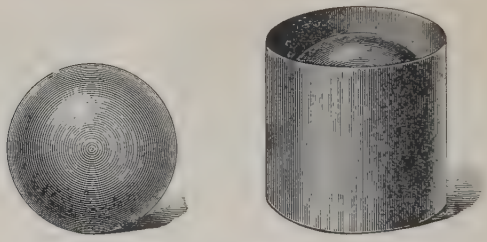
618. Illustrative problem. If the earth is a sphere of 4000 mi. radius, what is its area?

1. $4000^2 = 16{,}000{,}000$.
2. $4\pi \times 16{,}000{,}000 = 4 \times 3\frac{1}{7} \times 16{,}000{,}000 = 201{,}142{,}857\frac{1}{7}$.
3. Therefore the area is about 201,143,000 sq. mi.

WRITTEN EXERCISE

1. If a ball has a radius of $2\frac{1}{4}$ in., what is its area?
2. The sun's diameter is 866,500 mi. Find its area.
3. If a tennis ball is $2\frac{3}{8}$ in. in diameter, what is its area?
4. A gilded ball is to be put on top of a tower. The diameter of the ball is 2′ 6″. How many square inches are to be gilded, making no allowance for the support?

619. Sphere and cylinder compared. If we place this sphere in this cylinder of the same diameter and height, and fill



in the spaces with sand, the sand will fill one third of the cylinder when the sphere is removed. Therefore

A sphere equals two thirds of a cylinder of the same diameter and height.

620. Volume of a sphere. And since the volume of the cylinder is $\pi r^2 \times 2r$, or $2\pi r^3$ (§ 604), and the sphere is $\frac{2}{3}$ as large, or $\frac{2}{3}$ of $2\pi r^3$,

The volume of a sphere is $\frac{4}{3}\pi r^3$.

621. Illustrative problem. What is the volume of a sphere whose radius is 21 in.?

$$\tfrac{4}{3}\pi r^3 = \tfrac{4}{3} \times 3\tfrac{1}{7} \times 21^3$$

$$= \frac{4 \times 22 \times \cancel{21} \times 21 \times 21}{\cancel{3} \times \cancel{7}} = 38{,}808.$$

Therefore the volume is 38,808 cu. in.

In the following examples and on pages 525, 526, use $\pi = 3\frac{1}{7}$.

WRITTEN EXERCISE

What are the volumes of spheres with radii as follows?

1. $2\frac{3}{8}''$. 2. $10.5''$. 3. 1000 mi. 4. $7'\ 6''$. 5. $9'\ 7''$.

Also with diameters as follows?

6. $3'\ 6''$. 7. $4'\ 8''$. 8. 8000 mi. 9. $9.6'$. 10. $11.4'$.

Find the areas of the squares whose sides are as follows:

11. $\frac{23}{48}$ in. 12. $\frac{13}{16}$ in. 13. $\frac{17}{36}$ in. 14. $\frac{15}{64}$ in.
15. 2.87 ft. 16. 3.92 ft. 17. 17.5 rd. 18. 0.37 in.

Find the areas of the rectangles whose dimensions are:

19. 3.4 ft., 9.2 ft. 20. 6.4 in., 7.3 in.
21. $1\frac{1}{4}$ yd., $27\frac{1}{2}$ in. 22. 4.1 ft., 11.3 in.
23. 4.92 in., 0.97 in. 24. 14 ft. 3 in., 2 ft. 8 in.

Find the areas of the triangles whose bases and altitudes are respectively as follows:

25. 4.7 ft., 3.8 ft. 26. $6\frac{1}{4}$ yd., 27 in.
27. 0.37 in., 0.42 in. 28. 6.4 in., 4.9 in.
29. $4\frac{1}{2}$ yd., 3 ft. 9 in. 30. 6.8 ft., 11.9 in.

Given rectangles with areas and lengths as follows, to find the widths:

31. 306 sq. in., 17 in. 32. 90.3 sq. in., 43 in.
33. 14.62 sq. in., 4.3 in. 34. 4.18 sq. in., 2.2 in.
35. 473 sq. in., 3 ft. 7 in. 36. 850 sq. in., 4 ft. 2 in.

Given triangles with areas and bases as follows, to find the altitudes:

37. 864 sq. in., 27 in. 38. 14.72 sq. ft., 4.6 ft.
39. 101.4 sq. in., 26 in. 40. 25.62 sq. in., 6.1 in.
41. 0.0704 sq. in., 0.32 in. 42. 2911 sq. in., 3 ft. 5 in.

Find the sides of the squares whose areas are given in Exs. 43–51, carrying the results to two decimal places:

43. 2 A. 44. 171 sq. ft. 45. 385 sq. ft.
46. 153 sq. in. 47. 352 sq. rd. 48. 251 sq. yd.
49. 7.52 sq. in. 50. 6.04 sq. yd. 51. 8.51 sq. in.

Find the areas of the circles whose radii are :

52. 14 in. 53. 15.4 in. 54. 39.9 in.
55. 64.4 ft. 56. 71.4 ft. 57. 92.4 yd.
58. 123.9 rd. 59. 164.5 rd. 60. 235.9 yd.

Find the radii of the circles whose areas are :

61. 308 sq. in. 62. 770 sq. in. 63. 1386 sq. in.
64. 1694 sq. ft. 65. 1848 sq. ft. 66. 2618 sq. ft.
67. 40.04 sq. yd. 68. 44.77 sq. yd. 69. 141.68 sq. yd.

Find the volumes and surfaces of the spheres whose radii are as follows :

70. 181 in. 71. 172 in. 72. 625 in.
73. 31.8 in. 74. 27.6 in. 75. 34.2 in.

Find the volumes of the pyramids whose bases and altitudes are respectively :

76. 21.6 sq. ft., 3 ft. 77. 16.4 sq. in., 17.1 in.
78. 17.4 sq. in., 12.3 in. 79. 26.2 sq. ft., 5 ft. 1 in.

Find the volumes of the cylinders whose altitudes and the radii of whose bases are respectively :

80. 2.8 ft., 1.7 ft. 81. 2.7 in., 7.2 in.
82. 6.2 in., 3.4 in. 83. 14.2 ft., 8.1 ft.

Find the volumes of the cones whose altitudes and the radii of whose bases are respectively :

84. 6.2 ft., 4.1 ft. 85. 3.9 ft., 2.4 ft.
86. 3.4 in., 4.2 in. 87. 6.7 in., 7.8 in.
88. 2.8 in., 3.1 in. 89. 12.8 ft., 6.1 ft.
90. 4 ft. 7 in., 2 ft. 1 in. 91. 6 ft. 8 in., 3 ft. 2 in.

92. If a cubic foot of granite weighs 165 lb., what is the weight of a sphere of granite 1 ft. in diameter?

93. How many tons of water will fill a tank 14 ft. by 8 ft. by 8 ft., allowing 1000 oz. to a cubic foot?

94. A bowl is in the form of a hemisphere 4.2 in. in diameter. How many cubic inches does it contain?

95. If a cubic foot of marble weighs 173 lb., what is the weight of a cylindrical marble column 10 ft. high and 16 in. in diameter?

96. How many loads (cubic yards) of earth must be removed in digging a canal 2 mi. 1740 ft. long, 200 ft. wide, and 16 ft. deep?

97. In a cylindrical jar 4 in. high and 3 in. in diameter, half full of water, a marble 1 in. in diameter is dropped. It causes the water to rise how much?

98. What is the weight of a sphere of steel 7 in. in diameter, steel being 7.8 times as heavy as water, and 1 cu. ft. of water weighing 1000 oz.?

99. What is the weight of a sphere of marble 3 ft. in circumference, marble being 2.7 times as heavy as water, 1 cu. ft. of water weighing 1000 oz.?

100. The entire surface of a pyramid is made up of four equal-sided triangles 3 ft. on a side. What is the total area? (Carry the result to hundredths.)

101. The diameter of a sphere and the altitude of a cone are equal, and they have equal radii. The volume of the sphere is how many times that of the cone?

102. Taking the radius of the earth as 4000 mi., and the earth as an exact sphere, what is its volume? How many square miles on its surface? (Carry the answers only to 1000 square or cubic miles.)

MISCELLANEOUS PROBLEMS

622. Nature of the problems. The following list contains certain problems whose interest is rather in the reasoning exercised in solution than in their applicability to the life of to-day.

WRITTEN EXERCISE

1. Divide 91 into two parts having the ratio of 5 : 8.

2. The product of two numbers is 175 and their quotient is 7. What are the numbers?

3. How many bananas are there in a bunch of which 7 are decayed and 95% are not?

4. The sum of two numbers is 783 and their difference is 191. What are the numbers?

5. At 200 lb. per cubic foot, what is the weight of a block of stone $4'\ 3\frac{1}{4}'' \times 1'\ 1'' \times 9.6''$?

6. At \$1.95 a rod, how much will it cost to fence a rectangular 2-acre field that is 5 ch. long?

7. If iron weighs 444.56 lb. per cubic foot, find the weight of a 14-ft. bar of $2'' \times 1\frac{1}{2}''$ iron.

8. How much is gained by buying 63 gal. of molasses for \$25.20 and selling it at 12¢ a quart?

9. What is the rate of gain when oranges are bought at 40 for \$1, and sold at 50¢ a dozen?

10. What is the rate of gain when eggs are bought at 16 for a quarter and sold at 13 for a quarter?

11. What is the height of a block of marble $4'\ 2'' \times 2'\ 1''$, that weighs as much as one $3'\ 4'' \times 2'\ 1'' \times 2'\ 5''$?

12. A man owns 100 A. of coal land, the coal bed being 11 ft. thick. Allowing 35 cu. ft. to the ton, how many tons of coal in the bed?

13. The minute hand of my watch is $\frac{7}{10}$ in. long. Over what area does it pass in a day?

14. If 15 horses eat 11 bu. of oats in 9 da., how long will 44 bu. last 45 horses at the same rate?

15. In the year when Michigan produced 11,135,215 long tons of iron, how many pounds did it produce?

16. If telegraph poles are 198 ft. apart, and a train passes one every 3 sec., what is the train's rate per hour?

17. The pitch of a screw is 16 threads to the inch. How many turns must be made to move the screw $\frac{3}{4}$ in.?

18. A dealer bought hay at \$8.66 a ton and sold it at \$9.75, gaining \$408.75. How many tons did he buy?

19. What will it cost to fill in 3 acres of swamp land, raising the level on an average of 5 ft., at 45¢ a load?

20. A man used a bicycle for a month and sold it 15% below cost, receiving \$29.75 for it. How much did he lose?

21. What is the list price of some goods which cost \$68.40 after the discounts 10%, 5% have been deducted?

22. At what price must a dealer mark goods that cost him \$450 so as to take off 10% and still make 10% profit?

23. What is the shortest distance that can be measured exactly by a yardstick, a fathom rod, or a pole 1 rd. long?

24. A man sold two carriages at \$150 each. On one he gained 20%, and on the other he lost 20%. Did he gain or lose on both, and how much?

25. A man bought a horse for \$95 and sold it for \$114. What was his gain per cent? The purchaser sold the horse for \$95. What was his loss per cent?

26. In the year when Michigan produced 7,729,808 barrels of salt, this was 37.6% of the total amount produced in the United States. How many were produced in all?

27. At what distance from the end must a piece of timber 16″ square be sawed so as to cut off 9 cu. ft.?

28. Around a circular flower bed 28 ft. in diameter is a gravel walk 7 ft. wide. What is the area of the walk?

29. If 6 men can do a piece of work in 4 da., how many more must be employed to finish the work one day sooner?

30. A man can mow $\frac{2}{7}$ of a field in $1\frac{3}{4}$ hr. How many hours will it take to mow the whole field at this rate?

31. If 84 men can do a piece of work in 2 mo., how long will it take 12 men to do the same work at the same rate?

32. A train 156 ft. long is traveling 60 mi. an hour. How long will it take it to completely cross a bridge 295 ft. long?

33. One man can do a piece of work in 4 da. which another can do in 5 da. At these rates, how long will it take them to do the work together?

The first does what part in one day? The second does what part in one day? Together they do what part in one day? How many days will it take them?

34. A boy, after doing $\frac{3}{8}$ of a piece of work in 30 days, is assisted by a man, with whom he completes it in 6 days. How long would it take each to do the work alone?

35. If a certain number of men can do a piece of work in 15 da., how long will it take if twice as many men are added to this number?

36. If 19 men can do a piece of work in 76 da. of 7 hr. each, how many men, working at the same rate, will it take to do the work in 133 hr.?

37. An expert typesetter, working by the hour, receives \$32.40 a week, working 9 hr. daily except Sunday. How many hours a day must he work to earn \$19.20 in 4 da. at the same rate?

38. What two equal numbers produce, when multiplied together, 15,129?

39. The square root of a certain number is 42.3. What is 42.3% of the number?

40. A square is 300 ft. on a side. How long is the diagonal? Answer to two decimal places.

41. Multiply the square root of 13.7641 by the sum of 2.06 and 1.65, and extract the square root of the product.

42. A 51-ft. ladder rests against a building, its foot being 24 ft. from the wall. How high does it reach?

43. How long is a ladder that just reaches the top of a 4-story building, averaging 10 ft. to a story, the foot of the ladder being 9 ft. from the wall?

44. From the top of a 24-ft. flagstaff a rope is stretched to a point 7 ft. from the foot of the staff. How long is the rope?

45. A baseball "diamond" is a square 90 ft. on a side. How far must the second baseman throw from his base to the home plate? Answer to two decimal places.

46. How far must the catcher throw from the home plate to the shortstop, who stands halfway between the second and third bases of a baseball diamond? Answer to two decimal places. (See Ex. 45.)

47. If a locomotive travels 90 mi. in 2 hr., the piston making 162 strokes a minute, how many strokes per minute must be made for the same locomotive to travel 200 mi. in $4\frac{1}{2}$ hr.?

48. A city newspaper office must get out an edition of 126,000 papers. It has two presses, each with a capacity of 300 papers a minute. How long will it take to print the edition?

49. How many steps of 2 ft. 9 in. each would you have to take in a minute in order to walk at the rate of 4 mi. per hour?

50. At the rate of $\frac{2}{5}$ cu. yd. of earth in 15 min., how long would it take a man to excavate a cellar 15 ft. square and $7\frac{1}{2}$ ft. deep?

51. A machine does a certain amount of work in 12 hr. If its capacity be increased 20%, how long will it take it to do the work?

52. Of three partners, one is entitled to $\frac{5}{1\,8}$ and another to $\frac{7}{1\,8}$ of the profits. If they make \$4500, what is the share of each of the three?

53. If a certain gun metal is composed of 16% tin and the rest copper, how many pounds of tin will be needed with 46.2 lb. of copper?

54. Two cans have a capacity of 7 pt. and 10 pt. respectively. How would you measure out 16 pt. into a third can by using these alone?

55. A dealer marked a bedroom set 15% above cost, this representing a gain of \$9.30. He sold it at a discount of 10%. How much did he gain?

56. A strip of carpet 50 yd. long and 27 in. wide weighs 200 lb. What is the weight of such a carpet covering an assembly room 135 ft. $\times$ 150 ft.?

57. If a gallon of water weighs 10 lb., and a cubic foot of water weighs 1000 oz., how many gallons are there in a cistern which contains 16 cu. ft.?

58. A man bought a piece of alloy for \$72.25. The number of pounds purchased and the number of dollars per pound were the same. How many pounds did he purchase?

59. What is the number which multiplied by 80% of itself equals 4500?

60. The diagonal of one face of a cube is $\sqrt{162}$ in. Find the surface and the volume of the cube.

61. A horse is tied to a corner post of a square field by a rope 35 ft. long. Over what part of an acre can he feed?

62. If a horse is tethered by a 30-ft. rope, it can graze over how many times as much ground as it could if the rope were only 20 ft. long?

63. What is the area of the largest circle that can be inscribed in a square whose area is 196 sq. in.? of the largest square that can be inscribed in the circle?

64. A locomotive wheel 5 ft. in diameter makes 300 revolutions a minute, and travels at this rate for 4 hr. 10 min. without stopping. How far does it go? (Take $\pi = 3\frac{1}{7}$.)

65. A wheel 7 ft. 8 in. in circumference revolves 3762 times in going a certain distance. Another wheel revolves 3496 times in going the same distance. What is the circumference of the latter?

66. A piece of brickwork 279 ft. long is completed in 21 days. At this rate, how long would it take to complete a piece twice as high and 651 ft. long?

67. A boy buys apples at the rate of 5 for 2 ct., and sells them at the rate of 8 ct. a dozen. How many apples must he buy and sell in order to gain \$2?

68. The equatorial diameter of the earth is estimated at 20,926,202 ft., and the polar diameter at 20,854,895 ft. Express each in miles and find the difference.

69. A, B, and C bought a horse for \$112 and sold it at a profit of 25%, by which A gained \$6 and C twice as much. How much did each of the three pay for the horse?

70. How many acres in a square field whose diagonal is 120 rd.?

71. What is the square root of $\frac{1}{3}$ of the product of the square root and the cube root of 729?

72. If 145 bu. of turnips last 53 oxen a fortnight, how long will 435 bu. last 371 oxen?

73. A man computing the cost of 43 articles multiplied, by mistake, the cost of each by 34 instead of 43 and obtained \$84.32. What was the correct cost of the lot?

74. Two masts of a ship are 45 ft. 6 in. apart, one being 60 ft. high and the other 70 ft. Supposing them perpendicular to the deck, how far is it from the top of one mast directly to the top of the other? (Two decimals.)

75. Add $\frac{3}{7}$, $\frac{4}{5}$, $\frac{3}{8}$, and $\frac{3}{4}$. Reduce the result to a decimal fraction, to three decimal places. Also reduce each of the addends to a decimal fraction carried to four places, and add.

76. One man can do $\frac{1}{2}$ of a piece of work in a day, and another can do $66\frac{2}{3}\%$ as much. How long will it take them, working together, to do the whole work?

77. Two ships, sailing in opposite directions, pass in mid-ocean. One averages 7° 30′ a day, and the other 75% as much. What is their difference in ship time exactly 4 da. after passing?

78. In closing out a line of goods a merchant asked 20% less than a certain article cost, but at an auction sale he succeeded in getting 20% more than this asking price. What per cent did he lose on the cost?

79. If Michigan, one of the largest producers of Portland cement, made 1,602,370 bbl. in a certain year, an increase of 10% on the preceding year, how many barrels were produced in the preceding year?

80. Water is flowing into a tank whose base measure is $4'\,4'' \times 2'\,7''$ at the rate of 0.62 cu. ft. in 3 min. How long will it take the water to fill it to a depth of $2'\,8''$?

81. A 12-in. gun can fire a shell weighing 850 lb. every 30 sec. At this rate, how many pounds of shell could a battle ship fire from four 12-in. guns in 2 min.?

82. A train leaves a city at 5:15 P.M., and reaches a city $109\frac{1}{2}$ mi. distant at 7:51 P.M. Allowing 10 min. for stopping at stations, what is the rate of travel per hour?

83. The greatest known depth of the ocean is 5000 fathoms, and the greatest mountain height is 29,002 ft. Find the difference in level, in miles, to three decimal places.

84. A square park is surrounded by a gravel walk $7'\,6''$ wide. The park and walk together contain 1.6 acres. Find the cost of making the gravel walk at 10¢ per square yard.

85. A grocer bought 400 bunches of lettuce for \$8. He sold all but 43 heads at 5¢ each. Of the 43 heads he sold 30 at 1¢ each and the rest he threw away. How much did he gain?

86. The product of two numbers is 48 less than half the number of feet in a mile, and one of the numbers is the number of cubic inches in a cubic foot. What is the other number?

87. A farmer finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels will be contained in a bin 50% longer, twice as wide, and of the same depth?

88. How much must the terms of the fraction $\frac{1}{14}\frac{2}{5}\frac{3}{5}$ be increased to make the fraction equal $\frac{2}{2}\frac{7}{9}$? decreased to make the fraction equal $\frac{5}{6}\frac{0}{1}$? increased to make the fraction equal $\frac{6}{7}\frac{5}{6}$?

89. If 6 men can lay the tiles in a large railway station in 16 da., how many men must be added to the job to complete it 4 da. sooner?

90. How many times will a locomotive drive wheel 5 ft. 10 in. in diameter revolve in going from New York to Chicago, a distance of 983 mi.?

91. A man deposited some money in the bank, and then drew out 15% of the amount, then 20% of the remainder, and then 30% of what was still left, and there then remained $809.20. What was the deposit?

92. If the area of Illinois is 56,650 sq. mi., and if in a certain year 91.5% of the area was included in farms, and 84.5% of this part was improved farm land, how many acres of improved farm land were there?

93. When Adams County, Illinois, had a population of 67,100, Lasalle County had a population 31% larger, and Cook County had 1200% as much as both these counties together. What was then the population of Cook County?

94. When Port Arthur was captured it was found that there was food enough for the 45,000 soldiers and inhabitants for 6 wk. Suppose there had been 9000 more men, for how long would the food have been sufficient?

95. A village has a water tank 42 ft. in diameter. In 7 hr. 30 min., when no water is being pumped in, the water is lowered 6 ft. How many gallons are drawn out on an average each hour of this period? (Use 7.6 gal. to 1 cu. ft.)

96. When the manufactured products of Illinois were a billion and a quarter dollars a year, the only states with a larger product were New York, with 75% more, and Pennsylvania, with 48% more. The manufactures of New York then exceeded those of Pennsylvania by what per cent?

97. A rug-cleaning establishment charges 12¢ a yard for cleaning carpets 27 in. wide. At the same rate per square yard, how much will it charge for cleaning two rugs $7'\ 6'' \times 5'\ 6''$ and $5' \times 8'\ 6''$?

98. A grain of gold can be beaten out into a leaf of 56 sq. in. How many of these leaves, pressed one upon another, would make a pile 1 in. high, 1 cu. in. of gold weighing 10 troy ounces?

99. The quotient of two numbers is the sum of the numbers denoting the length of a mile expressed in feet and the length of a rod expressed in feet. The dividend is 21,186. What is the divisor?

100. A certain university library increases at the rate of 18,000 books a year. Allowing 300 working days to the year, and 10 hours to the day, how often, on an average, does a new book come to the library?

101. If $3\frac{1}{2}$ bbl. of lime are required for plastering 100 sq. yd., how many barrels will be required for a job of plastering equivalent to a wall surface $231' \times 9'$? How many for 6831 sq. ft.?

102. For a cellar floor 153 cu. ft. of concrete are needed. To make this there are used 2 parts of fresh powdered lime, 1 part of Portland cement, 6 parts of gravel. How many cubic feet of each are used?

103. If it takes 1 bbl. of lime and $\frac{5}{8}$ cu. yd. of sand to make the mortar for 1000 bricks, and if $22\frac{1}{2}$ bricks are allowed per cubic foot of wall, how much lime and sand are needed for a wall $30' \times 10' \times 1'\ 6''$?

104. If the more liberal estimate of 23 bricks to 1 cu. ft. is allowed, how many bricks would be needed for a wall $50' \times 100' \times 20''$?

105. A boy who can swim 1 yd. a second swims downstream for 10 minutes, the stream flowing 15 yd. a minute. How far does he go? At the same rates, how long will it take him to swim back to where he started?

106. I paid a bookseller \$5.05 for three books, a history, an atlas, and a dictionary. The history and atlas together cost \$2.85, and the atlas and dictionary together \$3.80. What was the price of each?

107. A tank $8' \times 3' \times 9''$ is filled with pulp for making paper. How long a strip of paper, $2'\ 6''$ wide, can be made from this pulp, the paper being $\frac{1}{250}$ in. thick, and half the volume of the pulp being lost in drying and rolling?

108. Two persons start at the same time from places $6\frac{1}{2}$ mi. apart, and go towards each other. One walks at the rate of 3 mi. an hour, and the other goes in an automobile at the rate of 10 mi. an hour. When and where will they meet?

109. A druggist has his clerk prepare 200 boxes of salve, the material costing \$4.90, the boxes \$2.20, the labels \$2, and the time of the clerk being worth \$3. He sells the salve at 15¢ per box. How much does he gain by the transaction?

110. Two water filters discharge into the same jar, one at the rate of 16 qt. an hour, the other at the rate of $3\frac{1}{2}$ qt. every 15 min. What is the capacity of the smallest jar that can be filled in an integral number of minutes by either filter?

111. A man wishes to go from one corner of a rectangular field 60 rd. long and 11 rd. wide, to the diagonally opposite corner. How many steps will he save by going directly across the field, instead of going along the two sides, allowing 2 ft. 6 in. to a step?

112. An ice company has an ice house 120 ft. long, 52 ft. wide, and 28 ft. high, after making allowance for packing. How many tons of ice will it hold, allowing $58\frac{1}{3}$ lb. to the cubic foot?

113. How many croquet balls 3 in. in diameter can be placed in a cubical can whose interior height is 2 ft. 3 in.? How many quarts of water can be poured in after the balls are in place?

114. At the time that Illinois had 11,427 miles of railway track it had more than any other state. If the weight of a rail averaged 80 lb. to the yard, how many tons of steel were used for the Illinois roads?

115. Two men rent an automobile for fifteen weeks during the summer season. One uses it 7 wk., and the other 8 wk. If they pay $345 rent, what is the share of each?

116. Three men own lots having frontages of 35 ft., 40 ft., and 50 ft. respectively, on the same street. They are assessed $150 for street improvement. If assessed according to the frontage, what should each pay?

117. Two houses rent for $300 a year, the rent being paid monthly in advance in one case, and at the end of each quarter (3 mo.) in the other. What is the difference in the amount of each rent in 2 yr., allowing 6% simple interest?

118. Three men own adjoining land fronting on the same street, the frontages being 561 ft., 357 ft., and 459 ft. respectively. They agree to cut each piece into building lots, and they wish all the lots to have the same frontage. What is the greatest possible frontage of each, and how many lots will each man have?

119. At what time between 1 and 2 o'clock will the hands of a clock be together?

Looking at the clock, it is evident that they are together at 12. We may also think of the hour hand as 60 minute spaces ahead of the minute hand at that time, so that the minute hand must gain 60 minute spaces to overtake the hour hand again. But in 1 hr. it evidently *gains* only 55 minute spaces, for at 1 o'clock the hour hand is 5 minute spaces ahead. Since it gains 55 in 1 hr., to gain 60 it will take $\frac{60}{55}$ hr., or $1\frac{1}{11}$ hr., or 1 hr. 5 min. $27\frac{3}{11}$ sec. (Draw the face of a clock to illustrate this.)

At what time between 2 and 3 o'clock will they be together?

120. At what time will the hands of a clock be together between 4 and 5 o'clock? between 7 and 8 o'clock?

121. At what times between 2 and 3 o'clock will the hour and minute hands be at right angles?

From noon the minute hand would have to gain on the hour hand 120 minute spaces for the two to be together, or 120 + 15 or 120 + 45 for them to be at right angles.

122. At what times between 5 and 6 o'clock will the hour and minute hands of a clock be at right angles? together?

123. At what times between 9 and 10 o'clock will the hour and minute hands of a clock make with each other an angle of 120°?

124. If a clock is 2 min. 30 sec. fast at noon, January 1, and gains 3 min. 30 sec. a week, what time will it indicate at noon on January 31?

125. Earth, Mars, and Saturn were in the same straight line on the same side of the sun in 1875. Taking the period of revolution of Mars about the sun as $1\frac{8}{9}$ yr., and that of Saturn as $29\frac{1}{2}$ yr., in what year will they again be in the same relative positions and in the same direction from the sun?

126. If the cotton crop of Mississippi decreased 5% in a certain year, amounting to 1,347,100 bales, how many bales were produced before the decrease?

127. The world's production of copper in a certain year was 575,000 long tons, of which the United States produced 308,000. Of the United States, the largest producers were Michigan, 192 million pounds, and Montana, 27% more than Michigan. What per cent of the world's production was that of the United States? of Michigan? of Montana?

128. What is the present worth of (or sum which put at interest will amount to) $230.25, due in 3 yr. 9 mo. 18 da., money being worth 6%? If we call the difference between the present worth and the given sum (or future worth) the true discount, as was formerly the custom, find the true discount in this case.

129. What is the present worth (see Ex. 128) of $151.33, due in 4 mo. 15 da., money being worth 6%?

130. What is the present worth and true discount (see Ex. 128) of $161.50, due in 1 yr. 3 mo. 10 da., money being worth 6%?

131. The following are some of our greatest coal-producing states, with the number of tons produced by each in a certain year, and the value of the coal at the mines. Find the value per ton in each case.

Alabama	11,700,000 T.	$14,375,000
Colorado	7,652,000 T.	$9,182,400
Illinois	34,955,000 T.	$36,353,200
Indiana	11,191,700 T.	$12,310,870
Kentucky	7,075,000 T.	$7,145,750
Ohio	24,574,000 T.	$29,488,800
Pennsylvania	103,000,000 T.	$115,360,000
West Virginia	26,883,000 T.	$28,227,150

132. Four bells toll at intervals of 3, 4, 5, and 8 seconds respectively, commencing at the same instant. At what intervals will they toll simultaneously?

133. Mr. Jameson has a balance of $2703 due him from King & Co., commission merchants. He writes them to close his account by buying for him as much as possible of a certain line of goods after deducting their commission of 2%. How much do King & Co. pay for the goods?

134. Two trains start at the same time, one from Albany for New York, and the other from New York for Albany. The first runs at the rate of 42.4 mi. per hour, and the second 52.8 mi. The distance between the cities being 147 mi., how far apart will the trains be in 1 hr. 15 min.?

135. A train leaves New York at the same time that one leaves Albany, the former traveling 54 mi. an hour and the latter 10 mi. less per hour. If they maintain this rate and travel towards each other, how long after starting and how far from Albany will they meet? (See Ex. 134.)

136. Slaters have this rule for finding the number of slates per *square* (100 sq. ft.): Subtract 3 in. (for overlapping) from the length of the slates; divide 14,400 sq. in. (the number in a square) by the area of each slate after making the deduction. Explain this by a drawing, and find the number of $8'' \times 14''$ slates needed per square.

137. In a certain gold mine 450 cans of explosives, of 25 lb. each, were used in a blast which broke up a mass of gravel $270' \times 180' \times 100'$. At this rate, how many pounds of explosives were used per 1000 cu. ft. of gravel broken up? The blast cost $6000. How much did it cost per 1000 cu. ft.? If the gravel yielded 23.4¢ per cubic yard, how much gold was taken out?

TABLES FOR REFERENCE

Length

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.).
320 rods, or 5280 feet = 1 mile (mi.).
4 inches = 1 hand.
6 feet = 1 fathom.
120 fathoms = 1 cable length.
1.15 common miles = 1 knot (nautical mile).

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd).
160 square rods = 1 acre (A.).
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
128 cubic feet = 1 cord (cd.).
$24\frac{3}{4}$ cubic feet = 1 perch (stone, etc.).

Weight

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).
100 pounds = 1 hundredweight (cwt.).
112 pounds = 1 long hundredweight.
2240 pounds = 1 long ton.

Troy Weight

24 grains (gr.) = 1 pennyweight (pwt. or dwt.).
20 pennyweights = 1 ounce (oz.).
12 oz. = 1 pound (lb.).

The avoirdupois pound contains 7000 gr., the Troy pound 5760 gr.

3.2 grains = 1 carat (diamond weight).

But "16 carats fine" means $\frac{16}{24}$ pure gold.

Apothecaries' Weight

20 grains (gr.) = 1 scruple (sc. or ℈).
3 scruples = 1 dram (dr. or ʒ).
8 drams = 1 ounce (oz. or ℥).
12 ounces = 1 pound (lb.).

Liquid Measure

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

231 cubic inches = 1 gallon.
31.5 gallons = 1 barrel (bbl.) (varies).
2 barrels = 1 hogshead (varies).
16 fluid ounces = 1 pint (apothecaries').
57.75 cubic inches = 1 liquid quart.

Dry Measure

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

2150.42 cubic inches = 1 bushel.
67.2 cubic inches = 1 dry quart.

Time

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
12 months (mo.) = 1 year (yr.).

Value

10 mills = 1 cent (ct. or ¢).
10 cents = 1 dime (d.).
10 dimes = 1 dollar ($).
10 dollars = 1 eagle (E.).

Angles and Arcs

60 seconds (60″) = 1 minute (1′).
60 minutes = 1 degree (1°).
90 degrees = 1 right angle.
360 degrees = 1 circumference.

Surveyors' Table of Length

7.92 inches = 1 link (li.).
100 links = 4 rods = 1 chain (ch.).
80 chains = 5280 ft. = 1 mile.

City property is usually measured by feet and decimal fractions of a foot; farm property, by rods or chains.

Surveyors' Table of Square Measure

16 square rods (sq. rd.) = 1 square chain (sq. ch.).
10 square chains = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).
36 square miles = 1 township (tp.).

A square rod was formerly called a *perch.*
A quarter of an acre (40 sq. rd.) was formerly called a *rood.*

Counting

12 units = 1 dozen.
12 dozen, or 144 = 1 gross (gr.).
12 gross, or 1728 = 1 great gross.
20 units = 1 score.

Paper

24 sheets = 1 quire.
20 quires = 480 sheets = 1 ream.

Now, for convenience in counting, 500 sheets are more often called a *ream;* and the word *quire* is used only for folded note paper, other paper being usually sold by the pound.

Metric Length

A myriameter	=	10,000 meters.
A **kilometer** (km. or Km.)	=	1000 "
A hektometer (Hm.)	=	100 "
A dekameter (Dm.)	=	10 "
Meter (m.)		
A decimeter (dm.)	=	0.1 of a meter.
A **centimeter** (cm.)	=	0.01 "
A **millimeter** (mm.)	=	0.001 "

1 meter = nearly 39.37 inches.
1 kilometer = nearly 0.6 mile.

Metric Square Measure

A square myriameter	=	100,000,000 square meters.
" **kilometer** (km^2.)	=	1,000,000 " "
" hektometer	=	10,000 " "
" dekameter	=	100 " "
Square meter (m^2.)		
A square decimeter (dm^2.)	=	0.01 of a square meter.
" centimeter (cm^2.)	=	0.0001 " "
" millimeter (mm^2.)	=	0.000001 " "

The abbreviation sq. m. is often used for m^2., sq. dm. for dm^2., etc.
A square dekameter is called an *are* (land measure).
100 ares = 1 hektare (about 2.47 acres).

Metric Cubic Measure

A cubic myriameter	=	10^{12} cubic meters.
" kilometer	=	10^9 " "
" hektometer	=	1,000,000 " "
" dekameter	=	1000 " "
Cubic meter (m^3.)		
A cubic **decimeter** (dm^3.)	=	0.001 of a cubic meter.
" centimeter (cm^3.)	=	0.000001 " "
" millimeter (mm^3.)	=	0.000000001 " "

A cubic meter is called a *stere* (in wood measure).

Metric Capacity

A **hektoliter** (hl. or Hl.)	= 100	liters.
A dekaliter (Dl.)	= 10	"
Liter (l.)		
A deciliter (dl.)	= 0.1	of a liter.
A centiliter (cl.)	= 0.01	"
A milliliter (ml.)	= 0.001	"

1 liter = 1 cubic decimeter = nearly 1 quart.

A liter of water weighs approximately 1 kilogram.

Metric Weight

A **metric ton** (t. or M.t.)	= 1,000,000	grams.
A quintal (q.)	= 100,000	"
A myriagram (Mg.)	= 10,000	"
A **kilogram** (kg. or Kg.)	= 1000	"
A hektogram (Hg.)	= 100	"
A dekagram (Dg.)	= 10	"
Gram (g.)		
A decigram (dg.)	= 0.1	of a gram.
A centigram (cg.)	= 0.01	"
A milligram (mg.)	= 0.001	"

The metric ton is nearly the weight of 1 m^3. of water at its greatest density; the kilogram, of 1 dm^3.; and the gram, of 1 cm^3.

A kilogram is about $2\frac{1}{5}$ lb.; a metric ton about 2205 lb.

English Money

12 pence (d.) = 1 shilling (s.) = $0.243 +.
20 shillings = 1 pound (£) = $4.8665.

French Money

100 centimes (c.) = 1 franc (fr.) = $0.193.

This system is used in several European countries.

German Money

100 pfennigs (pf.) = 1 mark (M.) = $0.238.

DEFINITIONS OF THE MOST COMMON TERMS USED IN ARITHMETIC

Abstract number. A number that does not refer to any particular kind of object or measure is called an *abstract number.* All *numbers* are, however, essentially abstract, whether the kind of thing numbered is mentioned or not.

Addends. Numbers to be added are called *addends.*

Addition. The process of finding a number that equals two or more numbers taken together is called *addition.*

Aliquot part. A number which is contained in a given number an integral number of times is called an *aliquot part* of that number. Thus $12\frac{1}{2}$ is an aliquot part of 100.

Amount of note. The sum of the principal (or face) and interest of a note is called the *amount.*

Analysis. In arithmetic the process of reasoning to find out a truth is called *analysis.*

Antecedent. The first term of a ratio is called the *antecedent.*

Arc. A portion of a circumference is called an *arc.*

Area. The number of square units contained in a surface is called its *area.*

Arithmetic. The science that treats of numbers and their applications is called *arithmetic.*

Assets. The property of an individual or corporation is called the *assets.*

Average. The sum of n given numbers, divided by n, is called their *average.* Thus the average of three numbers is $\frac{1}{3}$ of their sum.

Bank. An establishment for the transaction of the business of loaning sums of money, receiving money on deposit, and exchange, is called a *bank.*

Bank discount. The interest paid in advance on money borrowed at a bank is called *bank discount.*

Base. The number of which some per cent is to be found is called the *base.* The line on which a figure appears to stand is called the *base.*

Bill. A written statement of goods sold or of work done is called a *bill.*

Bill of exchange. A foreign draft is often called a *bill of exchange.*

Bond. A written or printed promise to pay a certain sum at a specified time, signed by the maker and bearing his seal, is called a *bond.*

Cancellation. Dividing both terms of a fraction by the same factor is called *cancellation.* The factor is then said to be *canceled.*

Capacity. The measure of a receptacle, indicating the number of units it contains, is called the *capacity.*

Capital. The money contributed by the stockholders to maintain a corporation is called its *capital* or *capital stock.*

Check. An order on a bank to pay money is called a *check.* An operation that tends to verify another one is called a *check.*

Circle. A figure bounded by a curve, every point of which is equidistant from a point within called the *center,* is called a *circle.*

Circumference. The bounding line of a circle is called its *circumference.* In advanced mathematics *circle* is used for *circumference.*

Commission. The percentage charged by an agent for transacting business is called *commission.*

Common denominator. A denominator common to two or more fractions is called a *common denominator* of the fractions.

Common divisor. A factor that is common to two or more numbers is called a *common divisor* or *common factor.*

Common factor. *See* Common Divisor.

Common fraction. A fraction having both terms fully expressed by figures is called a *common fraction.*

Common multiple. A multiple of each of two or more numbers is called a *common multiple* of the numbers.

Complex fraction. A fraction that has a fraction in either numerator or denominator, or both, is called a *complex fraction.*

Composite number. A number not prime is called a *composite number.*

Compound fraction. An expressed multiplication of an integer or a fraction by a fraction is called a *compound fraction;* for example, $\frac{2}{3}$ of 7 or $\frac{2}{3}$ of $\frac{4}{5}$.

Compound interest. If interest is not paid when due, but is added to the principal to form a new principal, and interest is reckoned on this amount, the obligation is said to draw *compound interest.*

Compound numbers. Denominate numbers in which two or more units are expressed are called *compound numbers.*

Concrete number. A number that refers to some particular kind of object or measure is called a *concrete number.*

Consequent. The second term of a ratio is called the *consequent.*

Consignee. The person, firm, or corporation to which goods are sent (consigned) is called a *consignee.*

Consignor. The one who sends (consigns) goods to another is called a *consignor.*

Corporation. A number of individuals organized by law to act as one body is called a *corporation;* also called a *company.*

Coupon. A note attached to a bond, promising to pay the regular interest, to be cut off and presented for payment when due, is called a *coupon.*

Creditor. A person to whom another owes money is called his *creditor.*

Cube. A regular solid with six equal squares as faces is called a *cube.* The third power of a number is called its *cube.*

Cube root. One of the three equal factors of a number is called its *cube root.*

Customhouse. The place where government duties on imported goods are received is called a *customhouse.*

Debtor. A person who owes money to another is called his *debtor.*

Decimal fraction. A fraction whose denominator is not written, but is 10 or some power of 10, is called a *decimal fraction.*

Decimal point. A period written before tenths in a decimal fraction is called a *decimal point.*

Decimal system. A system of numbers based upon the powers of ten is called a *decimal system;* for example, our common system of numbers and the metric system.

Denominate numbers. Concrete numbers denoting measure (including weight) are called *denominate numbers.*

Denominator. The number which shows into how many equal parts a unit has been divided is called the *denominator* of the fraction, — in a common fraction, the number below the line.

Difference. The number which added to one number makes another is called the *difference* between them. It is the result in taking one number from another.

Discount. A deduction made on a price or amount is called a *discount.* The difference between the face or par value and a less market value is also called a *discount,* as when stocks sell at 10% discount. Interest paid in advance on a note is called *discount.*

Dividend. The number to be divided is called the *dividend.* It is the given product of the quotient and divisor.

Dividends. The shares of the earnings divided at intervals among the stockholders of a corporation are called *dividends*, a stockholder being said to receive a *dividend.*

Division. The operation by which, given the product of two numbers and one of them, the other is found, is called *division.*

Divisor. The number by which the dividend is divided is called the *divisor.*

Draft. A check drawn by one bank on another, or by one individual on another, is called a *draft.*

Drawee. A person to whose order a draft is payable is called the *drawee.*

Drawer. A person who signs a draft is called the *drawer.*

Duties. Government taxes on imported goods are called *duties.* Some governments levy taxes on exported goods, and they are also called duties.

Equation. The expressed equality of two quantities is called an *equation.*

Equilateral triangle. A triangle whose three sides are equal is called an *equilateral triangle.*

Even number. A number that contains the factor 2 is called an *even number.*

Evolution. The process of finding a root of a number is called *evolution.*

Exact interest. Interest reckoned on the basis of 365 days to the year is called *exact interest.*

Exchange. The payment of money by means of checks, money orders, or drafts, is called *exchange.*

Exponent. A small figure placed at the right and a little above a number is called an *exponent.* If it is an integer, it shows how many times the other number is taken as a factor; thus, $3^2 = 3 \times 3 = 9$. There are also fractional exponents, where the denominator indicates a root and the numerator a power; thus, $8^{\frac{2}{3}}$ means $\sqrt[3]{8^2} = \sqrt[3]{64} = 4$.

Extremes and means. The first and last terms of a proportion are called the *extremes;* the second and third, the *means.*

Face of note. The principal mentioned in a note is called the *face of the note.*

Factors. The numbers which multiplied together make another number are called its *factors.* (This definition is limited to integers, except in speaking of roots.) In some lines of business agents are called *factors.*

Fraction. One or more of the equal parts of a unit is called a *fraction.* In general, any indicated division in the form $\frac{a}{b}$ is called a *fraction.*

Greatest common divisor. The greatest factor common to two or more numbers is called their *greatest common divisor.* It is also called *greatest common measure.*

Improper fraction. A fraction whose numerator equals or exceeds the denominator is called an *improper fraction.*

Index. A figure written in a radical sign to indicate what root is to be extracted is called an *index* of the root; thus, $\sqrt[3]{8}$ means the cube root of 8. In square root the index is not written, as $\sqrt{4}$.

Indorse. To write one's name on the back of a note or other document is to *indorse* it.

Insurance. An agreement to compensate any one for some specified loss is called *insurance.*

Integer. *See* Number.

Integral number. An *integer* is also called an *integral* or *whole number.*

Interest. Money paid for the use of money is called *interest.*

Involution. The process of finding a power of a number is called *involution.*

Isosceles triangle. A triangle two of whose sides are equal is called an *isosceles triangle.*

Latitude. The degree measure of the distance of a place from the equator is called its *latitude.*

Least common denominator. The least denominator common to several fractions is called their *least common denominator.*

Least common multiple. The least of all the common multiples of two or more numbers is called their *least common multiple.*

Length. The number of linear units a figure contains is called its *length.*

Liability. An obligation to pay money is called a *liability.*

Like numbers. Numbers referring to the same unit are called *like numbers;* thus, 2 ft. and 3 ft.

Longitude. The degree measure of the distance of a place from a prime (or standard) meridian (usually the one through Greenwich, England) is called its *longitude.*

Lowest terms. If the terms of a fraction are prime to each other the fraction is said to be reduced to its *lowest terms.*

Maker of a note. The one who signs a note is called the *maker.*

Maturity. The date on which a note or bond is due is called the date of its *maturity.*

Mean proportional. If the two means of a proportion are equal, either is called the *mean proportional* between the extremes.

Means. *See* Extremes.

Minuend. The number from which we subtract is called the *minuend.*

Mixed decimal. A number composed of an integer and a decimal fraction is called a *mixed decimal.*

Mixed number. The sum of a whole number and a fraction is called a *mixed number.*

Multiple. The product of two abstract integers is called a *multiple* of either.

Multiplicand. The number multiplied is called the *multiplicand.*

Multiplication. The process of taking one number as many times as there are units in another is called *multiplication* (by an integer).

Multiplier. The number by which we multiply is called the *multiplier.*

Mutually prime numbers. Numbers that have no common factor except one are said to be *prime to each other*, or *mutually prime.*

Net proceeds. The money that remains after all discounts and charges are paid is called the *net proceeds* of any sum.

Notation. The writing of numbers by means of symbols is called *notation.*

Note. *See* Promissory Note.

Number. A unit or a collection of units is called a *whole number* or an *integer. See also* Fraction.

Numeration. The naming of numbers is called *numeration.*

Numerator. The number which shows how many parts have been taken to make a fraction is called the *numerator*, — in a common fraction, the number above the line.

Obligation. A sum which a person is obliged to pay is called an *obligation.*

Odd number. A number that is not even is called an *odd number.*

Order. In writing numbers the place occupied by a figure is called its *order.* Thus we speak of units of the second order, meaning the tens.

Par value. The face or nominal value of stock is called its *par value.* In railroad stocks this is usually $100 a share.

Partial payment. A part payment on a note is also called a *partial payment.*

Payee. The person to whom the sum mentioned in a note, check, draft, or other similar document is payable is called the *payee.*

Per cent. Another name for hundredths is *per cent.* It originally meant "by the hundred."

Percentage. The result found by taking a certain per cent of the base is called the *percentage.* That part of arithmetic that deals with per cent is often called *percentage.*

Perimeter. The length of the boundary of a plane figure is called the *perimeter.*

Periods. Groups of three figures each, marked off by a separatrix, are called *periods.*

Perpendicular. A line that makes a right angle with another is said to be *perpendicular* to it.

Place value. The value of a period or of a figure, depending upon its position, is called its *place value.*

Policy. A written contract of insurance is called a *policy.*

Poll tax. A tax on a person (usually limited to male citizens over twenty-one years of age), without regard to the amount of property owned, is called a *poll tax.* (The old name for head was "poll.")

Power. The product of two or more equal factors is called a *power.*

Premium. Money paid for insurance for a certain time is called a *premium.* The excess of the market value above the par value of stock or other evidences of value is called a *premium.*

Present worth. The present value of a debt due in the future is called the *present worth.*

Prime factors. Factors that are prime numbers are called *prime factors.*

Prime number. A number that has no factors except itself and one is called a *prime number.*

Principal. A sum of money that draws interest is called the *principal.*

Proceeds. The face (or amount, if it draws interest) of a note less the discount is called the *proceeds.*

Product. The result of multiplying two or more numbers together is called the *product.*

Promissory note. A paper signed by a debtor, agreeing to pay money, is called a *promissory note.*

Proper fraction. A fraction whose numerator is less than the denominator is called a *proper fraction.*

Proportion. An expression of equality of two ratios is called a *proportion.*

Quotient. The number found by division is called the *quotient.*

Rate. The number of hundredths of the base to be taken is called the *rate;* thus the rate of interest may be 6%, the *rate per cent* of interest being then 6. The amount of tax on a dollar is often called the *rate of taxation.* The premium on each $100 of fire insurance or each $1000 of life insurance is often called the *rate* of insurance.

Ratio. The relation of one quantity to another of the same kind, as expressed by division of the first by the second, is called their *ratio.*

Reciprocal. The *reciprocal* of a fraction is the fraction with its terms interchanged. Unity divided by any number is called the *reciprocal* of the latter.

Reduction. Changing the form of a number without changing the value is called *reducing* the number. (This process is used in dealing with fractions and compound numbers.)

Reduction ascending. Reduction of compound numbers to a higher denomination is called *reduction ascending.*

Reduction descending. Reduction of compound numbers to a lower denomination is called *reduction descending.*

Remainder. The part of the dividend remaining when the division is not exact is called the *remainder.* The difference in subtraction is also called the *remainder.*

Root. One of the equal factors of a number is called a *root. See also* Square Root.

Scalene triangle. A triangle whose three sides are unequal is called a *scalene triangle.*

Separatrix. The comma used in separating periods in large numbers is called a *separatrix.*

Share. One of a specified number of equal parts of the capital stock of a company is called a *share.*

Similar fractions. If several fractions have the same denominator, they are said to be *similar fractions.*

Simple fraction. A common fraction whose terms are integral is called a *simple fraction.*

Solid. A magnitude that has length, breadth, and thickness is called a *solid.*

Square. A plane figure with four equal sides and four equal angles is called a *square.*

Square root. One of the two equal factors of a number is called its *square root.*

Stockholder. One who has stock in a corporation or company is called a *stockholder.*

Stocks. The shares of corporations generally are called *stocks.*

Subtraction. The operation of finding the difference between two numbers is called *subtraction.*

Subtrahend. The number which is subtracted is called the *subtrahend.*

Sum. The result in addition is called the *sum.*

Surface. A magnitude that has length and breadth, but not thickness, is called a *surface.*

Tax. Money assessed by a government for its support is called a *tax.*

Terms of a fraction. The numerator and denominator together are called the *terms of the fraction.*

Triangle. A plane figure formed by three straight lines is called a *triangle.*

Unit. Any one thing is called a *unit.* The number one is called *unity.*

Unit fraction. A common fraction whose numerator is 1 is called a *unit fraction.*

Volume. The number of cubic units a solid contains is called its *volume.*

Whole number. *See* Number.

INDEX

	PAGE
Abstract number	14
Accounts	403, 424
Acute-angled triangle	152
Addends	6
Addition	6, 46, 80, 98, 272, 292
Agriculture	51, 83, 220, 226
Aliquot parts	18
Altitude	154
America, extent, resources	7
historical facts	12
See Industries, Farms, Cities, etc.	
Amount of note	214
Analysis	160, 164, 168
Angle measure	94
Antecedent	218
Apothecaries' measure	92
Approximate answers	88
Arabic numerals	2, 265
Arc	94
Area	104, 154
Army	455
Assessed valuation	460
Assessors	460
Avoirdupois weight	92
Bank accounts	403
discount	215, 412
of deposit	407
savings	403
Banking	204, 215
Barometer	490
Base	137, 154
Bills discounted	193
and receipts	125
of exchange	438
of goods	422, 429
of lumber	235
Birds, value of	227, 391
Board measure	234
Bonds	478
Borrowing money	411
Broker	472
Brokerage	472
Buildings	190, 236
Business, entering	400
fractions	50, 114
Cancellation	39, 58, 363
Capacity, 91–93, 207, 228, 329, 444, 447	
Capital	247, 472
Carat	92
Carpenter	236
Carpeting	239
Cattle	28, 359
Centigrade	231
Checks	6, 22, 204, 409, 429
Cipher	2
Circle, circumference	510
area	512
Circular measure	94
Cities	19, 142
Clearing house	434
Clothing	143
Coal	242
Coffee	482
Collector of taxes	462
Commission	199, 472
Common divisor	30
fractions	36, 37, 114
multiple	35
Complex fraction	70
Composite number	31
Compound fraction	70
interest	405
numbers	95
proportion	386
Concrete number	14
Cone	520
Consequent	218
Corn	27

PAGE

Corporation 472
Cotton 116, 482
Creamery problems . . . 398
Credit, creditor . . . 125, 424
Cube numbers and roots, 494, 504
Cubic measure, 91, 207, 444, 447
Customhouse 457
Cylinder 516

Dairy problems 398
Days of grace 411
Debit, debtor . . . 125, 424
Decimal fractions . . . 76, 105
system 265
See the operations.
Definitions 548
Denominate numbers, 90, 149, 206
Denominator 37
Deposit, bank of 407
slips 407
Difference 10
Directors 472
Discount . . . 147, 191, 198, 215
bank 412
exchange 434
trade 420
Discounting notes 414
Dividend 20, 472, 478
Divisibility tests . . . 32, 33
Division . 20, 62, 86, 102, 110
Divisor 20, 30
Dozen 149
Drafts 430, 432
Drawee and drawer . . . 432
Dressmaking problems . . 353
Dry goods 197
measure 93
Duty 457, 459

Electricity 253
English money 436
Equation 176, 177
Equilateral 152
European money . . 436, 449
Even number 31
Exact interest 370
Excavations 332
Exchange 429
Extremes 384

PAGE

Face of note 214, 463
Factor 15, 30–33
Factory problems 376
Fahrenheit 231
Farm problems, 51, 83, 220, 226, 322, 390
Fertilizers 393, 394
Fisheries 145
F.o.b. 250
Food supply 26
Foreign exchange 436
money 436
Forests 232
Fractions 36, 76, 114
See the operations.
Freight problems 250
French money 436
Fruit 60

German money 436
Government expenses . . 452
Grain 26, 481
Greatest common divisor, 30, 34
Grocer's problems, 196, 354, 377
Grounds, laying out . . . 241

Height 154
Hindoo figures 265
House building 236
Hypotenuse 500

Improper fraction 37
Indorse, 204, 216, 409, 411, 430
Industrial problems, 322, 353, 354, 359, 360, 376, 377, 390, 483, 486, 487, 489
Insurance 463
Interest . . . 202, 211, 361
compound 405
exact 370
tables 369
Iron 244, 487
Isosceles triangle 152

Kitchen problems 65

Land measure 91
Lard, buying 482
Laws of operations . . . 291

PAGE

Least common denominator, 44
Least common multiple . . 35
Length 91, 315, 441
Life insurance 468
Liquid measure 92
Local taxes 460
Longitude and time . . . 337
Loss, profit 194
Lowest terms 43
Lumber 234

Magnitudes illustrated . . 120
Maker of note 216
Making change 11
Manufacturer . . 143, 246, 247
Marine insurance 467
Means 384
Measures, 91, 151, 157, 207, 228, 234, 311, 329
Mensuration 510
Merchants' rule 419
Meridian 327, 337
Metals 245
Meteorology 252, 490
Metric system . . . 440, 449
Mexican measures 207
Mines 242, 244
Minuend 10
Miscellaneous problems . . 254
Mixed numbers 37
See the operations.
Model bill 422
order 422
Money order 429
Mortgage 478
Multiple 15, 23, 35
Multiplicand, multiplier . . 14
Multiplication, 14, 52, 67, 84, 101, 106, 279, 299

Names of numbers 3
Naught 2
Navy 455
Notation 2
Note 214, 411
Numbers 2, 4, 31, 77
Numerals . . . 2, 31, 77, 265
Numeration 2
Numerator 37

PAGE

Obtuse-angled triangle . . 152
Ocean traffic 489
Odd number 31
Operations 291
See Addition, etc.
Oral analysis 160
Ordering goods 422
Orders 3
Oyster industry 360

Paper 150
Papering 240
Par of exchange 434
of stock 474
Parallelogram . 151, 155, 323
Partial payments 416
Partitive proportion . 381, 426
Payee 216, 409, 432
Per cent 129
Percentage . 129, 137, 181, 346
Periods 2
Perpendicular lines . . . 94
π 510
Pictures of magnitudes . . 120
Place value 2
Plastering 238
Policy 463, 469
Poll tax 460
Polygon 151
Population 142
Pork, buying 481
Post office 454
Postal money order . . . 429
Potatoes 83
Poultry 51
Power . 15, 17, 85, 87, 492, 494
Precipitation 252
Premium on exchange . . 434
on policy 463
Price lists 421
Prime number 31, 33
Principal 202
Prism 514
Problem solving . . 115, 159
Problems in one step . . . 174
Proceeds . . . 215, 412, 432
Produce . . . 228, 250, 481
Product 14
Profit and loss 194

PAGE

Promissory note . . 214, 411
Proper fraction 37
Proportion . . . 223, 379, 383
compound . . . 169, 386
Proportional quantities . . 224
sides 224
Public lands 327
Pyramid 519, 521

Quadrilateral 151
Quotient 20

Railways 105
Rainfall 252
Rate 137, 147, 202
of dividend 472
of exchange 438
of interest 214
of taxation 460
Ratio 217, 379
Receipts 125, 205
Reciprocal 64
Rectangle 151
Reduction 95
of fractions . . . 39, 78
Remainder 20, 22
Right-angled triangle . . . 152
Roman figures 265
Roots 492

Savings bank . . . 214, 403
Scalene triangle 152
Scales 267
School problems 82
Separatrix 2
Shares of stock 472
Shipper 250
Shipping . . . 119, 249, 250
Short methods 292
Similar figures 224
fractions 44
Simple interest . 202, 211, 361
proportion . . 223, 379, 383
Six per cent method . . . 364
Slant height 521
Solving equations 177
problems . . . 115, 159
Specific gravity 446
Sphere 523
Square 151

PAGE

Square measure . . . 91, 443
numbers 492
root 492
Standard time 342
State taxes 460
Stock 472
yards 29
Subtraction, 10, 47, 48, 80, 98, 105, 276, 296
Subtrahend 10
Sugar industry 486
Sum 6
Surveyors' tables . . 315, 316

Tables 91, 543
Tariff 457
Tax table 462
Taxes 452
Temperature . . 231, 252, 490
Terms of a fraction . . . 37
Thermometer . . . 231, 252
Timber 232
Time 93, 337, 367
Townships 327
Trade discount . 191, 198, 420
Transportation problems . . 489
Trapezoid 151, 325
Travel, problems of . . . 449
Triangle 152, 154, 324
Troy weight 92

Uniform scale 267
Unit 36, 269
fraction 37, 63
Unitary analysis 168
United States rule 416
Unknown quantity . . . 177

Value 93
Vara 207
Varying scale 267
Volume 156, 329, 514

Water, weight of . . 229, 230
Weather 252, 490
Weight . . 92, 228, 229, 445
Wheat 26
Wind 252
Wool 118
Written analysis 164

Zero 2, 265

ANSWERS

Page 3

1. 5,280,000 ft.
2. 60 sec., 3600 sec., 86,400 sec., 950,400 sec.
3. 8760 hr., 876,000 hr.
4. 4, 40, 40,000.

Page 6

1. 26,776.
2. 18,947.
3. 269,162.
4. 2,383,868.
5. 4,429,515.

Pages 7, 8

1. 76,303,387.
2. 6,429,474.
3. $4,289,626,071.
4. $2,272,400,000.
5. 3,756,884 sq. mi.
6. 1,740,032,459 bu.
7. 16,738,363.
8. 906,293,360 lb.
9. 293,298,516 T.
10. 13,157,000 bales.
11. $2,763,575,126.
12. 31,846 mi.
13. 3,753,986,804 bu.
14. $22,789,054.

Page 9

1. 1,241,016.
2. 383,759.
3. 7,902,936.
4. 934,695.
5. 7,119,003.
6. 4707.
7. 29,556.
8. 1,125,624.
9. 337,887.
10. 4,336,886.

Page 11

1. $16.27.
2. $88.75.
3. $190.75.
4. $186.25.
5. $191.75.
6. $126.70.
7. 307,890.
8. 254,648.

Page 12

2. 117 yr.
5. 44 yr.
7. 128 yr.
8. 1775.

Page 13

1. 14,338.
2. 4867.
3. 6789.
4. 4707.
5. 51,805.
6. 45,873.
7. 131,891.
8. 238,648.
9. 33,298.
10. 185,094.
11. $327.65.
12. $180.99.
13. $791.01.
14. $1832.63.
15. $49,355.25.
16. $22,725.42.
17. $16,393.25.
18. $8899.99.

Page 16

1. 863,928. 2. 57,591. 3. 232,232.
4. 244,720. 5. 62,212. 6. 525,625.
7. $9780.99. 8. $28,890.05. 9. $125,555.50.
10. $366,713.10. 11. $30,258.24. 12. $317,343.75.
13. $247,247. 14. $37,622,000. 15. $260,257.25.
16. $6,017,511.50. 17. $24,621,931.50. 18. $16,323,216.

Page 17

1. $72,660. 2. $134,720. 3. 745,600 ft. 4. 604,800 rd.
5. $41,280,000. 6. $35,475,000. 7. $107,940. 8. $195,750.
9. $81,500. 10. $85,305. 11. 964 mi. 12. $4425.

Page 18

1. 3615, $12,500, $237, $3333.
2. 11,850, 21,250 ft., $3425, $49,370.
3. 3350, 20,050 ft., $1240, $9610.
4. 8000, 32,000, 363,000, $21,400, $62,200.

Page 19

1. 76,303,388. 2. 1049.
3. $94,635,900, $221,106,010, $212,287,275. 4. 4,626,068.

Page 21

1. $135, $540. 2. $28, $196. 3. $580. 4. $540, $945.
5. $295.50, $492.50, $9850.

Page 22

1. $2.68, rem. $2.96. 2. $0.98. 3. $25.04.
4. $0.86, rem. $0.30. 5. $4.97, rem. $1.04. 6. 54.16\frac{2}{3}$.
7. $3.96, rem. $0.90. 8. $84.12, rem. $0.06. 9. $13.81\frac{9}{11}$.
10. $3.54\frac{5}{12}$. 11. $5.49\frac{1}{15}$. 12. $13.81\frac{1}{4}$.

Page 23

1. 58. 2. 52. 3. 14.
4. 32. 5. 30. 6. 70.
7. $39\frac{3}{7}$. 8. $271\frac{2}{5}$. 9. $4\frac{271}{375}$.
10. $7\frac{481}{630}$. 11. 20 da. 12. 21 sec.
13. $21,133.69\frac{61}{73}$. 14. $1,313,013.69\frac{61}{73}$.

Page 25

1. 44,528.	2. 34,342.	3. 302,133.	4. 578,154.
5. 254,347.	6. 66,717.	7. 201,501.	8. 200,802.
9. 478,813.	10. 144.	11. 408.	12. $2169\frac{4}{5}\frac{9}{1}$.
13. 912.	14. 67,910.	15. 37,864.	16. 204,100.
17. 781,312.	18. 390,208.	19. 4,402,496.	20. 12,975,035.
21. 12,899,583.			

Page 26

1. 565,825,000 bu. 2. 6864 bu. 3. 517,200,000 bu.
4. $383,112,100. 5. $7400.

Page 27

1. 84,204,096 A. 2. $701,700,800. 3. 695. 4. $3500.
5. $978,810, Conn.; $52,410,800, Ind. 6. $37\frac{5}{8}$ bu.

Page 28

1. $77,076,249. 2. $56,065,203. 3. $67,071,104.
4. $4,999,837.50. 5. $22. 6. $30.
7. 6,379,054, 26,147,604.

Page 29

1. $50,000,000. 2. 644 lb., $53.02. 3. 12,300.
4. $500. 5. $15,471,616. 6. 10 ct.

Page 31

1. 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97. Also odd numbers.
2. 2, 3, 3, 3; 2, 2, 2, 2, 3; 2, 2, 2, 2, 2, 2; 7, 11. 3. 12.

Page 32

1. Those ending in even numbers. 2. Those ending in 0 or 5.
3. All but 4004, 7117, 475,633.

Page 34

1. $5^2 \times 31$.	2. $2^2 \times 3 \times 5 \times 7$.	3. $5 \times 7 \times 11$.
4. $2^5 \times 11$.	5. $2^6 \times 5$.	6. $3^4 \times 11$.
7. $7^2 \times 3$.	8. $3^3 \times 11$.	9. $2^4 \times 11$.

10. $2^4 \times 3 \times 7$.
11. $2 \times 5 \times 13$.
12. $2 \times 3 \times 7 \times 11$.
13. $2 \times 3 \times 5 \times 13$.
14. $5 \times 3 \times 19$.
15. 3×5^3.
16. $2 \times 3^4 \times 5 \times 11$.
17. 2×5^4.
18. $2 \times 3^5 \times 5$.
19. $2^8 \times 5$.
20. 2^{10}.
21. $2 \times 3^3 \times 5 \times 11$.
22. $5^3 \times 11$.
23. $2 \times 5 \times 7^2 \times 11$.
24. $2^5 \times 7^2$.
25. 11^3.
26. $2^2 \times 5^2 \times 3^2 \times 7$.
27. 3×5^4.
28. $3^5 \times 5$.
29. $3^6 \times 5$.
30. $3^7 \times 5$.
31. 7.
32. 32.
33. 65.
34. 25.
35. 444.
36. 55.
37. 144.
38. 256.
39. 512.
40. 77.
41. 125.
42. 165.
43. 288.
44. 625.
45. 625.
46. 101.
47. 25.
48. 515.

Page 35

1. 210. **2.** 770. **3.** 224. **4.** 192. **5.** 900.
6. 96. **7.** 75. **8.** 1056. **9.** 50. **10.** 64.
11. 3780. **12.** 288. **13.** 1785. **14.** 756. **15.** 29,304.

Page 40

1. $\frac{3}{8}, \frac{5}{11}, \frac{6}{7}, \frac{1}{2}, \frac{2}{3}, \frac{4}{5}$.
2. $\frac{3}{15}, \frac{10}{15}, \frac{2}{15}, \frac{9}{15}, \frac{4}{15}, \frac{6}{15}, \frac{7}{15}, \frac{8}{15}$.
3. $\frac{12}{16}, \frac{14}{16}, \frac{8}{16}, \frac{5}{16}, \frac{7}{16}, \frac{8}{16}, \frac{3}{16}, \frac{5}{16}$.
4. $\frac{25}{50}, \frac{30}{50}, \frac{45}{50}, \frac{14}{50}, \frac{2}{50}, \frac{9}{50}, \frac{9}{50}, \frac{7}{50}$.
5. $\frac{8}{32}, \frac{20}{32}, \frac{6}{32}, \frac{28}{32}, \frac{14}{32}, \frac{9}{32}, \frac{9}{32}$.
6. $\frac{75}{100}, \frac{80}{100}, \frac{70}{100}, \frac{85}{100}, \frac{75}{100}, \frac{16}{100}, \frac{15}{100}, \frac{67}{100}$.
7. $\frac{84}{144}, \frac{96}{144}, \frac{8}{144}, \frac{132}{144}$.
8. $\frac{21}{24}, \frac{14}{24}$; $\frac{63}{72}, \frac{42}{72}$; $\frac{189}{216}, \frac{126}{216}$.

Page 41

1. $\frac{42}{6}$. **2.** $\frac{63}{7}$. **3.** $\frac{60}{10}$. **4.** $\frac{88}{11}$. **5.** $\frac{66}{6}$.
6. $\frac{52}{4}$. **7.** $\frac{18}{2}$. **8.** $\frac{180}{20}$. **9.** $\frac{45}{3}$. **10.** $\frac{17}{8}$.
11. $\frac{55}{8}$. **12.** $\frac{23}{3}$. **13.** $\frac{89}{10}$. **14.** $\frac{20}{3}$. **15.** $\frac{31}{8}$.
16. $\frac{44}{9}$. **17.** $\frac{58}{9}$. **18.** $\frac{147}{16}$. **19.** $\frac{81}{11}$. **20.** $\frac{69}{8}$.
21. $\frac{45}{7}$. **22.** $\frac{124}{15}$. **23.** $\frac{101}{14}$. **24.** $\frac{101}{12}$. **25.** $\frac{50}{8}$.
26. $\frac{25}{2}$. **27.** $\frac{100}{3}$. **28.** $\frac{200}{3}$. **29.** $\frac{43}{15}$. **30.** $\frac{1000}{3}$.
31. $\frac{500}{3}$. **32.** $\frac{2000}{3}$. **33.** $\frac{251}{2}$. **34.** $\frac{1403}{32}$. **35.** $\frac{733}{5}$.
36. $\frac{2120}{9}$. **37.** $\frac{5781}{7}$. **38.** $\frac{8992}{9}$. **39.** $\frac{2027}{3}$.

Page 42

1. 11. **2.** 6. **3.** $10\frac{5}{6}$. **4.** $9\frac{2}{3}$. **5.** $10\frac{1}{8}$.
6. $7\frac{5}{6}$. **7.** $15\frac{3}{5}$. **8.** $9\frac{6}{7}$. **9.** $7\frac{10}{11}$. **10.** $7\frac{2}{3}$.

11. 5. **12.** $7\frac{5}{33}$. **13.** $14\frac{1}{6}$. **14.** $1\frac{1}{92}$. **15.** 1.
16. $18\frac{5}{8}$. **17.** $10\frac{29}{32}$. **18.** $15\frac{1}{3}$. **19.** $12\frac{3}{8}$. **20.** $17\frac{11}{42}$.
21. $19\frac{3}{4}$. **22.** $26\frac{11}{16}$. **23.** $26\frac{5}{24}$. **24.** $5\frac{3}{16}$. **25.** $28\frac{1}{2}$.
26. 2. **27.** $12\frac{1}{2}$. **28.** $20\frac{1}{11}$. **29.** 25. **30.** 7.
31. $5\frac{2}{25}$. **32.** $1\frac{8}{17}$. **33.** $1\frac{9}{104}$. **34.** $1\frac{2}{3}$. **35.** 21.
36. 5. **37.** $10\frac{1}{11}$. **38.** 13. **39.** 182. **40.** 12.

Page 43

1. $\frac{4}{11}$. **2.** $\frac{7}{9}$. **3.** $\frac{8}{11}$. **4.** $\frac{3}{4}$. **5.** $\frac{8}{9}$. **6.** $\frac{9}{11}$. **7.** $\frac{9}{11}$.
8. $\frac{2}{5}$. **9.** $\frac{8}{13}$. **10.** $\frac{11}{13}$. **11.** $\frac{8}{9}$. **12.** $\frac{8}{13}$. **13.** $\frac{17}{31}$. **14.** $\frac{9}{19}$.
15. $\frac{21}{23}$. **16.** $\frac{17}{19}$. **17.** $\frac{11}{13}$. **18.** $\frac{23}{31}$. **19.** $\frac{5}{7}$. **20.** $\frac{13}{18}$. **21.** $\frac{9}{13}$.
22. $\frac{4}{7}$. **23.** $\frac{1}{2}$. **24.** $\frac{37}{41}$. **25.** $\frac{13}{14}$. **26.** $\frac{1}{12}$. **27.** $\frac{3}{5}$. **28.** $\frac{1}{18}$.
29. $\frac{2}{3}$. **30.** $\frac{21}{50}$. **31.** $\frac{5}{7}$. **32.** $\frac{5}{6}$. **33.** $\frac{3}{4}$. **34.** $\frac{11}{20}$. **35.** $\frac{3}{4}$.

Page 45

1. $\frac{21}{175}$, $\frac{30}{175}$. **2.** $\frac{18}{441}$, $\frac{35}{441}$. **3.** $\frac{82}{108}$, $\frac{15}{108}$.
4. $\frac{66}{275}$, $\frac{35}{275}$. **5.** $\frac{9}{186}$, $\frac{8}{186}$. **6.** $\frac{10}{72}$, $\frac{7}{72}$.
7. $\frac{8}{121}$, $\frac{11}{121}$. **8.** $\frac{144}{504}$, $\frac{448}{504}$, $\frac{189}{504}$. **9.** $\frac{42}{900}$, $\frac{55}{900}$.
10. $\frac{4}{625}$, $\frac{30}{625}$. **11.** $\frac{1}{12}$, $\frac{1}{12}$. **12.** $\frac{33}{1430}$, $\frac{40}{1430}$.
13. $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$. **14.** $\frac{350}{15540}$, $\frac{4077}{15540}$. **15.** $\frac{10}{16}$, $\frac{3}{16}$, $\frac{2}{16}$.
16. $\frac{18}{150}$, $\frac{40}{150}$, $\frac{45}{150}$. **17.** $\frac{100}{225}$, $\frac{120}{225}$, $\frac{54}{225}$. **18.** $\frac{5}{32}$, $\frac{6}{32}$, $\frac{28}{32}$.
19. $\frac{9}{60}$, $\frac{14}{60}$, $\frac{8}{60}$. **20.** $\frac{8}{50}$, $\frac{4}{50}$, $\frac{40}{50}$. **21.** $\frac{6}{81}$, $\frac{36}{81}$, $\frac{5}{81}$.
22. $\frac{78}{130}$, $\frac{20}{130}$, $\frac{7}{130}$. **23.** $\frac{140}{210}$, $\frac{90}{210}$, $\frac{168}{210}$, $\frac{21}{210}$. **24.** $\frac{7}{112}$, $\frac{70}{112}$, $\frac{9}{112}$.
25. $\frac{24}{222}$, $\frac{21}{222}$, $\frac{10}{222}$. **26.** $\frac{18}{156}$, $\frac{28}{156}$, $\frac{15}{156}$. **27.** $\frac{42}{441}$, $\frac{45}{441}$, $\frac{28}{441}$.
28. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$. **29.** $\frac{16}{32}$, $\frac{24}{32}$, $\frac{28}{32}$, $\frac{10}{32}$, $\frac{3}{32}$. **30.** $\frac{4}{36}$, $\frac{24}{36}$, $\frac{30}{36}$, $\frac{21}{36}$, $\frac{5}{36}$.
31. $31\frac{44}{297}$, $42\frac{21}{297}$. **32.** $29\frac{35}{77}$, $36\frac{13}{77}$. **33.** $42\frac{153}{162}$, $61\frac{50}{162}$.
34. $23\frac{16}{64}$, $67\frac{27}{64}$. **35.** $32\frac{44}{693}$, $77\frac{54}{693}$. **36.** $81\frac{21}{96}$, $27\frac{14}{96}$.
37. $71\frac{70}{150}$, $62\frac{33}{150}$. **38.** $68\frac{4}{74}$, $41\frac{11}{74}$. **39.** $18\frac{28}{260}$, $21\frac{55}{260}$.

Page 46

1. $1\frac{1}{8}$. **2.** $\frac{7}{8}$. **3.** $1\frac{3}{10}$. **4.** $1\frac{3}{8}$. **5.** $\frac{7}{10}$. **6.** $\frac{11}{16}$.
7. $1\frac{5}{16}$. **8.** $\frac{7}{12}$. **9.** $1\frac{5}{8}$. **10.** $19\frac{1}{2}$. **11.** $18\frac{1}{2}$. **12.** $7\frac{5}{7}$.

Page 47

1. $\frac{1}{12}$. **2.** $\frac{13}{16}$. **3.** $\frac{1}{8}$. **4.** $\frac{1}{4}$. **5.** $\frac{9}{16}$. **6.** $\frac{5}{12}$.
7. $\frac{2}{3}$. **8.** $\frac{1}{6}$. **9.** $\frac{7}{18}$. **10.** $\frac{1}{8}$. **11.** $\frac{7}{32}$. **12.** $\frac{44}{63}$.
13. $\frac{4}{21}$. **14.** $\frac{13}{18}$. **15.** $\frac{8}{22}$. **16.** $\frac{1}{24}$. **17.** $\frac{3}{10}$. **18.** $\frac{17}{36}$.

19. $\frac{1}{6}$. **20.** $\frac{41}{63}$. **21.** $\frac{14}{27}$. **22.** $\frac{4}{5}$. **23.** $\frac{17}{32}$. **24.** $\frac{16}{35}$.
25. $\frac{25}{42}$. **26.** $\frac{57}{64}$. **27.** $\frac{6}{25}$. **28.** $\frac{11}{14}$. **29.** $\frac{43}{51}$. **30.** $\frac{29}{39}$.
31. $\frac{19}{45}$. **32.** $\frac{2}{35}$. **33.** $\frac{1}{8}$ mi.

Page 48

1. $365\frac{7}{9}$. **2.** $122\frac{5}{18}$. **3.** $162\frac{9}{10}$. **4.** $266\frac{1}{15}$.
5. $88\frac{17}{24}$. **6.** $97\frac{1}{21}$. **7.** $212\frac{9}{14}$. **8.** $382\frac{2}{3}$.
9. $\frac{67}{70}$. **10.** $1\frac{11}{40}$. **11.** $1\frac{11}{48}$. **12.** $2\frac{1}{4}$.
13. $1\frac{19}{30}$. **14.** $1\frac{1}{8}$. **15.** $\frac{53}{60}$. **16.** $1\frac{17}{30}$.
17. $\frac{139}{210}$. **18.** $2\frac{9}{32}$. **19.** $\frac{955}{144}$. **20.** $37\frac{23}{24}$.
21. $2\frac{5}{24}$. **22.** $69\frac{29}{40}$. **23.** $2\frac{5}{14}$. **24.** $\frac{13}{16}$ in.
25. $1\frac{5}{8}$ in. **26.** $69\frac{1}{4}$ in. **27.** $1\frac{3}{8}$ in. **28.** $380\frac{7}{24}$ rd.

Page 49

1. 3 ft. $9\frac{1}{2}$ in. **2.** $10\frac{11}{12}$ yr. **3.** $3\frac{13}{16}$ in.
4. $78\frac{11}{12}$ lb. **5.** $196\frac{19}{24}$ yd. **6.** $36\frac{3}{8}$ in., $22\frac{1}{2}$ in.
7. 6 in. **8.** $26^{\wedge}9\frac{3}{8}''$. **9.** $2''$, $1\frac{3}{8}''$.

Page 50

1. $9\frac{49}{108}$. **2.** $3\frac{31}{120}$. **3.** $11\frac{13}{20}$. **4.** $7\frac{47}{144}$. **5.** $10\frac{121}{300}$. **6.** $14\frac{79}{144}$.
7. $17\frac{7}{8}$. **8.** $2\frac{29}{48}$. **9.** $4\frac{2}{5}$. **10.** $12\frac{7}{15}$. **11.** $9\frac{21}{32}$. **12.** $3\frac{23}{48}$.
13. $17\frac{29}{40}$. **14.** $22\frac{2}{35}$. **15.** $8\frac{21}{32}$. **16.** $84\frac{1}{2}$. **17.** $12\frac{7}{8}$. **18.** $12\frac{5}{8}$.
19. $\frac{1}{8}$ in., $\frac{1}{8}$ in., $\frac{1}{16}$ in., $\frac{1}{8}$ in., $\frac{1}{16}$ in.

Page 51

1. 171, 3, 15. **2.** 2 bu., \$1.50. **3.** $1\frac{3}{5}$ bu., \$1.44.
4. $1\frac{1}{4}$ bu., 60 ct. **5.** $\frac{1}{3}$ bu., 20 ct. **6.** 264 lb., $5\frac{11}{60}$ bu., $4\frac{12}{15}$ lb.
7. \$5.70, \$0.10, \$0.00$\frac{2}{3}$. **8.** 12 ct., \$8.55. **9.** \$1.30, $2\frac{16}{57}$ ct.

Page 54

1. $6\frac{2}{3}$. **2.** $4\frac{4}{5}$. **3.** $5\frac{1}{4}$. **4.** $3\frac{8}{9}$. **5.** 6. **6.** 44. **7.** $6\frac{3}{7}$.
8. $10\frac{1}{2}$. **9.** $1\frac{5}{9}$. **10.** $7\frac{1}{3}$. **11.** $7\frac{7}{8}$. **12.** $17\frac{7}{8}$. **13.** $1\frac{5}{9}$. **14.** $4\frac{10}{11}$.
15. $16\frac{1}{2}$. **16.** $10\frac{1}{12}$.

Page 56

1. $\frac{8}{15}$. **2.** $\frac{5}{14}$. **3.** $\frac{9}{14}$. **4.** $\frac{2}{7}$. **5.** $\frac{1}{2}$. **6.** $\frac{2}{3}$.
7. $\frac{18}{35}$. **8.** $\frac{11}{21}$. **9.** $1\frac{1}{2}$. **10.** $1\frac{1}{8}$. **11.** $1\frac{1}{2}$. **12.** 9.
13. $\frac{4}{11}$. **14.** $\frac{2}{5}$. **15.** 1. **16.** \$0.46$\frac{7}{8}$. **17.** $\frac{21}{32}$ min. **18.** $\frac{1}{4}$.
20. 323 sq. ft., 12 sq. in., 7 sq. in.

Page 57

1. $43\frac{1}{2}$. 2. 1122. 3. 4011. 4. 5067. 5. $1790\frac{2}{3}$.
6. 856. 7. 1701. 8. 1043. 9. $3391\frac{1}{2}$. 10. $487\frac{2}{3}$.
11. $3061\frac{1}{2}$. 12. $1669\frac{1}{2}$. 13. $1291\frac{1}{2}$. 14. $54\frac{9}{64}$. 15. $153\frac{11}{24}$.
16. $240\frac{1}{12}$. 17. $227\frac{41}{63}$. 18. $247\frac{4}{5}$. 19. $443\frac{11}{28}$. 20. $546\frac{11}{15}$.
21. $476\frac{4}{9}$. 22. \$4.12$\frac{1}{2}$.

Page 58

1. $\frac{1}{5}$. 2. $\frac{8}{21}$. 3. $1\frac{1}{2}$. 4. 1. 5. $3\frac{77}{81}$. 6. $\frac{8}{5}$. 7. $1\frac{1}{8}\frac{1}{4}$.

Page 59

1. $11{,}139\frac{8}{16}$. 2. $8812\frac{1}{2}$. 3. 24,990. 4. $213{,}511\frac{1}{9}$. 5. $9887\frac{5}{8}$.
6. 1824. 7. $262\frac{1}{2}$. 8. $5002\frac{113}{180}$. 9. 435. 10. $2914\frac{92}{147}$.
11. 16. 12. $1\frac{7}{8}$. 13. $\frac{9}{10}$. 14. 4. 15. 2.

Pages 60, 61

1. 140 cu. in.
2. $461\frac{1}{8}$ cu. in.
3. 3834 cu. in.
4. $70\frac{5}{8}$ cu. in.
5. $49\frac{1}{3}$ cu. in.
6. $159\frac{1}{2}$ cu. in. larger.
7. $489\frac{1}{2}$ cu. in. larger.
8. 9120 cu. in.
9. $370\frac{55}{72}$ cu. in. more in 1st.
10. $1235\frac{1}{4}$ cu. in.
11. $50\frac{55}{64}$ cu. in. more in 1st.
12. $869\frac{1}{2}$ cu. in., $427\frac{31}{32}$ cu. in., $1847\frac{11}{16}$ cu. in., $1059\frac{45}{64}$ cu. in.

Page 62

1. $\frac{5}{36}$. 2. $\frac{8}{85}$. 3. $\frac{1}{63}$. 4. $\frac{7}{36}$. 5. $\frac{7}{16}$. 6. $\frac{1}{12}$.
7. $\frac{5}{24}$. 8. $34\frac{1}{18}$. 9. $45\frac{1}{15}$. 10. $29\frac{2}{9}$. 11. $136\frac{9}{16}$. 12. $165\frac{6}{25}$.

Page 63

1. 648. 2. 144. 3. 161. 4. 192. 5. 216.
6. 87. 7. 240. 8. 441. 9. 243. 10. 204.
11. 1164. 12. 780. 13. 2000. 14. 4608. 15. 15,360.

Page 64

1. $1\frac{5}{16}$. 2. $1\frac{1}{9}$. 3. $1\frac{1}{8}$. 4. $\frac{10}{21}$. 5. $\frac{9}{10}$. 6. $4\frac{2}{3}$.
7. $\frac{5}{14}$. 8. $1\frac{1}{5}$. 9. $\frac{5}{16}$. 10. $\frac{1}{4}$. 11. $\frac{12}{17}$. 12. $\frac{39}{50}$.
13. $1\frac{384}{383}$. 14. $\frac{42}{155}$. 15. $1\frac{1}{2}$. 16. $\frac{41}{111}$. 17. $4\frac{1}{13}$. 18. $\frac{21}{55}$.

Page 65

1. $7\frac{3}{4}$ lb. **2.** 1 qt. $1\frac{1}{4}$ pt. **3.** 14. **4.** \$1.11. **5.** \$4.55.
6. 3 lb. 9 oz. **7.** $11\frac{1}{4}$ ct. **8.** 2 lb. 14 oz.; 1 lb. 8 oz. bread crumbs, 3 lb. blueberries, 1 lb. sugar, 4 oz. butter. **9.** $\$0.04\frac{1}{6}$.

Page 68

1. \$18.38. **2.** \$18.83. **3.** \$33.67. **4.** \$220. **5.** \$165.67.
6. \$126. **7.** \$156. **8.** \$6.

Page 69

1. $140\frac{5}{6}$. **2.** $155\frac{5}{12}$. **3.** $76\frac{19}{20}$. **4.** $97\frac{8}{15}$.
5. $102\frac{7}{15}$. **6.** $107\frac{1}{2}$. **7.** $96\frac{11}{15}$. **8.** $112\frac{13}{14}$.
9. $117\frac{3}{10}$. **10.** $68\frac{13}{20}$. **11.** $150\frac{3}{10}$. **12.** $55\frac{9}{14}$.
13. $57\frac{3}{10}$. **14.** $50\frac{5}{8}$. **15.** $24\frac{7}{8}$. **16.** $54\frac{17}{20}$.
17. $10\frac{19}{24}$. **18.** $13\frac{5}{8}$. **19.** $16\frac{11}{40}$. **20.** $24\frac{13}{24}$.
21. $25\frac{1}{3}$. **22.** 396. **23.** 2960. **24.** $4984\frac{1}{8}$.
25. $1741\frac{1}{2}$. **26.** 2074. **27.** 3872. **28.** 2754.
29. 4008. **30.** 1. **31.** $131\frac{1}{4}$. **32.** $45\frac{3}{5}$.
33. $34\frac{17}{20}$. **34.** $56\frac{7}{20}$. **35.** $18,422\frac{1}{4}$. **36.** $17,167\frac{1}{2}$.
37. 31,603. **38.** $31,348\frac{2}{7}$. **39.** $52,443\frac{3}{5}$. **40.** $1\frac{1}{5}$.
41. $\frac{5}{6}$. **42.** $\frac{9}{14}$. **43.** $1\frac{5}{9}$. **44.** $1\frac{3}{32}$.
45. $\frac{32}{35}$. **46.** $1\frac{1}{7}$. **47.** $\frac{7}{8}$. **48.** $\frac{3}{22}$.
49. $\frac{25}{77}$. **50.** $\frac{1}{7}$. **51.** $4\frac{24}{49}$. **52.** $2\frac{9}{10}$.
53. $3\frac{3}{35}$. **54.** 5. **55.** $87\frac{3}{5}$. **56.** $1\frac{47}{70}$.
57. $\frac{4}{5}$. **58.** $1\frac{53}{64}$. **59.** 4. **60.** $7\frac{31}{77}$.
61. $\frac{1}{4}$. **62.** $\frac{2}{5}$ cu. ft. **63.** $\frac{6}{25}$ cu. ft. **64.** $\frac{1}{3}$ cu. in.
65. 20 sq. ft. **66.** 432 sq. ft. **67.** 56 sq. in. **68.** 78 sq. in.

Page 70

1. $\frac{1}{6}$. **2.** $\frac{1}{5}$. **3.** $\frac{2}{49}$. **4.** $\frac{1}{24}$. **5.** $\frac{5}{7}$.
6. 4. **7.** $\frac{10}{13}$. **8.** $\frac{3}{4}$. **9.** $\frac{9}{10}$. **10.** $1\frac{52}{75}$.
11. $\frac{26}{49}$. **12.** $\frac{5}{9}$. **13.** $\frac{9}{20}$. **14.** $\frac{1}{2}$. **15.** $\frac{4}{45}$.
16. $1\frac{1}{4}$. **17.** 1. **18.** $1\frac{8}{35}$. **19.** $\frac{1}{2}$. **20.** $\frac{9}{44}$.
21. $\frac{27}{100}$. **22.** 18. **23.** $\frac{1}{24}$. **24.** 1. **25.** $\frac{5}{32}$.

Page 71

1. $\frac{4}{7}$. **2.** $\frac{7}{9}$. **3.** $\frac{16}{21}$. **4.** $\frac{21}{121}$. **5.** $\frac{1}{2}$.
6. $\frac{1}{2}$. **7.** $\frac{1}{9}$. **8.** $\frac{1}{8}$. **9.** $\frac{1}{4}$. **10.** $\frac{1}{9}$.

11. $\frac{5}{8}$. **12.** $\frac{1}{6}$. **13.** $\frac{1}{7}$. **14.** $\frac{1}{3}$. **15.** $\frac{9}{71}$.
16. $\frac{1}{2}$. **17.** $\frac{1}{6}$. **18.** $\frac{20}{27}$. **19.** $\frac{856}{1675}$. **20.** $\frac{8}{9}$.
21. $\frac{1}{7}$. **22.** $\frac{1}{2}$. **23.** $\frac{1}{9}$. **24.** $\frac{1}{11}$. **25.** $\frac{2}{3}$.
26. $\frac{1}{3}$. **27.** $\frac{1}{2}$, $\frac{3}{4}$.

Page 72

1. 69. **2.** 57. **3.** 72. **4.** 200.
5. 50. **6.** 250. **7.** 259. **8.** 160.
9. 256. **10.** \$250. **11.** \$3.60. **12.** \$18.45.
13. 153. **14.** 1394. **15.** \$2250. **16.** \$535, \$1605.

Page 74

1. \$51.06. **2.** \$0.54. **3.** \$1.92. **4.** \$4.40. **5.** \$0.62.
6. \$0.07. **7.** \$2.43. **8.** \$7.94. **9.** \$1.57.

Page 75

1. \$1.56, change. **2.** \$1.82. **3.** \$6.82.
4. \$8.65. **5.** \$1.68. **6.** \$0.25.

Page 77

1. 0.3, 0.5, 0.9. **2.** 4.4, 7.8. **3.** 0.75, $0.33\frac{1}{3}$, 300.03.
4. 0.145, 0.675, 0.0004. **5.** 5.007, 8.042, 0.025, 0.000001.

Page 78

1. $\frac{2}{5}$. **2.** $\frac{3}{5}$. **3.** $\frac{1}{5}$. **4.** $\frac{4}{5}$.
5. $\frac{1}{4}$. **6.** $\frac{3}{4}$. **7.** $\frac{9}{20}$. **8.** $\frac{11}{20}$.
9. $\frac{1}{8}$. **10.** $\frac{3}{8}$. **11.** $\frac{5}{8}$. **12.** $\frac{7}{8}$.
13. $1\frac{1}{4}$. **14.** $2\frac{3}{4}$. **15.** $5\frac{1}{8}$. **16.** $10\frac{3}{8}$.
17. $\frac{70}{100}$, $\frac{60}{100}$, $\frac{10}{100}$, $\frac{80}{100}$, $\frac{400}{100}$, $\frac{6300}{100}$. **18.** $\frac{100}{1000}$, $\frac{250}{1000}$, $\frac{370}{1000}$, $\frac{90}{1000}$, $\frac{31000}{1000}$.
19. $\frac{47}{100}$, $\frac{68}{100}$, $\frac{10}{100}$, $\frac{4}{100}$, $\frac{780}{100}$. **20.** $\frac{1}{8}$, \$200.

Page 79

1. 0.875. **2.** 0.15. **3.** 0.266+. **4.** 0.833+.
5. 0.857+. **6.** 0.555+. **7.** 0.636+. **8.** 0.107+.
9. 0.263+. **10.** 0.542+. **11.** 0.416+. **12.** 0.806+.
13. 0.288+. **14.** 0.610+. **15.** 3.770+. **16.** 3.075.
17. 4.052. **18.** 6.754. **19.** 4.005. **20.** 7.256.

Page 81

1. $47.96.	**2.** $55.59.	**3.** 177.63 ft.	**4.** 436.58.
5. 481.9.	**6.** 258.68.	**7.** 1072.44.	**8.** 761.34.
9. $0.18.	**10.** $5.75.	**11.** $6.85.	**12.** $2.75.
13. 57.225.	**14.** 35.93.	**15.** 41.084.	**16.** 539.325.
17. $121.36.	**18.** $137.08.	**19.** 27.884.	**20.** 108.398.
21. 68.9.	**22.** 79.93.	**23.** 60.44.	**24.** 39.64.
25. 108.9.	**26.** 70.09.	**27.** 533.46.	**28.** 124.75.
29. 9.08.			

Page 82

1. $3.80. **2.** $9.75. **3.** $17.50. **4** $10.15. **5.** $0.47.

Page 83

1. 333,070 bu.	**2.** $4,305,184.83.	**3.** 81.5 bu.	**4.** $13,261,680.
5. 173 bu.	**6.** $1887.50.	**7.** 204,960 lb.	

Page 84

1. 681.25.	**2.** 2087.36.	**3.** 968.40.	**4.** 2287.34.
5. 354.375.	**6.** 644.025.	**7.** 1895.625.	**8.** 3416.25.
9. 1500.375.	**10.** 3570.07.	**11.** 2614.85.	**12.** 2558.409.
13. 127.44 ft.	**14.** 85.32 in.	**15.** 44.16 in.	**16.** 2440.94 in.

Page 85

1. 269.2.	**2.** 146.1.	**3.** 588.6.	**4.** 7776.
5. 2920.	**6.** 43,771.	**7.** 52,336.	**8.** 33,372.
9. 24,720.	**10.** 1250.	**11.** 13,450.	**12.** 3,011,040.
13. 622,062.5.	**14.** 2,880,288.	**15.** 3,319,180.	**16.** 936,625.
	17. 1,428,280.	**18.** 43,750.	

Page 86

1. 42.75.	**2.** 3.217.	**3.** 21.11.	**4.** 7.77.
5. 20.02.	**6.** 47.04.	**7.** 70.11.	**8.** 311.21.
9. 212.75 in.	**10.** 412.6$\frac{1}{3}$ in.	**11.** 1.27 in.	

Page 87

1. 1.9.	**2.** 0.18.	**3.** 0.47.	**4.** 7.49.
5. 7.9.	**6.** 0.63.	**7.** 2.28.	**8.** 0.746.

9. 0.816.	**10.** 0.025.	**11.** 0.031.	**12.** 0.042.
13. 1.44.	**14.** 0.0024.	**15.** $0.039\frac{7}{11}$.	**16.** 2.904.
	17. $0.012\frac{5}{6}$.	**18.** 0.031.	

Pages 88, 89

1. 80¢.	**2.** $1.13.	**3.** $1.79.	**4.** $1.09.
5. $7.	**6.** $1.12\frac{1}{2}$.	**7.** $6.87\frac{1}{2}$.	**8.** 7.2 mi.
9. $1.20.	**10.** 23 gross.	**11.** $0.09\frac{1}{2}$.	**12.** $2.69.
13. $2.40.	**14.** $92.50.	**15.** $129\frac{1}{4}$ mi.	**16.** 23¢.
17. 28¢.	**18.** $40.	**19.** $21.60.	**20.** $1.12.
21. $4.50.	**22.** $339.	**23.** 24 da. 3 hr.	**24.** 20.95 mi.
	25. 32 yd.	**26.** $4035, $2690.	

Page 93

It is desirable at this point to give a great deal of oral and written drill upon the tables which have just been reviewed. Pupils should be required to write the tables rapidly, the tables themselves being the answers to this work.

Page 96

1. 34 pk.	**2.** 1035 rd.	**3.** 168 ft.
4. 320 sq. rd.	**5.** $2896\frac{1}{2}$ ft.	**6.** 66 in.
7. 34 oz.	**8.** 16 ft.	**9.** 11 qt.
10. 440 sq. rd.	**11.** 1280 cu. ft.	**12.** 630 cu. ft.
13. 169 cu. ft., 292,032 cu. in.		**14.** 1092 sq. ft., 157,248 sq. in.

Page 97

1. 80 ft.	**2.** 41 ft.	**3.** 30 lb.
4. 11 yd.	**5.** 11 lb.	**6.** 2 hr.
7. 60 hr.	**8.** 16 rd. 11 ft.	**9.** 68 bu.
10. 1 T. 1255 lb.	**11.** 8 bu. 7 qt. 1 pt.	**12.** 3 sq. ft. 68 sq. in.
13. 0.27 mi.	**14.** $0.062\frac{1}{2}$ ft., $0.0208\frac{1}{3}$ yd.	**15.** $\frac{5}{6}$ mi.

Page 98

1. 50 lb.	**2.** 100 rd.	**3.** 50 mi.
4. 83 yd.	**5.** 71 ft. 1 in.	**6.** 51 lb. 2 oz.
7. 71 rd. 2 ft.	**8.** 151 mi. 5 rd.	**9.** 51 yd. 2 in.
10. 1 yr. 3 mo. 13 da.	**11.** 6 wk. 4 da.	**12.** 62 bu. 1 pk.
13. 165 yd.	**14.** 206 gal.	**15.** 104 mi. 88 rd.

Page 99

1. 19 yd. **2.** 2 ft. 8 in. **3.** 9 lb. 12 oz.
4. 5 rd. 10 ft. **5.** 9 mi. 300 rd. **6.** 9 yd. 16 in.
7. $8\frac{3}{5}$ yd. **8.** 9 ft. $7\frac{3}{4}$ in. **9.** 8 lb. $14\frac{2}{3}$ oz.
10. 7 rd. $15\frac{1}{4}$ ft. **11.** 46 mi. $319\frac{1}{4}$ rd. **12.** 5 yd. $31\frac{3}{4}$ in.

Page 100

1. 179 ft. 2 in. **2.** 170 lb. 9 oz. **3.** 229 rd. 11 ft.
4. 85 qt. 1 pt. **5.** 229 yd. 9 in. **6.** 182 bu. 1 pk.
7. 154 gal. 2 qt. **8.** 179 mi. 262 rd. **9.** 334 mi. 1657 ft.
10. 200 cu. ft. **11.** 255 sq. ft. 132 sq. in. **12.** 31 ft. 11 in.
13. 20 lb. 15 oz. **14.** 5 rd. $15\frac{1}{2}$ ft. **15.** 15 mi. 229 rd.
16. 17 yd. 32 in. **17.** 16 ft. 9 in. **18.** 48 lb. 14 oz.
19. 28 rd. **20.** 19 mi. 240 rd. **21.** 4 yr. 10 mo. 29 da.
22. 4 yr. 4 mo. 18 da.

Page 101

1. 1187 ft. 6 in. **2.** 2283 lb. 12 oz. **3.** 4709 yd. 1 ft.
4. 3916 mi. 267 ft. **5.** 5134 ft. 9 in. **6.** 9560 ft. 10 in.
7. 11,155 lb. **8.** 20,723 lb. 15 oz. **9.** 27,273 yd. 2 ft.
10. 30,712 yd. 11 in.

Page 103

1. 30 lb. $2\frac{3}{8}$ oz. **2.** 157 yd. $1\frac{2}{3}$ ft. **3.** 11 ft. $2\frac{10}{11}$ in.
4. 3 yd. 2 in. **5.** 73 ft. 6 in. **6.** 20 lb. 3 oz.
7. 20. **8.** 4. **9.** $3\frac{2}{25}$.
10. 10. **11.** 3. **12.** 3.
13. 11. **14.** $15\frac{1}{20}\frac{3}{97}\frac{8}{7}$. **15.** 92 hr. 42 min.
16. 150 bu. **17.** 60,237 mi. 1940 ft. **18.** 39 T. 600 lb.
19. 393 min. 36 sec. **21.** 29,492 mi. 5110 ft. **22.** 920 ft. 10 in.
23. 1145 ft. 10 in. **24.** 35 ft. **25.** 4 ft. 8 in.
26. 8. **27.** 600.

Page 104

1. 45 mi. **2.** 20 mi., 135 mi. **3.** $16{,}666\frac{2}{3}$.
4. \$9750. **5.** \$2148.30. **6.** $2\frac{13}{16}$ T.
7. 280 ft. **8.** 55 bu. **9.** \$66.67.

Page 105

1. 7932.73 mi. **2.** 2600.97 mi. **3.** 2264.69 mi.
4. 435.19 mi. **5.** 6511.68 mi., 1637.29 mi. **6.** 1195.03 mi.
7. 1763.25 mi., 6656.19 mi., 7602.7 mi., 6381 mi.

Page 106

1. 1.58. **2.** 2.88. **3.** 3.12. **4.** 6.51. **5.** 6.56.
6. 6.03. **7.** 3.32. **8.** 1.89. **9.** 5.25. **10.** 12.10.
11. 43.38. **12.** 73.84. **13.** 139 in.

Page 107

1. 67.2. **2.** 59.2. **3.** 28.8. **4.** 75.6. **5.** 7.84.
6. 3.43. **7.** 4.23. **8.** 2.97. **9.** 100.5. **10.** 105.
11. 174.8. **12.** 164.5. **13.** 9.18. **14.** 4.94. **15.** 16.92.
16. 65.61. **17.** 110.236 in., 692.912 in.

Page 108

1. 4259.52. **2.** 4392.63. **3.** 2782.35. **4.** 2424.558.
5. 6348.861. **6.** 154.8504. **7.** 280.0027. **8.** 1815.268.
9. 6912.121. **10.** 372.6437. **11.** 715.3543. **12.** 3598.4167.
13. 1876.2345. **14.** 0.3. **15.** \$136.69. **16.** \$1829.23.
17. \$12,144.50. **18.** \$100. **19.** 230. **20.** 170.2.
21. 38.4. **22.** 921.6. **23.** 56.05. **24.** 35.04.
25. 12.06. **26.** 18.9. **27.** 1563.25. **28.** 1244.35.
29. 3238.17. **30.** 2165.44. **31.** 408.04. **32.** 1744.6.
33. 5517.54. **34.** 5431.69. **35.** 52.5525. **36.** 32.7096.
37. 227.151. **38.** 453.22. **39.** 15.76204. **40.** 53.9352.
41. 14.63775. **42.** 282.9. **43.** 53.63205. **44.** 0.170625.
45. 2.3672. **46.** 23.21. **47.** $0.069\frac{1}{8}$.

Page 109

1. \$29.75. **2.** 138.8 ft. **3.** \$1925.25.
4. \$24.15. **5.** \$9037.50. **6.** \$13,100, \$1663.70, \$245.625.
7. \$496.40. **8.** 15.836 gal. **9.** 17.444 in.
10. 42.432 ft. **11.** 3494.54 sq. ft., \$5241.81. **12.** 132.795432 ft.
13. 9128 sq. rd., 57.05 A., \$4278.75. **14.** 7.232 mi.

Page 110

1. 25.	**2.** 2500.	**3.** 100.	**4.** 10,000.
5. 17.	**6.** 2.1.	**7.** 1700.	**8.** 2.3.
9. 2.2.	**10.** 1.7.	**11.** 5000.	**12.** 3.4.
13. 50.	**14.** 7.6.	**15.** 2.46.	**16.** 7.6.

Page 111

1. 2666.9.	**2.** 246.96.	**3.** 238.59.	**4.** 266.43.
5. 19.809.	**6.** 1.3263.	**7.** 15.151.	**8.** 1.0087.
9. 85.46.	**10.** 160.6.	**11.** 5.281.	**12.** 74.06.
13. 0.159.	**14.** 0.122.	**15.** 79.572.	**16.** 27.006.
17. 57.529.	**18.** 30.09.	**19.** 10.06.	**20.** 0.35.
21. 9.296.	**22.** 830.025.	**23.** 225.024.	**24.** 7099.488.
25. 2546.2116.	**26.** 329.4308.	**27.** 317.2.	**28.** 196.54.
29. 74.	**30.** 185.	**31.** 235.68.	**32.** 7.198.

Page 112

1. 0.0345.	**2.** 0.01624.	**3.** 1.168.	**4.** 0.01031.
5. 0.0001052.	**6.** 0.00826.	**7.** 0.00131.	**8.** 0.0121.
9. 0.0311.	**10.** $7.605.	**11.** $14.6755.	**12.** $4.1567.
13. 11.853.	**14.** 270.565.	**15.** 6.213—.	**16.** 1568.889—.
17. 45.351.	**18.** 3.114.	**19.** 7810.72.	**20.** 11.326—.

21. 66.104.

Page 113

1. 0.21.	**2.** 3.	**3.** 56.	**4.** $1\frac{1}{3}$.	**5.** 3.5.	**6.** 1.
7. $\frac{6}{35}$.	**8.** $3\frac{1}{3}$.	**9.** 10.	**10.** 0.1.	**11.** 0.1.	**12.** 0.3.
13. 1.	**14.** 7.	**15.** 5.	**16.** 9.	**17.** 5.	**18.** 2.

Pages 116, 117

1. 14 A., $203. **2.** 3430 lb., 7 bales. **3.** $8.75, $2.80.
4. $3.50. **5.** 10 cu. ft., 17,280 cu. in. **6.** $3.50.
7. $240.10. **8.** 24 da., 6 da.
9. 2,675,363$\frac{1}{4}$ T. **10.** 5,992,306 bales.
11. $92,196,000, Texas; $46,098,000, Ga.; $40,976,000, Miss.; $35,854,000, Ala.
12. $5,430,780. **13.** 2,214,103 A., 1,107,051$\frac{1}{2}$ A. **14.** $267,878,240.
15. $90,240,000, $60,160,000, $30,080,000, $15,040,000.

Page 118

1. $39. **2.** $50.75. **3.** 261,740,817 lb. **4.** 103,989,126 lb.
5. 317,627,100 lb. **6.** 2,722,518,000 lb.
7. 9,627,000 lb., 7,701,600 lb., 6,931,440 lb., 6,931,440 lb.

Page 119

1. $298.90. **2.** $2.25. **3.** $13,800. **4.** $1.26.
5. Better by car lot. $50.35, straight freight; $46, car lot.

Page 122

1. 199,902,328 lb. **2.** 144,801 sq. mi. **3.** 1,625,240 sq. mi.
4. 12,224,102 T. **5.** 672,146 sq. mi. **6.** $19.36.
7. 8,750,000 bales. **8.** $68,225,214.

Page 123

1. 1,098,861. **2.** 25,230,000. **3.** 2,442,549 oz.
4. 4,939,538 sq. mi. **5.** 65,500,000 lb. **6.** 54 sq. mi., 32,003,069.
7. 43,532,445.

Page 124

1. $985.03.	**2.** $1614.02.	**3.** $1535.04.	**4.** $928.70.
5. $1663.92.	**6.** $3553.04.	**7.** $1762.10.	**8.** $2330.81.
9. $341.71.	**10.** $572.88.	**11.** $775.14.	**12.** $701.25.
13. $2447.13.	**14.** $397.38.	**15.** $18,155.34.	**16.** $18,969.21.
17. $4790.02.	**18.** $12,841.09.	**19.** 526.26.	**20.** 464.69.
21. 20,672.	**22.** 32,985.	**23.** 87,896.4.	**24.** 26,015.9.
25. $4.06.	**26.** $1.19.	**27.** $6.85.	**28.** $1.05.
29. $7.11.	**30.** $1.25.	**31.** $2\frac{4}{18}$r.	**32.** 1.
33. 5.	**34.** 3.75.	**35.** 7.	**36.** 2.

Page 126

1. $58.75.	**2.** $6.90.	**3.** $3.83.	**4.** $4.42.	**5.** $19.65.
6. $4.37.	**7.** $20.26.	**8.** $4.63.	**9.** $6.86.	

Page 127

1. 4.91.	**2.** 2.	**3.** 3.57.	**4.** 2.708.	**5.** 1.01.	**6.** 8.34.
7. 1.01.	**8.** 3.94.	**9.** 24.88.	**10.** 7 wk.	**11.** 5 yr.	

Page 128

1. $33\frac{1}{3}$, 50. 2. 10, 15, 25. 3. 1, 2. 4. 1, 6, 7.
5. 896.5. 6. 2496.9375. 7. 2153.4. 8. 10.8106.
9. 724.098. 10. 525.9375. 11. 10.5. 12. 115.41.
13. 91.4. 14. 3.16. 15. 1.46. 16. 27,500.
17. 3. 18. $2\frac{4}{5}\frac{6}{8}\frac{7}{7}$. 19. 2. 20. 42.
21. 2. 22. 60.

Page 129

1. 9%, 0.09. 2. 17%, 0.17. 3. 0.28, $\frac{28}{100}$. 4. 37%, 0.37.
18. 1, 100 per cent.

Page 131

1. 316. 2. 148. 3. 166.5. 4. $854.
5. $2458.50. 6. 108 mi. 7. 489 bu. 8. 24 ft. 2 in.
9. $11.41. 10. $9.21. 11. 9 lb. 1 oz. 12. 1 lb. 1 oz.
13. $2.70. 14. $0.98. 15. 19, 133. 16. 88, 1320.

Page 132

1. 297. 2. 474. 3. 84. 4. $6.85.
5. $1.30. 6. $13.31. 7. 498 rd. 8. 1629 bu.
9. 74 ft. 3 in. 10. 159 men. 11. 121 bu. 1 qt. 12. 94 lb. 10 oz.

Page 133

1. 0.25. 2. $0.33\frac{1}{3}$. 3. 0.01. 4. 0.025. 5. 0.075.
6. 1.25. 7. 0.005. 8. 0.0075. 9. 2.00. 10. 0.003.
11. $\frac{8}{25}$. 12. $\frac{6}{25}$. 13. $\frac{9}{25}$. 14. $\frac{17}{100}$. 15. $\frac{33}{40}$.
16. $\frac{17}{25}$. 17. $\frac{1}{16}$. 18. $1\frac{1}{4}$. 19. $3\frac{3}{4}$. 20. $\frac{1}{8}$.
21. 50%. 22. 80%. 23. $87\frac{1}{2}$%. 24. $33\frac{1}{3}$%. 25. 140%.

Page 134

1. $14\frac{2}{7}$%, $28\frac{4}{7}$%, $42\frac{6}{7}$%, $57\frac{1}{7}$%, $71\frac{3}{7}$%, $85\frac{5}{7}$%, 1.
2. $11\frac{1}{9}$%, $22\frac{2}{9}$%, $33\frac{1}{3}$%, $44\frac{4}{9}$%, $55\frac{5}{9}$%, $66\frac{2}{3}$%, $77\frac{7}{9}$%, $88\frac{8}{9}$%, 1.
3. $8\frac{1}{3}$%, $16\frac{2}{3}$%, 25%, $33\frac{1}{3}$%, $41\frac{2}{3}$%, 50%, $58\frac{1}{3}$%, $66\frac{2}{3}$%, 75%, $83\frac{1}{3}$%, $91\frac{2}{3}$%, 1.
4. $6\frac{1}{4}$%, $12\frac{1}{2}$%, $18\frac{3}{4}$%, 25%, $31\frac{1}{4}$%, $37\frac{1}{2}$%, $43\frac{3}{4}$%, 50%, $56\frac{1}{4}$%, $62\frac{1}{2}$%, $68\frac{3}{4}$%, 75%, $81\frac{1}{4}$%, $87\frac{1}{2}$%, $93\frac{3}{4}$%, 1. 5. $1\frac{1}{5}$ or 1.2.
6. $1\frac{1}{3}$ or $1.33\frac{1}{3}$. 7. $\frac{1}{4}$ or 0.25. 8. $\frac{1}{2}$ or 0.5. 9. $1\frac{4}{5}$ or 1.6.
10. $\frac{1}{10}$ or 0.1. 11. $\frac{1}{5}$ or 0.2.

Page 135

1. 1204.5. 2. $5980. 3. $149.76. 4. 4089 ft.
5. $593.75. 6. $2754. 7. $248.40. 8. $700.
9. 1848 ft. 10. 1525.2 lb. 11. 2170 mi. 12. $12.60.

Page 136

1. 123, 764.5, $22.22.
2. 243, $307.74, 1760 ft.
3. 880 ft., $205.76, 288 cu. in.
4. $14.
5. $74.70.
6. $64.
7. $595.
8. $466.50 gain.

Page 137

1. 50.75. 2. $216. 3. 187 da. 4. $980.
5. 2250 lb. 6. 191.1 lb. 7. $1500. 8. $85,808,000.

Page 138

1. 5%, $3\frac{1}{3}$%.
2. 3%, 20%.
3. 4%, 5%.
4. 6%, $16\frac{2}{3}$%.
5. $33\frac{1}{3}$%, $6\frac{2}{3}$%.
6. 25%, 1%.
7. $266\frac{2}{3}$%, 150%.
8. $33\frac{1}{3}$%, 20%, 10%.
9. 11%.
10. 12%.
11. $275, 11%.

Page 139

1. 1180. 2. 40. 3. $190. 4. 220.

Page 140

1. 20. 2. 200. 3. 100. 4. 220.
5. 1000. 6. 2200. 7. 172. 8. 250.
9. 33. 10. 7000. 11. $35. 12. $65.

Page 141

1. $0.48. 2. $210. 3. $27. 4. $9, $8.
5. $0.204, $0.17.

Page 142

1. 1,120,000 dark, 2,080,000 light.
2. 483,000 dark, 1,817,000 light.
3. 735,000 dark, 765,000 light.
4. 3.9%.
5. 36.8%.
6. 1.3%.
7. 18.6%.

Pages 143, 144

1. \$310,500,000, \$103,500,000. 2. 360,000, 9,054,612.
3. 10,003. 4. 209,700, 1,922,700. 5. \$259,776,000. 6. \$5150.
7. \$5370. 8. \$6⅔. 9. \$6.60. 10. \$1.58. 11. \$1.20, \$60.
12. \$10,734,319⅓, \$18,240,660; \$26,181,266⅔, \$64,855,680; \$2,356,314, \$12,160,440; \$6,080,220.

Pages 145, 146

1. 111,367,809 lb. 2. 65%. 3. 42%.
4. 37%. 5. 36,808. 6. 248 bbl., \$12,152.
7. 79 bbl., \$2765. 8. 6700 lb., \$18,090. 9. 11,180 men, 250%.
10. \$315,900. 11. \$549,900. 12. \$364,000.
13. 21,000 sq. ft. 14. 2333⅓ sq. ft., ⅓. 15. 187 lb. 8 oz., \$3.75.

Pages 147, 148

1. \$11.20. 2. \$1.05. 3. \$2.20. 4. \$5.60.
5. \$64.50. 6. \$56.20. 7. \$42.55. 8. \$209.95.
9. \$329. 10. \$14.42. 11. \$10.66. 12. \$396.
13. \$39.09. 14. \$220. 15. \$546.25. 16. \$448.14.
17. \$568.75. 18. \$374. 19. \$275.20. 20. \$10.20.
21. \$5333.72. 22. \$44.03⅓. 23. \$13.16. 24. \$3303.30.
25. \$9.62. 26. \$14.88. 27. \$1.53. 28. \$0.51.
29. \$23.25. 30. \$1.08, \$0.96, \$0.90. 31. \$0.81, \$0.72.
32. \$125. 33. \$650.25.

Page 149

1. 12,096, 1392. 2. \$27. 3. \$0.60. 4. 132.

Page 150

1. 408, 8. 2. 36,240. 3. \$1.25.
4. \$1.19, \$0.81, \$1.50. 5. \$0.32.

Page 152

1. 827 ft. 2. 117 ft. 8 in. 3. 129.5 ft. 4. 2 right angles.

ANSWERS

Page 153

1. 37′ 9″. **2.** 42⅓ yd. **3.** 29′ 5″. **4.** 27′ 7″.
5. 15 yd. 1½ ft. **6.** 29′ 8″. **7.** 10′ 6⅞″. **8.** 2′ 10″.
9. 56 mi. 250 rd. **10.** 33½′. **11.** 17.6′, 35.2′, 35.2′. **12.** 9′, 8′, 7′.

Page 155

1. 331.8 sq. in. **2.** 242.55 sq. ft. **3.** 4662.08 sq. ft.
4. 8.75 sq. in. **5.** 28½ sq. ft. **6.** 136.125 sq. ft.
7. 425.88 sq. in. **8.** 118.925 sq. in. **9.** 195 sq. ft.
10. 142.725 sq. ft.

Page 156

1. 1292 cu. in. **2.** 1830 cu. ft. **3.** $83.20.
4. 168 sq. ft., 468 sq. ft., 168 sq. ft., 1512 cu. ft.

Pages 157, 158

1. 134⅖ bu. **2.** 108 bu. **3.** 7⅕ bu. **4.** 900 gal.
5. 6 gal. **6.** 382$\frac{3}{16}$ gal.; using 231 cu. in. = 1 gal., 381$\frac{9}{11}$ gal.
7. 43$\frac{3}{29}$ bbl. **8.** 59$\frac{8}{29}$ bbl. **9.** 2$\frac{1}{58}$ bbl. **10.** 65$\frac{3}{5}$ T.
11. 24$\frac{2}{7}$ T. **12.** 5.544 T. = 5$\frac{68}{125}$ T. **13.** 14.88 T. **14.** 7$\frac{7}{64}$ cords.
15. 8$\frac{15}{16}$ cords. **16.** 30. **17.** 11,000 cu. ft.
18. 28½ gal. **19.** 7$\frac{367}{1232}$ gal. **20.** $1.80.
21. 85$\frac{55}{99}$ perches. **22.** 8662½, or, say, 8700 bricks. **23.** $12.50, $11.67.

Pages 164-167

1. $5.81. **2.** 111. **3.** 17 cars. **4.** 19 cars.
5. $4166.67. **6.** 354 bu. **7.** 625 bu. **8.** 125 chests.
9. 8064 bricks. **10.** 4 mi. **11.** 54,912 ties. **12.** 6 min. 25 sec.
13. 7. **14.** 337½ T. **15.** 13⅓ min. **16.** $3,000,000.
17. $15,625,000, $12,625,000. **18.** 14.67 ft., 117.33 ft., 10 mi., 80 mi.
19. $408. **20.** $1.52. **21.** $1.26. **22.** 56 yd.
23. 90 pieces. **24.** 29 yd. **25.** $4.75. **26.** $2665.
27. $125, $175. **28.** $5760. **29.** $270. **30.** $164.25.
31. $0.48. **32.** $9. **33.** $3.76. **34.** $202.50 gain.

Page 168

1. $518.42. **2.** $3250. **3.** $2100.
4. 16 bbl., 170 bbl., 1860 bbl. **5.** $7.77, $4.41, $5.18, $9.03, $18.06.
6. $0.56, $0.91, $1.16, $1.59. **7.** 12,300 lb., 11,300 lb., 77,000 lb.

Pages 169, 170

1. 64 da. **2.** 8 da., 32 da., $10\frac{2}{3}$ da., $6\frac{2}{5}$ da. **3.** 40 da. **4.** $7\frac{1}{5}$ da.
5. 189 bu., 189 bu. **6.** 15 da. **7.** 3159 bricks. **8.** $960.
9. 15 da., 3 men, 5 men, 2 men. **10.** 75 men. **11.** 10 da.
12. 30 A. **13.** $14\frac{2}{5}$ lb. **14.** 22 A. **15.** 394.2 A. **16.** 177 A.

Page 171

1. $66. **2.** $5400. **3.** $504, $36, $198, $720.
4. $30, $75, $112.50, $8.25. **5.** 1470 lb., 3430 lb., 4410 lb., 17,150 lb.
6. $105, $300, $4200. **7.** 294 mi., $367\frac{1}{2}$ mi., $477\frac{3}{4}$ mi.
8. 195 lb., 455 lb., 585 lb.

Pages 172, 173

1. $47.50. **2.** $11,500. **3.** 7840 ft., 10,640 ft., 117,600 ft.
4. 1948 lb., 3409 lb., 7792 lb., 12,175 lb. **5.** 54 ft. **6.** 15,000 bricks.
7. 24 ft. **8.** 55 ft. **9.** $4\frac{1}{11}$ hr. **10.** $9.60, $9, $9.60.
11. 8 men. **12.** $9\frac{3}{5}$ T. **13.** 900, 800. **14.** 1,950,000,000.

Page 174

1. $362,700. **2.** $2\frac{161}{200}$ mi. **3.** $3\frac{2}{11}$ mi. **4.** 376,320. **5.** $13.20.

Page 175

1. 124.8 lb. **2.** $1704. **3.** $6.30. **4.** 213.6 mi. **5.** 122. **6.** 284 A.

Page 176

1. 548. **2.** 222.5. **3.** $169\frac{1}{4}$. **4.** 276.25. **5.** 41.83.
6. 634.11. **7.** 129. **8.** 383.7. **9.** 23.5. **10.** 16.

Page 178

1. 64. **2.** 7. **3.** 30. **4.** 10. **5.** 30. **6.** 20.

Page 179

1. $350. **2.** 625. **3.** 30. **4.** 119. **5.** $337. **6.** 13 yr.

Page 180

1. $2812.50. **2.** $90. **3.** $6250. **4.** $76.50. **5.** 3420. **6.** $62.

Page 181

1. \$61.50. **2.** \$2.10. **3.** \$925. **4.** \$48.20. **5.** \$12.10.
6. \$69.55. **7.** \$5025. **8.** \$3971. **9.** \$3267. **10.** \$760.80.
11. \$9205. **12.** \$381.88.

Page 182

1. \$1071.40. **2.** \$22.68. **3.** \$30,776.80. **4.** \$35,850. **5.** \$15.
6. \$4936.25. **7.** \$591.10. **8.** \$21.04. **9.** \$780. **10.** 535.
11. 114 bbl.

Page 183

1. $3\frac{1}{3}\%$. **2.** 20%. **3.** 25%. **4.** 60%. **5.** $13\frac{1}{3}\%$.
6. $33\frac{1}{3}\%$. **7.** 5%. **8.** $16\frac{2}{3}\%$. **9.** $11\frac{1}{9}\%$. **10.** $12\frac{1}{2}\%$.
11. 58%. **12.** 48%, 52%. **13.** 75%. **14.** $66\frac{2}{3}\%$, 50%, $58\frac{1}{3}\%$. **15.** 2, 4.

Page 184

1. \$7. **2.** \$6.50. **3.** 28 ft. 4 in. **4.** $1\frac{1}{4}$ yd.
5. 181 lb. 4 oz. **6.** 250 ft. **7.** \$2110. **8.** 145.
9. Yes. **10.** 120%, \$50. **11.** 140 girls, 259 pupils. **12.** \$451.

Page 185

1. \$1250. **2.** 150 A. **3.** 55 lb. **4.** 61,000. **5.** 7134.

Page 186

1. 12. **2.** 75. **3.** \$900. **4.** 390. **5.** 52 ft. **6.** 650 ft., 65 ft.

Pages 187-189

1. 132. **2.** 371. **3.** \$125.
4. 19.18 mi. **5.** 54. **6.** \$450, \$900.
7. 84. **8.** 76. **9.** $0.8\frac{3}{12}\%$.
10. \$1512. **11.** \$280.80. **12.** $78\frac{2}{3}\%$.
13. $60\frac{5}{17}\%$. **14.** $23\frac{16}{51}\%$, $26\frac{38}{543}\%$. **15.** 50,309,000 sq. mi.
16. 90%, 71 lb. **17.** $2755\frac{10}{27}\%$. **18.** $19\frac{35}{800}\%$, $12\frac{29}{401}\%$.
19. $125\frac{52}{803}\%$, $27\frac{2641}{4017}\%$. **20.** 5%. **21.** 4.21 lb.
22. \$3915. **23.** 35.3675 lb. **24.** 32%.
25. $9\frac{1}{2}\%$. **26.** 20 cu. ft., 25 cu. ft. **27.** \$1515.63.
28. 40%. **29.** \$100, \$1600, \$500, \$300, \$320.

Page 190

1. $51\frac{409}{741}\%$. 2. $57\frac{193}{501}\%$. 3. $317\frac{1}{7}\%$. 4. 15%. 5. 0.75%.

Page 191

1. \$596.90. 2. \$297.50. 3. \$233.75. 4. \$276.25.
5. \$19.55. 6. \$36.40. 7. \$360.24. 8. \$140.40.
9. \$180.60. 10. \$878.13. 11. \$1466.40. 12. \$186.
13. \$22.22. 14. \$2654.31.

Page 192

1. \$112. 2. \$432.25. 3. \$318.75. 4. \$431.20.
5. \$526.50. 6. \$252. 7. \$229.50. 8. \$222.75.
9. \$140. 10. \$450. 11. \$50. 12. No difference.

Page 193

1. \$79.63. 2. \$15.31. 3. \$147.05. 4. \$2.43.

Page 195

1. 20%. 2. $22\frac{22}{49}\%$. 3. $2\frac{2}{49}\%$. 4. $11\frac{1}{9}\%$ gain.

Page 196

1. $15\frac{1}{5}\%$. 2. \$0.10. 3. \$0.20. 4. 25%. 5. 20%. 6. \$7.50 gain.

Page 197

1. $33\frac{1}{3}\%$. 2. \$0.24. 3. $29\frac{3}{5}\%$. 4. \$72, 25%. 5. $6\frac{1}{4}\%$.
6. \$2.52 gain.

Page 198

1. 50% above. 2. $66\frac{2}{3}\%$ above. 3. $85\frac{5}{7}\%$ above. 4. $78\frac{4}{7}\%$ above.
5. 100% above. 6. 120% above. 7. $16\frac{2}{3}\%$ above. 8. $29\frac{1}{31}\%$ above.
9. 30%, 80%, above. 10. 50% above; at cost.

Page 199

1. \$21.25. 2. \$66. 3. \$32.10. 4. \$29.20, \$262.80. 5. \$196.

Page 200

1. \$11.44. 2. \$17.38. 3. \$784. 4. \$2.10, \$2.95. 5. \$253.20.

Page 201

1. \$473.60. 2. \$21.48. 3. \$168. 4. $5\frac{1}{8}\%$.
5. \$4.88. 6. \$66.70. 7. \$787.50. 8. \$25.53.

Page 202

1. \$22.50, \$15. 2. \$47.50, \$28.50. 3. \$9.40, \$11.75.
4. \$0.81. 5. \$503.50.

Page 203

1. \$26.08. 2. \$18.75. 3. \$0.95. 4. \$9.75. 5. \$13.
6. \$1.89. 7. \$11. 8. \$18.01. 9. \$1.88. 10. \$43.50.
11. \$13.76. 12. \$3.78. 13. \$26.75. 14. \$31.26. 15. \$2.40.
16. \$2.05. 17. \$3.50. 18. \$4.38. 19. \$2.25. 20. \$9.38.
21. \$11.25. 22. \$0.90. 23. \$13.50. 24. \$10.13. 25. \$0.35.
26. \$4.88. 27. \$14.40.

Page 204

2. \$4.12. 3. \$47.96. 4. \$212.75.

Page 205

1. \$64.75. 2. \$132.50. 3. \$68.25. 4. \$145.60, \$445.20, \$393.75.
5. \$3.45. 6. \$257.50, \$307.50, \$153, \$386.25. 7. \$1753.10.

Page 206

1. 2880 cu. ft., $5420\frac{5}{8}$ cu. ft. 2. 288 sq. ft., $551\frac{1}{4}$ sq. ft.
3. 680 sq. ft., $924\frac{1}{3}$ sq. ft. 4. $5\frac{5}{32}$ cd., $18\frac{3}{64}$ cd.
5. $1781\frac{9}{11}$ gal., $2025\frac{2}{77}$ gal.

Pages 207, 208

1. 3,537,440.9 cu. in., $5,899,318.86\frac{2}{3}$ cu. in. 2. 1,015,000 cu. ft.
3. \$18,000. 4. $21\frac{1}{8}\%$, $17\frac{5}{7}\%$. 5. 45 lb. 6. $50\frac{35}{177}\%$, $53\frac{141}{177}\%$.
7. 2%, $2\frac{2}{45}\%$. 8. 25%, 20%. 9. $525\frac{5}{7}$ gal. 10. $1\frac{2}{7}$ ft.
11. 72 ft. 12. Any two factors of 1155 give the number of inches.
13. \$13.96. 14. $1520\frac{7}{8}$ cu. in., $132\frac{1}{4}$ sq. in.

Pages 209, 210

1. 1234′ 9″. 2. 919′ 5″. 3. 1043′ 6″.
4. 759 yd. 9 in. 5. 617 gal. 2 qt. 6. 1014 bu. 1 pk.
7. 282′ 4″. 8. 10 lb. 1 oz. 9. 123 lb. 9 oz.

10. 4′ 4″. **11.** 74 yd. 29 in. **12.** 9 rd. $3\frac{1}{8}$ yd.
13. 275 bu. 2 pk. **14.** 8 bu. 1 pk. **15.** 48 gal. 2 qt.
16. 6 mi. $303\frac{1}{3}$ rd. **17.** 4286 lb. 10 oz. **18.** 1704 lb. 6 oz.
19. 7972 lb. **20.** 1257 lb. **21.** 18,237′ 6″.
22. 31,640′. **23.** 4456 yd. 2 ft. **24.** 25,146 yd. 2 ft.
25. 9900 gal. **26.** 4552 bu. **27.** 21 lb. 14 oz.
28. 31 lb. $11\frac{1}{8}$ oz. **29.** 162. **30.** 109.
31. 151 lb. 4 oz. **32.** 107 lb. $1\frac{1}{2}$ oz. **33.** 25′ 2″.
34. 17′ $\frac{1}{5}$″. **35.** 11 lb. $\frac{3}{4}$ oz. **36.** 25 lb. $\frac{1}{5}$ oz.
37. 476. **38.** $1061\frac{1}{8}$. **39.** 370.
40. 740. **41.** 2 yr. 25 da. **42.** 1 yr. 9 mo. 27 da.
43. 4 yr. 10 mo. 8 da. **44.** 143 yr. 5 mo. 27 da. **45.** 1 yr. 6 mo. 28 da.
46. 2 yr. 2 mo. 25 da. **47.** 1 yr. 6 mo. 28 da. **48.** 1 yr. 5 mo. 29 da.
49. 6 min. 43 sec. **50.** Monday. **51.** 38 hr.
52. 11 hr. 51 min. 13 sec. A.M. **53.** $2\frac{5}{11}$.
54. $63\frac{7}{11}$ min. **55.** 90 da., 230 da., 100 da.
56. $99\frac{2}{3}$ sq. yd.

Page 211

1. $26.16, $13.08, $8.72, $17.44, $21.80.
2. $18, $9, $15, $1.50, $13.50.
3. $23.46, $46.92, $70.38.
4. $3.04, $15.20, $30.40.
5. $21.60, $1.80, $0.90, $0.30, $0.06, $0.42.
6. $7.02, $3.51, $2.34, $1.76, $0.59, $0.20.

Page 212

1. $7.50. **2.** $11. **3.** $1127.50. **4.** $87.50. **5.** $3.15.
6. $4.10. **7.** $87.50. **8.** $11.33. **9.** $47.25. **10.** $286.88.

Page 213

1. $27.31. **2.** $24.75. **3.** $42.33. **4.** $9.79. **5.** $22.78.
6. $100.94. **7.** $99.45. **8.** $244.38. **9.** $42.50. **10.** $5.69.
11. $19.53. **12.** $109.50. **13.** $53.70. **14.** $357.58.

Page 214

1. $76.88. **2.** $26.50. **3.** $260.42. **4.** $424.

Page 215

1. \$0.38, \$74.62. **2.** \$0.45, \$89.55. **3.** \$3.13, \$246.87.
4. \$3.44, \$271.56. **5.** \$7.08, \$417.92. **6.** \$4.50, \$295.50.
7. \$11.25, \$363.75. **8.** \$6, \$394. **9.** \$1.13, \$148.87.
10. \$1.83, \$398.17. **11.** \$2, \$198. **12.** \$1.31, \$173.69.
13. \$5.50, \$294.50. **14.** \$2.92, \$497.08. **15.** \$26.03, \$2948.97.
16. \$65.88, \$4184.12. **17.** \$2.08, \$1247.92. **18.** \$6.25, \$2493.75.
19. \$5, \$1495. **20.** \$7.81, \$1242.19. **21.** \$12.60, \$2737.40.
22. \$39.60, \$3480.40.

Page 216

1. \$371. **2.** \$386.25. **3.** \$825. **4.** \$78.75. **5.** \$293.75.
6. \$133.75. **7.** \$282.08. **8.** \$369.69. **9.** \$311.25.

Page 218

1. 20. **2.** 9. **3.** 8. **4.** 15. **5.** 10. **6.** 5. **7.** 3. **8.** 4.
9. 4. **10.** 6. **11.** 50. **12.** 10. **13.** 30. **14.** 25. **15.** 2 ft. **16.** 4 ft.

Page 219

1. \$80, \$20; \$70, \$30; \$10, \$90. **2.** 33 : 40. **3.** 18, 12.
4. 1,008,000 sq. mi.

Pages 220-222

1. $\frac{9}{10}$. **2.** \$650. **3.** \$550. **4.** \$300.
5. 16 lb., 16 lb., 8 lb. **6.** 8 lb., 4 lb., 9 lb., 3 lb., 8 lb.
7. 850%, 770%, 1200%, 840%. **8.** $6\frac{1}{4}$ gal.
9. $357\frac{1}{7}$ lb., $257\frac{1}{7}$ lb. **10.** 9 lb., 3.3 lb., 2.7 lb.
11. $12\frac{1}{2}\%$, $12\frac{1}{2}\%$, 75%. **12.** $11\frac{1}{9}\%$. **13.** \$13.33.
14. \$13.25, first. **15.** \$20.25. **16.** 4760 bu.
17. 1 : 6. **18.** \$626.25. **19.** 50%.
20. \$0.02 per lb., ducks; \$0.04$\frac{2}{3}$ per lb., chickens.

Page 223

1. 36. **2.** 18. **3.** 82. **4.** 175. **5.** 4. **6.** 56.
7. 160. **8.** 455. **9.** 114. **10.** 13. **11.** 5. **12.** 15 boys.

Page 225

1. 17,000,000. **2.** 27 ft. **3.** 1 : 160 = 50,000 : 8,000,000.
4. 3 in. **5.** $12\frac{1}{2}$ in. **6.** 27 ft.

Page 226

1. 1350 lb. **2.** $12\frac{1}{2}$ lb. **3.** 3.675 lb. **4.** 2.28 lb.
5. 0.18 lb. **6.** 1034.8 lb. **7.** $167\frac{2}{5}$ hhd. **8.** 300 lb.
9. 2180. **10.** 5.655 gal. **11.** 10.85 ft.

Page 227

1. 19%. **2.** $87\frac{1}{2}$%. **3.** 93%. **4.** 110.4 oz.
5. 90%. **6.** 4%. **7.** 875 T.

Page 228

1. 30 T., 16 T. **2.** $33\frac{1}{3}$ bu. **3.** 11. **4.** 80%.
5. $6\frac{2}{3}$%, $6\frac{1}{4}$%. **6.** $16\frac{2}{3}$%, $24\frac{4}{5}$%, 75%. **7.** $33\frac{1}{3}$%, 25%.
8. 70 : 56 = 60 : 48. **9.** 33 : 26, 26 : 33, 1, 11 : 12.

Page 229

1. $\frac{125}{216}$ oz. **2.** 125 lb. **3.** 15 lb. **4.** 1206.25 lb.
5. $5250. **6.** 70 lb. **7.** 15 lb. **8.** 4975 lb.

Page 230

1. 261.45 lb. **2.** 8.549 lb., 2.13725 lb., 1.068625 lb.
3. 6.557 lb., 206.5455 lb. **4.** 5.81 lb., 183.015 lb.
5. 559.375 lb., 74.285 lb. **6.** 8.217 lb., 258.8355 lb.
7. 864 gal. **8.** 10.292 lb.

Page 231

1. 34°. **2.** $2\frac{1}{2}$°. **3.** $45\frac{1}{2}$°, 35°, 12°.

Page 232

1. 1,050,000 sq. mi. **2.** 94.42%. **3.** $410.
4. $2000. **5.** $19,828,200. **6.** 2500 trees.

Page 233

1. 29 ft. **2.** 43 ft. 2 in. **3.** 55 ft., 51 ft. **4.** 66 ft. **5.** 40 ft.

Page 234

1. $13\frac{1}{2}$ bd. ft. **2.** $5\frac{5}{6}$ bd. ft. **3.** $9\frac{1}{3}$ bd. ft. **4.** $16\frac{1}{2}$ bd. ft.
5. $19\frac{1}{6}$ bd. ft. **6.** $18\frac{5}{12}$ bd. ft. **7.** $9\frac{3}{4}$ bd. ft. **8.** 12 bd. ft.

9. $11\frac{2}{3}$ bd. ft. **10.** $32\frac{2}{3}$ bd. ft. **11.** 21 bd. ft. **12.** $14\frac{2}{3}$ bd. ft.
13. 1 bd. ft. **14.** $2\frac{2}{3}$ bd. ft. **15.** 4 bd. ft. **16.** $5\frac{1}{3}$ bd. ft.
17. $8\frac{1}{3}$ bd. ft. **18.** 10 bd. ft. **19.** 6 bd. ft. **20.** 28 bd. ft.
21. 15 bd. ft. **22.** 9 bd. ft. **23.** 30 bd. ft. **24.** 15 bd. ft.

Page 235

1. $8.64. **2.** $17.14. **3.** $12.96.
4. $71.68. **5.** 2700 bd. ft. **6.** $46.51+.

Pages 236, 237

1. 900 bd. ft. **2.** $15.19. **3.** $283\frac{1}{2}$ bd. ft. **4.** 3375.
5. 188 bd. ft., $5.64. **6.** 735 bd. ft. **7.** 2552 bd. ft. **8.** 4560 bd. ft.
9. $149.09. **10.** $184.58. **11.** $973.10. **12.** $1381.77.

Page 238

1. $93\frac{5}{9}$ sq. yd. **2.** $852\frac{5}{8}$ sq. yd. **3.** $41.81. **4.** $667.80.

Page 239

1. 20 yd. **2.** $32\frac{5}{6}$ yd. **3.** $55\frac{1}{12}$ yd. **4.** $46\frac{1}{6}$ yd.
5. $24\frac{1}{3}$ yd. **6.** $48.93. **7.** $56.93. **8.** $46.53.
9. $39.67. **10.** $29.47. **11.** $19.07. Answers 6–11 to nearest cent.

Page 240

1. $7.20. **2.** 21 rolls, $7.35. **3.** $4.50.

Page 241

1. 3200 sq. ft., 4800 sq. ft. **2.** 32. **3.** 30. **4.** 100.
5. 144 sq. ft., 108 sq. ft., 108 sq. ft., 216 sq. ft., 216 sq. ft.
6. 9800 sq. ft., 8865 sq. ft. without house.

Page 243

1. 1521 lb., 46.4 lb. **2.** 91.4 lb., 91.4%. **3.** $6\frac{2}{3}\%$.
4. $20,439,000. **5.** $80,000,000. **6.** $351,204,000.
7. $23,618\frac{2}{3}$ loads. **8.** $36\frac{2}{3}\%$. **9.** 12,806,000 T.

Page 244

1. 12%. **2.** $1.357. **3.** 842,600,000 lb.
4. $34,090\frac{10}{11}$ mi. **5.** $15.84. **6.** 363 T., 265 T., $36\frac{52}{53}\%$.
7. 15,186,400 T.

Page 245

1. \$26.25. **2.** \$400. **3.** \$0.066 per lb. **4.** $53\frac{1}{2}$ ct., or $53\frac{12}{257}$ ct.
5. $109\frac{5}{7}$. **6.** \$0.60. **7.** \$36, $66\frac{2}{3}\%$.

Page 246

1. 51,171,360 gr. **2.** \$15,072,000. **3.** 100%.
4. \$2,231,075. **5.** \$5,550,000, 555. **6.** \$3,333,333$\frac{1}{3}$, \$0.03$\frac{1}{3}$.
7. $85\frac{1}{2}\%$.

Pages 247, 248

1. \$1,650,000. **2.** \$63,000, 21%. **3.** \$1950.
4. \$2.50. **5.** \$0.15, $6\frac{18}{17}\%$. **6.** Rate is greater on second.
7. 1.8%. **8.** 4%. **9.** \$5580.
10. \$7155. **11.** \$1,027,000. Yes. **12.** \$2107.50, \$0.75, \$210.75.
13. \$15,000. **14.** \$13.44, \$13.09, \$8.68, \$11.20.

Page 249

1. 20,880 T. **2.** $5\frac{16}{21}\%$. **3.** 1340. **4.** 20,000 ft. **5.** $118\frac{3}{4}\%$.

Pages 250, 251

1. \$66.406. **2.** \$67.50. **3.** \$12.50 loss. **4.** \$138 gain. **5.** \$524.
6. x=\$50.04, \$56.77, \$18.14; y=\$142.03; z=\$25.54, \$57.77, \$58.72.
7. \$129.92, \$28.57, \$75.56, \$25.79.
8. \$105.60, \$21.72, \$47.21, \$27.07, \$7.20, \$2.40.

Page 252

1. 36°. **2.** 29.7°, 39.6°. **3.** −11°.
4. 114.519°, 109.74°. **5.** 9 mi., 7 mi. **6.** 70,848 lb.
7. 50 in., 3 in., 30 in.

Page 253

1. 1700%. **2.** \$85,500,000. **3.** \$20,000,000.
4. 6¢ gain is $31\frac{11}{19}\%$ of 19¢, $26\frac{54}{71}\%$. **5.** 2,855,395,000. **6.** 1 : 130.

Page 255

1. $1\frac{66}{161}$.	**2.** $1\frac{83}{104}$.	**3.** $1\frac{82}{231}$.	**4.** $1\frac{121}{340}$.	**5.** $1\frac{43}{83}$.
6. $\frac{127}{460}$.	**7.** $\frac{19}{77}$.	**8.** $\frac{47}{105}$.	**9.** $\frac{223}{420}$.	**10.** $2.01.
11. $1.40.	**12.** $2.25.	**13.** $35.10.	**14.** $35.65.	**15.** $3.54.
16. $1.60.	**17.** $0.12\frac{1}{2}$.	**18.** $0.32.	**19.** 42.6 mi.	**20.** $18.
	21. $0.03\frac{3}{8}$.	**22.** $137.50.	**23.** $0.02\frac{1}{2}$.	

Page 257

1. 1.	**2.** $1\frac{2}{25}$.	**3.** $\frac{6}{7}$.	**4.** $1\frac{2}{7}$.	**5.** $1\frac{7}{8}$.
6. $\frac{11}{36}$.	**7.** 1.	**8.** $\frac{13}{35}$.	**9.** $\frac{5}{7}$.	**10.** $\frac{4}{5}$.
11. $1\frac{1}{2}$.	**12.** 6.	**13.** $1\frac{21}{44}$.	**14.** $1\frac{29}{35}$.	**15.** $21\frac{13}{72}$.
16. 504 bbl.	**17.** 20%.	**18.** $40.	**19.** 171 da.	**20.** $115.20
	21. $22.50.		**22.** $660.	
	23. $1450, $1413.75.		**24.** $33\frac{1}{3}$%, 25%, 20%.	

Page 259

1. $10\frac{13}{24}$.	**2.** $15\frac{17}{63}$.	**3.** $10\frac{17}{20}$.	**4.** $36\frac{9}{32}$.	**5.** $17\frac{14}{27}$.
6. $12\frac{41}{128}$.	**7.** 7.	**8.** $5\frac{9}{19}$.	**9.** 3.	**10.** 3.
11. 4.	**12.** 6.	**13.** $\frac{2}{3}$%.	**14.** $\frac{3}{5}$%.	**15.** 1%.
16. $0.16\frac{3}{4}$.	**17.** 1.4%, $0.7035.		**18.** $85\frac{195}{313}$%, $14\frac{118}{313}$%.	
19. $1.968.	**20.** $1100.	**21.** Loss, $49\frac{47}{72}$ sq. yd., $43\frac{1}{3}$%.		

Page 261

1. 0.09.	**2.** $0.16\frac{2}{3}$.	**3.** $0.33\frac{1}{3}$.	**4.** $0.30\frac{10}{23}$.	**5.** 0.5.
6. $0.42\frac{6}{7}$.	**7.** $20.	**8.** $32.50.	**9.** $195\frac{38}{43}$ bu.	**10.** $2\frac{2}{3}$.
11. $570.	**12.** $7.90.	**13.** 50%.	**14.** $1200.	**15.** 80 T.
	16. $113.67\frac{3}{16}$.		**17.** $880.	

Pages 263, 264

1. $33\frac{1}{3}\%$. **2.** 5%. **3.** \$420. **4.** \$250. **5.** \$364. **6.** \$16.50. **7.** \$11.75. **8.** \$850. **9.** \$250. **10.** \$475. **11.** \$259. **12.** \$11.55. **13.** \$105. **14.** $22\frac{1}{2}$ A. **15.** \$19. **16.** \$4.67. **17.** $337\frac{1}{2}$ mi. **18.** \$3200. **19.** $87\frac{1}{2}$ ft. **20.** 120 ft. **21.** \$55.20.

Page 266

1. 1669, 1344. **2.** XLIX, LXXIX, XCIV, XCVI, XCIX, CXLVI.
4. 21,000,475. **5.** 1,001,001,001.0001.
6. 1,000,958,326.0001, 1,001,000,988.7551, 1,000,705,999.8731.

Page 268

3. 111, 222, 333, to 999.
4. 111,111,111, 222,222,222, to 999,999,999
5. 142,857, 714,285, 571,428, to 428,571.
6. 111,111, 222,222, to 999,999.
7. 81, 9801, 998,001, 99,980,001, 9,999,800,001.
8. 121, 12,321, 1,234,321, 12,345,654,321.
9. 1001, 11,011, 135,135, 234,234, 345,345, 789,789.
10. 1, 11, 111, 1111, 11,111. 8, 88, 888, 8888, 88,888.

Page 269

1. $2\frac{3}{4}\frac{3}{2}\frac{3}{7}$, 233 ft., 0.233.
2. 450 in., 192 qt., 7360 rd., 144 ft., 1200 oz., 12,000 lb., 11,664 sq. in.
3. $3\frac{1}{1\frac{1}{2}}$ ft., 14 yd., $2\frac{1}{4}$ lb., 3 mi., 2 T., 25 gal., 144 ft.
4. 1782 in., 252 in., 380,160 in., 48 in., 108 in., 84 in., 42 in., 270 in., 1287 in., 158,400 in.

Page 271

1. 288 doz., 24 gross, 2 great gross.
2. 204 in., 258 in., 108 in., 792 in.
3. 3 ft., 12 ft., 144 ft., 162 ft., 9.9 ft.
4. 5000 lb., 6500 lb., 9 lb., 7520 lb., 427 lb., 99 lb.
5. 212 qt., 832 qt., 71 qt., 49 qt., 1008 qt., 126 qt., 303 qt., 560 qt.
6. 900, 1080, 184, 10,368, 189, 2, 3960.

Pages 273-275

1. 3586. **3.** 32 mi. 40 yd. **5.** $1\frac{7}{12}$.
2. 358 ft. 6 in. **4.** 359 bu. 2 pk. **6.** $2\frac{1}{8}$, $2.125 = 2\frac{1}{8}$.
7. $\frac{1}{6}\frac{2}{2} = \frac{2}{1}\frac{4}{2} = \frac{3}{1}\frac{6}{8} = 2$. **8.** $3_{\frac{1}{1}\frac{1}{2}\frac{1}{0}}$. Common fractions. **9.** 3.175. Decimals.
10. $31x + 23y$, 3123, 32 ft. 11 in., 32 lb. 7 oz.
11. \$250.56. **15.** \$1244.53. **19.** \$2421.38. **23.** \$8858.80.
12. \$317.66. **16.** \$1245.30. **20.** \$3703.20. **24.** \$14,585.48.
13. \$338.45. **17.** \$1831. **21.** \$2601. **25.** \$23,135.89.
14. \$443.10. **18.** \$793.50. **22.** \$3314.63. **26.** \$18,237.87.
27. 80,467, \$2,769,197,978, 1,415,539, \$565,103,062, \$2,646,580,941, \$3,968,616,096.
28. Columns: 407,600,000, \$4,469,100,000, \$1,593,400,000, \$881,800,000.
Rows: \$2,272,400,000, \$1,457,800,000, \$761,000,000, \$1,123,700,000, \$817,800,000, \$376,900,000, \$134,700,000.

Pages 277, 278

1. \$154.55. **15.** 88 T. 1987 lb. **29.** 8 yr. 5 mo. 18 da.
2. \$83.56. **16.** \$3078. **30.** 10 yr. 9 mo. 19 da.
3. \$200.43. **17.** 30 mi. 5258 ft. **33.** $1438\frac{5}{7}$.
4. \$297.04. **18.** 30 T. 1978 lb. **34.** $6924\frac{1}{3}\frac{2}{5}$.
5. \$103.59. **19.** \$1,092,855,499. **35.** $2303\frac{1}{2}\frac{1}{1}$.
6. \$331.85. **20.** \$10,219,191,055. **36.** $1244\frac{7}{1}\frac{7}{5}$.
7. \$681.35. **21.** \$1,142,901,501.75. **37.** $4304\frac{9}{1}\frac{9}{0}$.
8. \$228.95. **22.** 219. **38.** $6597\frac{2}{2}\frac{3}{4}$.
9. \$308.25. **23.** 2.19. **39.** 12 ft. $7\frac{1}{6}$ in.
10. \$1806.55. **24.** 21 ft. 11 in. **40.** 102 lb. $10\frac{5}{8}$ oz.
11. \$1664.50. **25.** 21 lb. 15 oz. **41.** 228 gal. $2\frac{1}{2}\frac{1}{4}$ qt.
12. \$3987.20. **26.** 2 ft. **42.** $1894\frac{7}{6}\frac{7}{0}$.
13. \$88.87. **27.** 1999 lb. **43.** 58 sq. ft. $54\frac{5}{6}$ sq. in.
14. 88 mi. 5267 ft. **28.** 11 mo.
44. 78 cu. ft. $1421\frac{2}{1}\frac{1}{0}$ cu. in. **45.** 195 da. 23 hr. 1 min. 53.25 sec.
46. 103 da. 6 hr. 2 min. $33\frac{3}{4}$ sec.
47. \$147,546,695, \$153,082,710, \$185,027,555, \$124,973,766, \$610,630,726.
48. 966,779, 2,447,344, 3,111,349, 4,162,229.
49. 1,480,565, 2,144,570, 3,195,450.
50. 664,005, 1,714,885.

Page 280

1. 1225, 121 ft. 11 in., 7 mi. 525 ft.
2. 15,799, 1570 lb. 3 oz., 151 mi. 39 rd.
3. 12,168, $121.68, 1215 ft. 6 in., 117,468
4. 10,700, 104 sq. ft. 124 sq. in., 100,700.
5. $83,806.25, 83,578 mi. 165 rd., 83,508,125.
6. $1132.80, 1150 gal. 2 qt., 1160 yd. 1 ft.
7. $2,551,900.50, 255,042 da. 7 hr. 30 min.
8. 1,080,000, 10,746 sq. ft. 76 sq. in., 10,642,500.
9. 351,155.
10. 1,183,896.
11. 1,244,078.
12. 1,394,196.
13. 4,700,094.
14. 2,672,768.
15. 2,713,634.
16. 5,921,013.
17. $56,962.75.
18. $179,762.50.
19. $158,397.76.
20. $274,533.
21. $224,820.80.
22. $248,718.25.
23. $409,149.78.
24. $706,362.93.
25. 470 ft. 4 in.
26. 952 ft.
27. 1562 ft. 4 in.
28. 5615 ft. 8 in.
29. 25 lb.
30. 380 lb. 15 oz.
31. 484 lb. 5 oz.
32. 558 lb.
33. 1188 ft.

Pages 282, 283

1. 486.
2. 247.
3. 696.
4. $36\frac{3}{4}$.
5. $11\frac{1}{6}$.
6. $15\frac{3}{5}$.
7. $9\frac{1}{6}$.
8. $30\frac{1}{4}$.
9. $33\frac{13}{40}$.
12. 105.3.
13. 147.84.
14. 246.33.
15. 810.46.
16. 788.42.
17. 0.9625.
18. 9.35.
19. 65.4481.
20. 5.214.
21. 1.17304.
22. 20.9934.
23. 5601.76.
24. 1064.45.
25. 4.784.
26. 8.27891.
27. 15.50625.
28. 4.59375.
29. 5.72832.
30. 123.6552.
31. 189.3758.
32. 355.1936.
33. 3.67536.
34. 5.18616.
35. 1015.6708.
36. $35.062.
37. $53.865.
38. $43.2717.
39. $591.2136.
40. 4823.
41. 10873.5.
42. 23636.8.
43. 222.3056.
44. 2.118924.
45. 4.017263.
46. 58037.58.
47. $\frac{1}{10}$.
48. $\frac{1}{10}$.
49. $\frac{2}{5}$.
50. $\frac{3}{4}$.
51. $\frac{1}{6}$.
52. $\frac{1}{12}\frac{3}{2}$.
53. $4424.76.
54. $5266.93.
55. $5645.75.
56. $12,952.14.
57. $53,609.20.
58. $82,084.86.
59. 335 ft. 10 in.
60. 905 lb. $7\frac{1}{2}$ oz.
61. 1610 yd. 21 in.
62. $17266\frac{1}{12}$.
63. $49{,}063\frac{5}{24}$.
64. $531{,}701\frac{7}{25}$.
65. $49{,}366\frac{11}{16}$.
66. $109{,}222\frac{2}{3}$.
67. $32{,}781\frac{27}{128}$.
68. $23.01.
69. $47.84.
70. $95.90.
71. $42.19.
72. $225.47.
73. 1211 ft. 9 in.
74. 69 T. 600 lb.
75. $115\frac{5}{16}$ in.
76. $406\frac{2}{3}$.

Pages 286, 287

1. $28\frac{1}{5}$.
2. 23.
3. 23.
4. 138 yd., $\$0.26\frac{1}{2}$.
8. 42.
9. 37.
10. 49.
11. 62.
12. $114\frac{1}{7}$.
13. 37.
14. 146.
15. 42.
16. $68\frac{1}{18}$.
17. 33.
18. $189\frac{1}{2}$.
19. 11.
20. 43.
21. 209.
22. 313.
23. 21.
24. $211\frac{10}{49}$.
25. 309.
26. 129.
27. 32.
28. $70\frac{208}{209}$.
29. $248\frac{84}{121}$.
30. 92.
31. 22.
32. 99.
33. 84.
34. $22\frac{1}{2}$.
35. 13.
36. 23.
37. 41.
38. 63.
39. 91.
40. 91.
41. 23.
42. 32.
43. 44.
44. 52.
45. 3.25 lb.
46. $129.
47. $146.25.
48. 0.75 A.
49. 13.
50. 4.

Page 288

1. 81 gal. 3 qt.
2. 39 yd. $4\frac{2}{11}$ in.
3. 9.
4. 8.
5. 12.
6. 12.
7. 23 hr. 42 min. 15 sec.
8. 211 cu. ft. $1179\frac{3}{23}$ cu. in.
9. 4.
10. 14 lb. 9 oz.
11. $30\frac{1029}{1181}$.
12. 1 hr. 26 min. 39 sec.
13. 2 yd. 2 ft. 11 in.
14. 62 lb. 8 oz.
15. 42 min. 37 sec.

Page 289

1. $1\frac{1}{6}$.
2. $4\frac{1}{2}$.
3. $\frac{10}{21}$.
4. $1\frac{1}{2}$.
5. $\frac{3}{4}$.
6. $\frac{16}{105}$.
7. $\frac{39}{55}$.
8. $\frac{60}{77}$.
9. $3\frac{1}{3}$.
10. $1\frac{1}{55}$.
11. $\frac{11}{27}$.
12. $1\frac{1}{2}$.
13. $\frac{1}{2}$.
14. $\frac{1}{3}$.
15. $3\frac{3}{5}$.
16. 6.
17. $\frac{35}{276}$.
18. $1\frac{214}{3757}$.
19. $1\frac{35}{1472}$.
20. 6.
21. $\frac{1859}{5491}$.
22. $\frac{984}{1127}$.
23. 6.
24. $2\frac{4}{11}$.

Page 290

1. $6\frac{6}{13}$.
2. $8\frac{35}{89}$.
3. $10\frac{18}{23}$.
4. $6\frac{21}{25}$.
5. $6\frac{6}{7}$.
6. $5\frac{21}{23}$.
7. $6\frac{41}{62}$.
8. $4\frac{44}{71}$.
9. $3\frac{32}{39}$.
10. $4\frac{3}{13}$.
11. $2\frac{9}{19}$.
12. $6\frac{17}{20}$.
13. $3\frac{4}{7}$.
14. $4\frac{13}{20}$.
15. $8\frac{152}{293}$.
16. $4\frac{1}{37}$.
17. $7\frac{56}{95}$.
18. 45.
19. $2\frac{125}{128}$.
20. $10\frac{2}{131}$.
21. $11\frac{77}{83}$.
22. $11\frac{53}{77}$.
23. $\frac{63}{40}$.
24. $5\frac{9}{76}$.
25. $24\frac{1}{34}$.
26. $\frac{73}{149}$.
27. $4\frac{37}{368}$.
28. 2.
29. $3\frac{7}{44}$.
30. 17.
31. 6.
32. $8\frac{73}{231}$.
33. 3.
34. 7.
35. 7.
36. $4\frac{37}{47}$ yd.

Pages 294, 295

1. $392.20.
2. $1028.92.
3. $1291.59.
4. $2286.32.
5. $2649.64.
6. $2772.96.
7. $2122.51.
8. $2880.56.
9. $30,328.04.
10. $30,670.29.
11. $20,735.76.
12. $19,521.82.
13. 255 ft.
14. 77 lb. 5 oz.
15. 17 yr. 5 mo. 6 da.
16. $934.81.
17. 6,218,295.
18. 9,799,016.
19. 21,264,047.
20. $52,962,410.

Pages 297, 298

1. $93.64.
2. $61.89.
3. $109.31.
4. $330.28.
5. $100.09.
6. $278.78.
7. $121.49.
8. $239.83.
9. $732.84.
10. $106.26.
11. $572.93.
12. $573.99.
13. $386.49.
14. $588.89.
15. $12.40.
16. $196.42.
17. $279.89.
18. $178.87.
19. $188.33.
20. $278.40.
21. $123.60.
22. $737.98.
23. $857.04.
24. $1993.79.
25. 7 ft. 11 in.
26. 11 lb. 12 oz.
27. 11 yd. 29 in.
28. 2 mo. 23 da.
29. 3 yr. 277 da.
30. 4° 41′ 39″.
31. 6 mi. 4755 ft.
32. 64 ft. 6 in.
33. 12° 47′ 33″.
34. 47 gal. 2 qt.
35. 2 yr. 3 mo.
36. 45° 40′ 49″.
37. $64.21.
38. $237.12.
39. $161.41.
40. 277,896,484 bu.
41. 426,408,355 A.
42. 201,926.
43. $7,399,509,782.
44. 1,098,861, 2,837,488, 3,975,171.
45. $7,422,907.
46. 635.83 mi., 927.14 mi.

Page 301

1. 604,000, 659,375, 861,500, $6,170,625.
2. 33,700, 240,900, 103,400, $1,446,400.
3. $856, $409, $5.31, $4410.50.
4. 100, $5.57, $2381.11, $5388.56.
5. $27.40, $22.44, $9522.08, $45,594.14.
6. 1, $24.86, $316.90, $24.72.
7. 2961, 6714, $8829, 5652 ft., 13,284, $211,329.
8. 67,353, 41,382, $53,207, 43,692 ft., $344,157.

age 307

4. 97.26. 7. 176.22. 10. 788.96. 13. 206.67. 16. 39.488.
5. 109.2. 8. 289.86. 11. 591.68. 14. 148.77. 17. 539.2.
6. 139.14. 9. 548.04. 12. 654.96. 15. 266.76. 18. 319.104.
19. 537.12, $1505.68, $5424.84, $17,107.72.
20. 137.6, $546.40, $2211.20, $2540.
21. 2091.6, $988.80, $1462.50, $8289.60.
22. 149 yd. 23. 294 yd. 24. 149,707.88.

Page 310

1. $36.55. 11. $1\frac{5}{8}$. 21. $1383\frac{1}{8}$. 31. 259.2.
2. $1\frac{5}{8}$. 12. 79 yd. 22. $9.05. 32. 21.44.
3. $187\frac{1}{2}$ ft. 13. $41.40. 23. $\frac{13}{16}$. 33. 7008 yd.
4. $15.30. 14. $1\frac{7}{12}$. 24. $467\frac{5}{8}$ ft. 34. 142 gross.
5. $1\frac{9}{32}$. 15. $85\frac{1}{3}$ mi. 25. 825. 35. 549.
6. $59\frac{3}{4}$ ft. 16. $13.46. 26. 804. 36. 36 gross.
7. $20.55. 17. $1\frac{13}{15}$. 27. 2478. 37. 72.
8. $4\frac{19}{20}$. 18. $175\frac{4}{5}$ yd. 28. 11,860. 38. 48 mi.
9. $222\frac{3}{4}$ lb. 19. $214\frac{1}{5}$. 29. 8175. 39. 10 mi.
10. $23.80. 20. $2058\frac{1}{3}$. 30. 100.8.

Page 314

1. 1521 ft., 18,252 in.
2. 98 ft., 1176 in.
3. 3 A. 80 sq. rd.
4. 3106 sq. in.
5. $3\frac{14}{27}$ cu. yd., 164,160 cu. in.
6. $820\frac{3}{4}$ sq. yd., $7386\frac{3}{4}$ sq. ft.
7. 79 sq. ft., 11,376 sq. in.
8. $35\frac{5}{9}$ cu. yd., 960 cu. ft., 1,658,880 cu. in.
9. 4.8675 T., 9735 lb., 155,760 oz.
10. 19.713 T., 19 T. 1426 lb., 630,816 oz.
11. 6.875 gal., 27.5 qt., 55 pt.

Page 315

1. 80 ch., 320 rd., 8000 li.
2. 1 mi. 62 ch., 568 rd.
3. $4\frac{1}{4}$ ch., 425 li., 3366 in.
4. 39.6 ft., 660 ft., 82.5 ft.
5. 17.4 mi., 1392 ch., 5568 rd.
6. 7 ch. 61 li., 24 ch. 41 li.
7. 1416 li., 14.16 ch.
8. 63,360 in.
9. 14 ch. 1 rd.
10. 58 ch. 71 li.
11. 18 mi. 147 rd.
12. 8 ch. 2 rd.
13. 5 ch. 28 li.
14. 8 mi. 98 rd.
15. 121 ch. 2 rd.
16. 476 ch. 48 li.
17. 318 mi. 64 rd.

Pages 317, 318

1. 22.4 A.
2. 18.4 A.
3. 66.7 A.
4. 108.8 A.
5. 6.05 A.
6. 0.0625 A.
7. 9.01 A.
8. 5.832 A.
9. 8.84 A.
10. 1.813 A.
11. 2.025 A.
12. 10.104 A.
13. 1.7625 A.
14. 8.632 A.
15. 19.60875 A.
16. 20.2170 A.
17. 90.0625 A.
18. 86.6472 A.
19. 146.77841 A.
20. 18.63555 A.
21. 4840 sq. yd., 43,560 sq. ft.
22. $272\frac{1}{4}$ sq. ft., 39,204 sq. in.
23. 102,400 sq. rd., 3,097,600 sq. yd., 27,878,400 sq. ft.
24. 3.05 sq. mi., 1952 A.
25. 7.8 A., 78 sq. ch., 1248 sq. rd.
26. 3421.68 sq. ft.
27. 380 sq. rd.
28. $x = 15$ ch., $y = 8$ ch., 100 ch., 48 A.
29. 22,440 sq. ft.
30. $3825.
31. The second, $140.
32. $x = 60$ ch., $y = 15$ ch., 190 ch., 140 A.
33. No, 400 sq. ch., 300 sq. ch.
35. $23,383.50.
34. $5400.
36. $9618.75.

Pages 320, 321

1. 20 sq. ft.
2. $7.6\frac{3}{17}$ ch.
3. 68 ch.
4. $12\frac{1}{3}$ ft.
5. 71 ft.
6. 41 ch.
7. 18.6875 sq. in.
8. 16.8 ft.
9. 16 ft. 6 in.
10. 32 ft.
11. $13,612.50.
12. 16 ft. 4 in.
13. 32 ft. 6 in.
14. $30\frac{1}{8}$ in.
15. $91\frac{10}{152}$ sq. ft.
16. 13 yd. 8 in.
17. $36\frac{3}{4}$ sq. yd.
18. 117 rd., 8307 sq. rd., 95 rd., 8835 sq. rd.
19. $118\frac{3}{4}$ rd., 95 rd., $317\frac{1}{2}$ rd., 290 rd.
20. 98 ft.

Page 322

1. 102.24 A., 204.52 A., 109.2 A., 415.96 A.
2. $20,174.06.
3. 1.596 A., 414.364 A.
4. $1105.
5. $371.70.
6. $571.20.
7. 363 ft., 966 ft.
8. 42,060 sq. ft.
9. 200 sq. rd.

Page 323

1. 26.13 sq. in.
2. 1181 sq. ft.
3. 121.68 sq. ft.
4. $24\frac{13}{24}$ sq. ft.
5. $73\frac{3}{4}$ sq. yd.
6. $39\frac{3}{4}$ sq. ft.
7. $64\frac{1}{8}$ sq. ft.
8. 43974.4925 sq. ft.
9. $16\frac{11}{16}$ sq. ft.
10. $99\frac{103}{168}$ sq. yd.
11. 7 in.
12. 11 ft.
13. 12 ft.
14. $266\frac{2}{3}$ ft.
15. 7 yd.
16. 6 ft.
17. $5\frac{3}{4}$ ft.
18. 9 ft.

Page 324

1. 688 sq. ft.
2. 22.785 sq. ft.
3. $34.36\frac{1}{2}$ sq. in.
4. $791.56\frac{1}{4}$ sq. ft.
5. 27.405 sq. in.
6. 1 sq. yd. 108 sq. in.
7. 7614 sq. in.
8. 640 sq. in.
9. 67.5 sq. ft.
10. 25 rd., 20 in.
11. 600 sq. rd., $3\frac{3}{4}$ A.
12. 3900 sq. ft.

Page 326

1. $38\frac{37}{66}$ sq. rd.
2. 154,721 sq. ft.
3. $17,589\frac{1}{2}$ sq. ft.
4. 52,274 sq. in.
5. $1031\frac{1}{3}$ sq. ft.
6. $4264\frac{1}{8}$ sq. ft.
7. $17\frac{1}{2}$ sq. yd.
8. 3 ft. 2 in.
9. 4056.25 sq. ft.
10. 6084.375 sq. ft.
11. 9731.25 sq. ft.
12. 4461.875 sq. ft.
13. 1623.75 sq. ft.
14. 1082.5 sq. ft.
15. 2706.25 sq. ft.
16. 4330 sq. ft.
17. 2706.25 sq. ft.
18. 2165 sq. ft.

Page 328

1. 160 A.
2. 160 A.
3. 80 A.
4. 80 A.
5. 80 A.
6. 80 A.
7. 40 A.
8. 40 A.
9. 40 A.
10. $10,400.
11. $3000.
12. 160 A.
13. 320 A.
14. 480 rd.
15. 2 mi.
16. 3 A., $14.25.

Pages 330, 331

1. $8\frac{3}{4}$ cd., 24 cd.
2. 112 loads.
3. $\$33\frac{1}{3}$, $\$41\frac{2}{3}$.
4. $7486\frac{1}{9}$ cu. ft.
5. 129,600 cu. ft.
6. $\$111\frac{1}{9}$.
7. 1904 cu. ft., 65,280 lb.
8. 6000 cu. ft.
9. 60 cu. ft., 65 cu. ft., 103,680 cu. in., 112,320 cu. in.
10. $54\frac{1}{6}$ bu.
11. 7, 3.
12. $457\frac{1}{6}$ cu. ft.
13. $16\frac{41}{49}$ T.
14. $12\frac{123}{196}$ T.
15. $7\frac{1}{2}$ T., $5\frac{1}{2}$ T.
16. 300 cu. ft., 8.

Page 332

1. 13,858 cu. yd.
2. $19\frac{1}{2}$ cu. yd., 234 cu. yd.
3. $9\frac{3}{4}$ hr., 39 min.
4. $710\frac{2}{3}$, $78\frac{26}{27}$, 21,320 ft.
5. 5.19675 da.
6. $2.67.
7. $6,880,346\frac{2}{11}$.
8. $3, $3.50.

Pages 335, 336

1. 112,000 cu. ft.
2. 6 ft.
3. 15 in.
4. 112.5 sq. ft.
5. $43\frac{1}{3}$ ft.
6. $12\frac{1}{2}$ ft.
7. 27.5 ft.
8. 3.9 in.
9. 7.2 in.
10. 11 ft.
11. 12 ft.
12. 9.5 ft. $\times$ 12 ft. $\times$ 12 ft.
13. $30.
14. 9 ft. $\times$ 14.3 ft. $\times$ 16.4 ft.
15. $68\frac{68}{215}$ bu.
16. 160.
17. $3280.50.
18. $1615\frac{61}{77}$ gal.
19. $26.58.
20. $7\frac{1}{2}\frac{2}{5}\frac{3}{5}$ T.
21. $59\frac{45}{77}$ gal.
22. $691.20.
23. $46.91.
24. $291.55.

Page 337

1. First, 2 to right, 4 up.
2. First, 2 to left, 4 up.
3. First, 2 to left, 3 down.
4. First, 3 to right, 2 down.

Page 339

1. 1 hr. 48 min. 17 sec.
2. 5 hr. 2 min. 2 sec.
3. 11 hr. 8 min. 10 sec. A.M.
4. 105°, 9 P.M.
5. 4 A.M., 8 P.M.
6. 8 hr. 22 min. 42 sec. A.M., 8 hr. 2 min. 42 sec. A.M.

Pages 340, 341

1. 30° 47′ 30″.
2. 105° W.
3. 2° 5′ 40.5″.
4. 22° 45′ W.
5. 135° E.
6. 150° E.
7. 37° 30′ W.
8. Fast, 3 hr. 30 min. 3 sec.
9. 12 hr. 9 min. $38\frac{2}{15}$ sec. P.M.
10. 7 : 30 A.M.
11. 46° 53′ 45″ W.
12. 110° 11′ 12.2″ W.
13. 17° 26′ 15″ E.

Page 344

1. 6 hr. 9 min. 21 sec. P.M.
2. 9 P.M., 6 A.M., 4 A.M.
3. 4 hr. 20 min. 54 sec.
4. 9 min. $33\frac{1}{3}$ sec.
5. 2 : 30 P.M., 1 : 30 P.M.
6. 1 hr. 9 min. 21 sec. P.M.
7. Standard, 9 min. 42.72 sec.
8. 6 min. 25.1 sec.
9. 9 hr., 9 hr. 3 min. 17.62 sec.
10. 9 P.M. Dec. 31, 10 P.M. Dec. 31, 11 P.M. Dec. 31, 5 A.M. Jan. 1, 6 A.M. Jan. 1.

Page 345

1. 4 min. $6\frac{3}{16}$ sec., 5 min. $\frac{4}{5}$ sec.
2. 44 min. 13.9 sec., 5 hr. 49 min. $28\frac{3}{5}$ sec.
3. 3 hr. 14 min. $43\frac{13}{25}$ sec., 5 hr. 4 min. $20\frac{1}{5}$ sec.
4. 54 min. $33\frac{1}{10}$ sec., 2 hr. 19 min. $15\frac{23}{25}$ sec.
5. 11 A.M., 10 hr. 50 min. 17.28 sec. A.M.
6. 11 hr. 54 min. 59.2 sec. A.M., 10 hr. 54 min. 59.2 sec. A.M.
7. 32 min. $32\frac{8}{15}$ sec.
8. 1 hr. 1 min. 50 sec.
9. 29 min. $35\frac{1}{5}$ sec.
10. 1 hr. 36 min. $40\frac{1}{5}$ sec.
11. 1 hr. 10 min. 49 sec.
12. 33 min. $18\frac{2}{5}$ sec.
13. 1 hr. 41 min. $4\frac{3}{5}$ sec.
14. 2 hr. 11 min. $13\frac{4}{15}$ sec.
15. 25 min. $9\frac{4}{15}$ sec.
16. 3 hr. 13 min. $5\frac{2}{15}$ sec.
17. 4 hr. 13 min. 18 sec.
18. 36 min. 55 sec.
19. 46° 2′.
20. 151° 1′.
21. 79° 16′ 15″.
22. 107° 19′ 30″.
23. 124° 46′ 45″.
24. 87° 43′ 45″.

Page 348

1. \$560.80.
2. \$246.24.
3. $57\frac{1}{7}\%, 62\frac{1}{2}\%, 27\frac{3}{11}\%, 26\frac{2}{3}\%, 15\frac{5}{8}\%, 41\frac{2}{3}\%$.
4. \$308.50, \$1543.
5. \$26.25.
6. \$579.39, \$255.75, \$325.50.
7. \$142.50.
8. 1533.
9. $\frac{12}{25}$.
10. $\frac{2}{5}$.
11. $\frac{17}{20}$.
12. $\frac{2}{7}$.
13. $\frac{67}{400}$.
14. $\frac{9}{40}$.
15. $\frac{5}{9}$.
16. $\frac{3}{8}$.
17. $\frac{7}{9}$.
18. $\frac{3}{11}$.
19. $\frac{1}{7}$.
20. $\frac{89}{500}$.
21. $\frac{259}{800}$.
22. $\frac{2791}{1000}$.
23. $\frac{473}{700}$.
24. $\frac{341}{640}$.
25. 17.
26. \$37.07.
27. \$486.
28. \$42.06.

Page 350

1. 292 da.
2. \$55.80.
3. \$25.69, \$51.38, \$12.85, \$64.23.
4. 3933.
5. 4 da.
6. \$252.
7. \$161.20, \$145.08.
8. \$656.08.
9. 235.

Page 352

1. $33\frac{1}{3}\%$.
2. $11\frac{1}{9}\%$.
3. 20%.
4. 15%.
5. $16\frac{2}{3}\%$.
6. 12,626.12 sq. mi.
7. 90%.
8. 15%.

Page 353

1. $13\frac{1}{18}\%$.
2. $20\frac{8}{8}\%$.
3. $26\frac{3}{8}\%$.
4. $5\frac{13}{15}\%$.
5. 11.98+%.
6. $21\frac{3}{7}\%$.
7. $18\frac{58}{169}\%$.

Page 354

1. 15.
2. 50%.
3. $0.21.
4. $21\frac{3}{7}\%$.
5. 25%.
6. $20.
7. The first, $23\frac{2}{3}\frac{1}{3}\%$.
8. 15.
9. $135.20.

Page 357

1. $42.50, $17.
2. $35.75, $28.60.
3. $29.25, $17.55.
4. $324, $113.40.
5. $175.
6. $225.
7. 400 ft.
8. 28,200.
9. 11%.
10. $1650.

Page 358

1. $3162.50, $3080.
2. $3080, $3220.
3. $371, $367.50, $362.25, $365.75.
4. 238.
5. $891, $907.50, $924, $899.25.
6. $13,175.
7. $12\frac{1}{2}\%$.
8. $8308.75.

Page 359

1. $3\frac{1}{3}\%$.
2. 60%.
3. $12.50.
4. $31\frac{1}{4}$ sq. mi.
5. $8.40.

Page 360

1. 60%, $166\frac{2}{3}\%$, $37\frac{1}{2}\%$, $62\frac{1}{2}\%$.
2. 22,500, 50,000.
3. $175.
4. $124.08.
5. $62.50, $92.50.

Page 362

1. $48.13.
2. $593.75.
3. $88.73.
4. $38.96.
5. $21.95.
6. $25.08.
7. $105.08.
8. $268.54, $219.71, $292.95.
9. $261.25.
10. $372.75.
11. $546.75.

Page 363

1. $21.98.
2. $58.04.
3. $43.90.
4. $28.27.
5. $9.19.
6. $37.08.
7. $18.85.
8. $45.10.
9. $33.98.
10. $106.91.
11. $78.07.
12. $36.28.
13. $622.71.

Page 366

1. $12.80.
2. $7.
3. $7.88.
4. $23.13.
5. $15.40.
6. $35.25.
7. $90.30.
8. $46.43.
9. $221.58.
10. $252.62.
11. $271.25.
12. $17.73.
13. $33.16.
14. $84.40.
15. $187.43.
16. $380.67.
17. $1933.29.
18. $2560.16.
19. $345.14.
20. $466.61.
21. $408.50.
22. $147.33.
23. $263.18.
24. The second plan, $4.20.

Pages 367, 368

1. 123 da.
2. 158 da.
3. 110 da.
4. 224 da.
5. 122 da.
6. 127 da.
7. $4.90.
8. $11.59.
9. $18.19.
10. $8.22.
11. $7.48.
12. $4.45.
13. $10.27.
14. $8.99.
15. $7.01.
16. $1003.96.
17. $152.44.
18. $76.73.
19. $455.89.
20. $283.07.
21. $896.15.
22. $645.40.
23. $696.18.
24. $844.80.
25. $1042.66.
26. $2098.68.
27. $7.64.
28. $471.59.

Page 369

1. $37.50.
2. $52.90.
3. $3.80.
4. $80.60.
5. $7.37.
6. $364.50.
7. $5.38.
8. $18.75.
9. $10.08, same.
10. $12.01, $11.63, $11.88.

Page 370

1. $61.15.
2. $584.40, $596.28, $578.46, $566.58.
3. $15.33, $15.12.
4. $1.82, $3.95.
5. $50 − $49.32 = $0.68.
6. $6.42, $6.51.
7. $34.19, $34.67.
8. $2.37, $2.41.
9. $2.49, $2.53.
10. $4.75, $4.81.
11. $46.23, $46.88.
12. $34.72, $35.20.
13. $17.17, $17.41.
14. $40.11; $40 at 60 da., $40.67 at 61 da.
15. $13.95, $14.15.

Page 371

1. 5%.
2. 6%.
3. 4%.
4. 5%.
5. 4%.
6. 4%.
7. 6%.
8. 4%.
9. 4%.
10. $3\frac{1}{2}$%.
11. 3%.
12. $5\frac{1}{2}$%.
13. 3%.
14. $3\frac{1}{2}$%.
15. 2%.
16. $4\frac{1}{2}$%.
17. 4%.
18. 4%.
19. 6%.
20. 5%.

Page 372

1. 2 yr. 6 mo.
2. 2 yr. 6 mo.
3. 2 yr. 7 mo.
4. 1 yr. 8 mo.
5. 1 yr. 9 mo.
6. 1 yr. 6 mo.
7. 1 yr. 5 mo. 3 da.
8. 2 yr.
9. 1 yr. 2 da.
10. 4 mo.
11. 16 yr. 8 mo.
12. 2 yr. 6 mo.
13. 11 yr.
14. 14 yr.

Pages 374, 375

1. $41.33.
2. $126.
3. $2.68.
4. $24.59.
5. $56.11.
6. $105.75.
7. $102.62.
8. $1022.71.
9. 5%.
10. 4%.
11. $3\frac{1}{2}$%.
12. $3\frac{1}{2}$%.
13. 1 yr. 2 mo. $10\frac{10}{17}$ da.
14. 2 yr. 4 mo. 28 da.
15. 5 yr.
16. 1 yr. 6 mo. 15 da.
17. 3 yr. 6 mo.
18. $840.
19. $30.
20. $55 less.
21. The first, $3.13.
22. $3\frac{13}{14}$%.
23. $405.
24. $1035.
25. $3327.50 loss.
26. $5302.50.
27. Net income $190 in each case.
28. $19\frac{2}{7}$%.

Page 376

1. $4\frac{1}{8}$ A., 1000.
2. $266\frac{2}{3}$ lb., $8\frac{8}{9}$ lb.
3. 7200.
4. 1920, 40.
5. 10 da., $18.
6. 1500.

Pages 377, 378

1. $12\frac{1}{2}$%.
2. $440, $17\frac{1}{6}$%.
3. $11\frac{7}{18}$%, gain.
4. $37\frac{1}{2}$%, $56\frac{1}{4}$%.
5. $16.81, gain.
6. $13.40, $13.10, $13.20, $13.35.
7. $0.25.
8. $3.59.
9. $38\frac{8}{9}$%.
10. $59.40, $30\frac{5}{9}$%.
11. $0.29.
12. $25.50, $10\frac{10}{19}$%.
13. $2, $4\frac{1}{6}$%.
14. $6, 25%.
15. $1.44 gain, $8\frac{16}{23}$%.
16. $2.88 gain.
17. $42, $3.99.

Page 380

1. 32.83.
2. 457.104.
3. 19.032.
4. 217.98.
5. 65.1.
6. 2.38.
7. 3.65.
8. 8.37.
9. 0.51.
10. 1.1.
11. 2.1.
12. 8.
13. 9.775.
14. $1.34\frac{5}{7}$.
15. $0.9\frac{1}{7}$.
16. 4.
17. 1.54.
18. 1.25.
19. $\frac{14}{15}$.
20. 2.
21. 2.

Page 382

1. 187, 341.
2. 169, 533.
3. 189, 261.
4. 476, 574.
5. 6.51, 6.72.
6. 4.41, 11.13.
7. 33, 55, 77.
8. 65, 91, 117.
9. 147, 231, 273.
10. 583, 689, 901.
11. 8.19, 10.01, 15.47.
12. .891, 1.701, 1.863.
13. 12.07, 13.49, 14.91.
14. 260, 130.
15. $1600, $1200.
16. $28.75, $40.
17. $13.20, $12.70.

Page 383

1. $22.6\frac{6}{7}\frac{4}{3}$.
2. 4.135.
3. $15.7\frac{9}{11}$.
4. 0.24.
5. $2.003\frac{9}{11}$.
6. $1.281\frac{7}{9}$.
7. $2.3\frac{75}{107}$.
8. $77\frac{1}{1}\frac{3}{9}$.
9. 1.5.
10. 3120.
11. $106.7\frac{8}{9}\frac{8}{7}$ in.
12. $1.4\frac{8}{17}\frac{9}{1}$.
13. 5473.

Pages 384, 385

1. 46 bbl.
2. $2142.
3. 302 ft.
4. $101\frac{1}{2}$ ft.
5. 4 hr.
6. 4.
7. $21\frac{1}{15}\frac{3}{5}$ sec.
8. 12 min. 40 sec.
9. 75 mi.
10. 3.
11. 5.
12. 8 min. 23.2 sec.
13. 29 min. $59\frac{1}{3}$ sec.
14. $14.70.
15. $4.

Pages 388, 389

1. 15 bbl.
2. $2\frac{2}{3}$ bu.
3. $992.79.
4. $7.20.
5. $5\frac{5}{6}$.
6. $132.50.
7. 50.
8. $9.37\frac{1}{2}$.
9. $5\frac{1}{3}$ da.
10. 6, 2.
11. $31\frac{1}{2}\frac{3}{6}\frac{9}{7}$ da.
12. 0.75 mo.
13. 180 T.
14. 5 da.
15. $4\frac{3}{8}$ da.
16. $48, $35.
17. 10.
18. 40.
19. $17.50.
20. 14.
21. 1350.

Pages 390-392

1. $1\frac{163}{1280}$ A.
2. 21 ft.
3. $3.
4. $4.55.
5. $0.82\frac{1}{2}$.
6. $8.05.
7. $30.37\frac{1}{2}$.
8. $11.25.
9. $26.25.
10. $10.
11. $138.
12. $242\frac{3}{16}$ lb.
13. 6300.
14. 12,675,000.
15. 2,281,500.
16. 20.24 oz.
17. 12 lb. $9\frac{3}{5}$ oz.
18. 19 lb.
19. 84%.
20. 6,445,312$\frac{1}{2}$ lb., 2,352,539,062$\frac{1}{2}$ lb.
21. 189 bbl., 891 bu.
22. 2,038,405,600 bu.
23. $17.50.
24. $450,000,000, $45,000,000.
25. 34 bu., 25¢, $8.50.
26. $1100.
27. 15 bu., $58,500,000.
28. $10\frac{1}{3}$ T.
29. 30 min.

Pages 394-399

1. $\frac{100}{241}$%, $99\frac{141}{241}$%.
2. 3.125 lb., 3.125 lb., 243.75 lb.
3. 299.04 lb.
4. 139.32 lb.
5. 138,750.
6. 28,224.
7. 80 lb., 300 lb., 60 lb., 225 lb.
8. 875 lb., 950 lb., 175 lb.
9. 1075 lb., 800 lb., 125 lb.
10. $12.07\frac{1}{2}$.
11. 262.5 lb., 37.5 lb., 112.5 lb., 337.5 lb., 700 lb., 100 lb., 300 lb., 900 lb., 1050 lb., 150 lb., 450 lb., 1350 lb.
12. $18.82.
13. $18.72.
14. $16.86.
15. $60.50.
16. $269\frac{3}{13}$%.
17. $137\frac{1}{27}$%.
18. 96 bu., $18\frac{3}{7}$ bu.
19. 37.625 lb.
20. 43.895$\frac{5}{9}$ lb.
21. $10.53\frac{1}{2}$.
22. 21,000 lb.
23. $3\frac{1}{8}$%.
24. 66 lb.
25. $48.
26. $0.54, the second.
27. A. 152 lb., 44.08 lb., $12.34. B. 185 lb., 62.9 lb., $17.61. C. 277 lb., 74.79 lb., $20.94. D. 114 lb., 34.2 lb., $9.58. E. 212 lb., 55.12 lb., $15.43. F. 421 lb., 147.35 lb., $41.26. G. 521 lb., 166.72 lb., $46.68.

28.

	a	*b*	*c*	*d*
1.	672 lb.	168 lb.	196 lb.	$45.08.
2.	672 lb.	168 lb.	196 lb.	$45.08.
3.	528 lb.	132 lb.	154 lb.	$35.42.
4.	408 lb.	102 lb.	119 lb.	$27.37.
5.	528 lb.	132 lb.	154 lb.	$35.42.

29. 30.72 lb., 35.84 lb.
30. 4%, 34 lb., $39\frac{2}{3}$ lb., $9.92.
31. 5 lb.
32. 500 lb.
33. $181\frac{9}{11}$ lb.

Pages 401, 402

1. $1200.
2. $1150.
3. 83⅓%.
4. $1350.
5. 300 da.
6. $2.
7. $1690.
8. $9 decrease.
9. $4, second plan.
10. $5.16, second place.
11. $270, $324, $450, $600.
12. $4.60, $4.37, $4.83, $6.13⅓, operating department.
13. $25.

Pages 403, 404

1. $91.25.
2. $465.
3. $375.
4. $231.
5. $168.30.
6. $911.10.
7. $107.18.
8. $1852.50.
9. $73.20
10. $167.

Page 406

1. $3000, $3038.77.
2. $4720, $4764.06.
3. $3920, $3937.02.
4. $2520, $2532.40.
5. $3080, $3095.15.
6. $1933.75, $1940.25.
7. $713.07, $717.77.
8. $12,400, $12653.19.
9. $502.09, $507.42.
10. $312.69, $314.40.
11. $225.23.
12. $324.73.
13. $731.59.
14. $765.41.
15. $900.94.
16. $541.21.
17. $1093.08.
18. $1126.16.
19. $2143.72.
20. $3152.83.
21. $9.46 more at simple interest.
22. $35.13.
23. $1380.86.

Page 408

1. $509.
2. $1176.92.
3. $524.23.
4. $2462.35.
5. $930.75.
6. $211.44.
7. 715.23.
8. $1668.
9. $246.80.
10. $272.

Page 410

1. $155.25.
2. $156.
3. $159.23.
4. $517.43.
5. $810.40.
6. $848.74.

Page 413

1. $4.38, $870.62.
2. $12.19, $962.81.
3. $2.13, $422.87.
4. $21.40, $2546.60.
5. $12.50, $1237.50.
6. $18.75, $1481.25.
7. $32.06, $2532.94.
8. $44.69, $3205.31.
9. $15.94, $4234.06.
10. $1.63, $323.87.
11. $3.56, $423.94.
12. $0.99, $236.51.
13. $6.25, discount.
14. $3.38, discount.
15. $18.75, discount.
16. $1500.
17. $22.50, $1477.50; $15, $1485.
18. $700, discount = $3.50.

Page 415

1. \$0.75, \$300.25; \$0.80, \$300.20.
2. \$6.86, \$799.81; \$6.99, \$799.68.
3. \$2.78, \$552.72; \$2.87, \$552.63.
4. \$1.39, \$331.74; \$1.43, \$331.70.
5. \$5.03, \$999.97; \$5.19, \$999.81.
6. \$21.59, \$1252.54; \$21.77, \$1252.36.

The second set of answers in Exs. 1–6 are the results obtained when both the first and last days of the discount period are counted.

7. \$31.25, \$3718.75.
8. \$35.21, \$4189.79.
9. \$99.75, \$9400.25.
10. \$65.63, \$7434.37.
11. \$54.57, \$4170.43.
12. \$86.41, \$5488.59.
13. \$34.38, \$7465.62.
14. \$26.13, \$4723.87.
15. \$0.21, \$42.29.
16. \$0.14, \$27.46.
17. \$0.18, \$35.32.
18. \$0.14, \$28.61.
19. \$1.26, \$124.24.
20. \$2.75, \$272.50.
21. \$5.63, \$369.87.
22. \$6.76, \$443.99.
23. \$1.91, \$455.59.
24. \$1.23, \$295.27.
25. \$3.60, \$284.
26. \$4.69, \$370.71.
27. \$31.97, \$2718.03.
28. \$38.07, \$3236.93.
29. \$45.82, \$4714.18.
30. \$26.42, \$2718.58.
31. \$6.37, \$1538.63.
32. \$5.05, \$1569.95.
33. \$1250.
34. \$1747.07.

Pages 417, 418

1. \$393.75.
2. \$337.28.
3. \$102.52.
4. \$651.97.
5. \$191.96.
6. \$178.85.
7. \$211.08.
8. \$476.18.
9. \$453.49.
10. \$375.58.
11. \$60.20.
12. \$408.34.
13. \$297.09.

Page 419

1. \$189.95.
2. \$106.42.
3. \$558.99.

Page 421

1. \$1.34, \$0.67, \$2.68.
2. \$5.67.
3. \$5.42, \$1.17.
4. \$5.87, \$8.80.
5. \$1.03, \$0.20.
6. \$6.43, \$12.86.
7. \$3.74, \$11.22.

Page 423

1. $65.87.
2. $540.
3. $95.42.
4. $62.04.
5. $121.87.
6. $345.
7. $3317.28.
8. $3375.25.
9. $304.77.

Pages 424, 425

1. $187.70.
2. $29.30.
3. $1.39.
4. $1.35.
5. $63.07.
6. $4.40.
7. $4.65.
8. $481.41.

Pages 427, 428

1. $31.05, $56.25, $65.70.
2. $440, $670, $330.
3. $810, $975, $615.
4. $75, $54, $66.
5. $210, $315, $560.
6. $110, $88, $66.
7. $65.10, $36.
8. $240, $315.
9. $11.10, $14.40.
10. $5446.45, $3853.55.
11. $175, $2325.
12. $3925, $2475.
13. $3900, $900, $2400.
14. $288, $189, $378.
15. $8.16, $6.36, $10.20.

Page 431

1. $300.60.
2. $3203.20, $2501.25.
3. Add 15¢, 25¢, 10¢, 30¢, 30¢.
4. $2751.10.
5. 0.3%.
6. $2750, $2.75.
7. $3750, $1.50.
8. $250.25, $150.15, $100.10, $350.35.
9. Draft, 10¢.
10. Registered letter, 2¢.

Pages 432, 433

1. $749.25.
2. $549.45.
3. $149.70.
4. $\frac{4}{15}\%$.
5. $\frac{1}{5}\%$.
6. $74.85.
7. $\frac{1}{10}\%$.
8. $249.75.
9. $0.75.

Page 435

1. $3756.70
2. $3496.50.
3. $751.50.
4. $6753.38.
5. $2452.45.
6. $17,465.
7. $5161.
8. $3408.50.
9. Discount, $\frac{1}{8}\%$.
10. Discount, $\frac{1}{5}\%$.
11. $249.75.
12. $2447.50.
13. $379.75.
14. $3240.

Page 437

1. 720d.
2. 528d.
3. 786d.
4. 44d.
5. 219d.
6. 1214d.
7. 8.66⅔s.
8. 9.25s.
9. 66.75s.
10. £2.1.
11. £3.2.
12. £6.225.
13. 2500 c.
14. 3700 c.
15. 3530 c.
16. 2.75 fr.
17. 4.75 fr.
18. 12.75 fr.
19. 3.5 M.
20. 4.8 M.
21. 92.75 M.
22. 17,500 pf.
23. 20,000 pf.
24. 1075 pf.
25. $365.25.
26. $331.16.
27. $80.36.
28. £8.
29. £5.
30. £9.
31. $17.85.
32. $29.75.
33. $892.50.
34. 40 M.
35. 60 M.
36. 300 M.
37. $16.41.
38. $44.39.
39. $144.75.

Page 439

1. $433.50.
2. $58.75.
3. $176.02, $145.51.
4. £15.
5. 650 fr.
6. 322.12 M.
7. $610.73.
8. $111.
9. Demand draft $1.37 better.
10. $130.69.

Page 442

1. 0.2753 m.
2. 4.764 m.
3. 29.38 m.
4. 4.862 km.
5. 0.12758 km.
6. 0.628341 km.
7. 78.33⅓ km.
8. 44.16⅔ km.
9. 3.249 km.
10. 450.6 mi.
11. 171.6 mi.
12. 20.94 mi.
13. 3.168 mi.
14. 1.7856 mi.
15. 0.88686 mi.
16. 55¼ ft.
17. 208 ft.
18. 351 ft.
19. 224.055 ft.
20. 97.045 ft.
21. 264.212 ft.
22. 1850.39 in.
23. 127.5588 in.
24. 184.2516 in.
25. 1358.265 in.
26. 1.114171 in.
27. 118.11 in.
28. 2.7559 in.
29. 659.75 ft.
30. 121.875 ft.
31. 125.4 mi.
32. 106.8 mi.
33. 61 fr. 20 c.
34. 27.9⅜ mi.

Page 443

1. 7,500,000,000,000 cm².
2. 370,000 cm².
3. 42.96 cm².
4. 62,500 cm².
5. 80,583,750 A., using 2½ A. = 1 ha.

Page 444

1. [illegible],750 m^3.
2. 0.00042765384 m^3.
3. 4161.63 m^3.
4. 1 m^3.
5. 100 m^3.
6. 1 m^3.

Page 446

1. 170 kg.
2. 10 kg.
3. 3.275 kg.
4. $7012\frac{8}{11}$ kg.
5. 22 kg.
6. 7.275 kg.
7. $1001\frac{9}{11}$ kg.
8. 3 kg.
9. 79 kg.
10. 77 lb.
11. 200 5-ct. pieces.
12. 0.0352 oz., 0.0022 lb.
13. 55 lb.
14. $11\frac{1}{4}$ t., 86.7 t.
15. 30.94 t., 8.4 t.
16. 28.2906 kg.
17. 0.6075 t.
18. 3174.8706 g.
19. 521.1 g., 0.5211 kg.
20. 165 lb.

Page 448

1. 7.224 kg.
2. 5.81 kg., 12.782 lb.
3. 11,811 in., 984.25 ft., 0.3 km.
4. 337.75 g., 5201.35 grains.
5. 2.25 kg., 4.95 lb.
6. 2929.5 g.
7. 5138 g.
8. 29.625 kg.
9. 160.2 kg.
10. 0.06 ha., 0.124 A.
11. 0.77 lb., 12.32 oz., 0.35 kg.
12. 30,000 kg., 1875 lb.

Page 451

1. $1052.18.

Pages 452, 453

1. $42\frac{2}{23}\%$.
2. $22\frac{58}{61}\%$, $18\frac{2}{61}\%$, $28\frac{17}{61}\%$.
3. $4\frac{33}{88}\%$.
4. $2\frac{8}{11}\%$.
5. $97\frac{3}{11}\%$.
6. $3566\frac{2}{3}\%$.
7. 32%.
8. $40\frac{8}{11}\%$.
9. $92,049,300.
10. $11\frac{1}{10}\%$.
11. $229,926,355.40.
12. $53,590,000, $179,410,000.
13. $9,180,000.

Page 454

1. $0.0086\frac{2}{23}\%$.
2. $7.80.
3. 9.1%.
4. $0.08, $1,826,512.
5. $2000, $296.
6. 507,500 mi., 475,020,000 mi.
7. $514.89.

Pages 455, 456

1. \$3,220,000, \$3,590,000.
2. 474 lb.
3. 459 lb.
4. 4,000,000 lb., 31,120,000 lb.
5. 24 hr.
6. 15,000.
7. 23, $4\frac{8}{23}\%$.
8. 23 knots.
9. 10.1 in.
10. 130,206 lb.
11. 72 hr.
12. 20, 20.
13. 27,200 T.
14. \$696, \$8352.
15. $10{,}420\frac{3}{4}$ ft.

Page 458

1. \$1957.50, *ad valorem*.
2. \$1050, specific.
3. \$127.84.
4. \$131.49.
5. \$1620.
6. \$2830.
7. \$144.75, \$36.19, \$180.94.
8. \$1905.88, \$762.35, \$2668.23.
9. \$790.75.

Page 459

1. \$900.
2. \$250.
3. \$1125.
4. \$62.50.
5. \$39.
6. \$216.
7. \$37.50.
8. \$80.
9. \$197.
10. \$75.
11. \$278.50.
12. \$15.
13. \$64.
14. \$262.50.
15. \$139.50.
16. \$27.

Page 461

1. \$40.63.
2. \$1125.
3. \$0.0055.
4. \$64.75.
5. \$0.0075.
6. \$55.60.
7. \$0.004.
8. \$0.003.
9. \$0.0055.
10. \$0.003.
11. \$0.0045.
12. \$0.006.
13. \$2,750,000.
14. \$3,500,000.
15. \$8,250,000.
16. \$9,125,000.
17. \$19,035.
18. \$47,625.
19. \$39,875.
20. \$83,070.

Page 462

The second answer is the net tax plus the collector's 1%, calling a fraction of a cent one cent.

1. \$35.75, \$36.11.
2. \$48.13, \$48.62.
3. \$26.81, \$27.08.
4. \$42.21, \$42.65.
5. \$66, \$66.66.
6. \$90.75, \$91.66.
7. \$134.20, \$135.55.
8. \$129.25, \$130.55.
9. \$171.05, \$172.77.
10. \$127.88, \$129.16.
11. \$103.82, \$104.86.
12. \$248.88, \$251.37.

Pages 465, 466

1. $23.75.
2. $30.80.
3. $43.13.
4. $160.
5. $75.
6. $3500.
7. $2052.
8. $202.13.
9. $1.54, $0.51.
10. $1.20, $0.40.
11. $10,000.
12. $10,500, $16,800.
13. $6050.
14. $31.25, $1500.
15. 3-year policy, $10.50.
16. $5880.
17. $2972.98.
18. $7785.
19. $25,000.
20. $1200, $3.
21. $16\frac{2}{3}$ yr.
22. $280.

Page 467

1. $9000.
2. $44,900.
3. $123.35.
4. $102, $163.20.
5. $54, $162, $108.
6. $16.92, $25.38, $84.60.

Pages 470, 471

1. $136.95.
2. $410.40, $4104.
3. $2739.
4. $171, $3420.
5. $243.75, $4875.
6. $66, $82.25.
7. $263.60, $6590.
8. $4076.
9. $2739, $3834.60.
10. $24.60.
11. $951.50.
12. $1912, $922.
13. $402.
14. $1072.80.
15. $331.20.
16. $1917.

Page 473

1. 15,000.
2. $1,250,000.
3. $500.
4. $270.
5. $135.
6. $1\frac{1}{4}\%$.
7. $2,750,000.
8. $12,500, $250.
9. $2\frac{1}{4}\%$.
10. 5%.

Page 475

1. $9553.13.
2. $3415.63.
3. $8512.50.
4. $7843.75.
5. $16,387.50.
6. $43,125.
7. $10,730.
8. $31,937.50.
9. $29,553.13.
10. $44,468.75.
11. $20,667.50.
12. $17,085.
13. $68\frac{1}{2}$.
14. 40.

Pages 475, 477

1. $150.
2. $306.25.
3. $2.50 loss.
4. $16,467.50.
5. $31.25 gain.
6. $125 loss.
7. $275 loss.
8. $75 gain.
9. Neither gain nor loss.
10. $131.25 loss.
11. $12.50 gain.
12. $50 gain.
13. $106.25 loss.
14. Neither gain nor loss.
15. $18.75 gain.
16. $93.75 gain.
17. $68.75 gain.
18. $87.50 loss.
19. $287.50 loss.
20. $100 loss.
21. $124\frac{7}{8}$.
22. $142\frac{3}{8}$.
23. $24,393.75.
24. 250.
25. 200.
26. $272.

Pages 479, 480

1. $7218.75, $7706.25, $5146.88, $7500.
2. 4%.
3. 5% stock @ $139\frac{7}{8}$.
4. Same.
5. $3\frac{1}{2}$%.
6. $3\frac{49}{117}$%.
7. $5\frac{1}{2}$% note.
8. 7% stock @ $149\frac{7}{8}$.
9. 5% bond @ 121, $\frac{100}{117\frac{37}{37}}$%.
10. The same.
11. The same.
12. The same.
13. 5% bond @ 108, $\frac{5}{351}$%.
14. 6% bond @ 140, $\frac{10}{819}$%.
15. $3\frac{1}{2}$% bond @ 107, $\frac{25}{2481}$%.
16. 5% bond @ 111, $\frac{1}{222}$%.
17. The same.
18. 4% bond @ 86, $\frac{20}{559}$%.
19. The same.
20. 5% stock @ 95, $\frac{5}{19}$%.
21. 5% stock @ 114, $\frac{50}{7809}$%.
22. $1487.50, stock; $1103.75, at interest.
23. $83.79 gained by stock.

Page 481

1. $2760.
2. $84.38.
3. $33.75.
4. $318.75.

Page 482

1. $13,732.50.
2. $5283.75.
3. $184,650.
4. $7000.
5. $147,000.
6. $150.
7. $93.75 gain.
8. $53,345.
9. $540 gain.

Pages 483-485

1. 120.
2. Wage earners, $285; others, $1511.
3. Materials, $181,170,000; labor, $56,430,000; salaries, $5,940,000; miscellaneous. $17,820,000; profit, $35,640,000.
4. White pine, 543,950 M; hemlock, 247,940 M; spruce, 106,260 M; yellow pine, 703,340 M; oak, 323,840 M; other woods, 604,670 M.
5. $10,373,000.
6. $3,542,000.
7. $29\frac{1}{31}\%$.
8. $172,000,000.
9. $222,000,000.
10. $98,000,000.
11. 31%.
12. 376,441 doz.
13. Decreased, $30.
14. Population, $8\frac{4}{7}\%$; ice, $34\frac{26}{111}\%$.
15. $786,600,000.
16. $255,280,000.
17. 10¢.
18. Canada, 103,105,926 bu.; U.S., 687,372,840 bu.; both, 790,478,766 bu.
19. 42,500,000 cu. ft.
20. 21,900,000 bbl.
21. $4\frac{6}{11}\%$, $4\frac{16}{21}\%$, $4\frac{8}{23}\%$, $4\frac{1}{6}\%$.

Page 486

1. $112, $109.20.
2. $4, $4.30.
3. 1.775%.
4. $3,258,048.60 @ $4.10.
5. 672,000,000 lb., 436,800,000 lb., $60.6+\%$, $39.4-\%$.
6. 5,717,333 T.
7. 322.

Page 487

1. $0.47+.
2. $1126\frac{1}{4}$ lb.
3. 730 lb.
4. 275,968 T.
5. $2\frac{3}{4}$ in.
6. 290,400 long T.
7. $2.47 or $2.52, depending on how cross pieces run.

Page 488

1. 2437.
2. 255,000,000 cu. ft.
3. 1750 cu. ft.
4. 944 : 5775 = nearly 1 : 6.12.
5. 5.
6. $1.44-m$.
7. 3 ft. $11\frac{1}{2}$ in.
8. $761,400.
9. $25,698,336.

Page 489

1. 691.15 mi.
2. 2850 passengers, 342 crew.
3. $41\frac{81}{109}\%$.
4. 2590.
5. 5600 T., 12,544,000 lb.
6. 15 times, $3\frac{3}{4}$ times.

Pages 490, 491

1. $113\frac{7}{16}$ T., $22{,}687\frac{1}{2}$ T.
2. 1.8°.
3. $39\frac{1}{5}$° F., $16\frac{2}{3}$° C.
4. 29.871 in., 29.433 in.
5. 4,515,720 T., 1,517,340 T., 1,052,700 T., 3,121,800 T., 2,403,060 T., 1,996,500 T., 2,983,860 T., 3,397,680 T.
6. 1,517,450 mi.
7. 6.94375 lb.
8. 80%.
9. 1.765— mi.
10. 19,488 ft.
11. 44 min.
12. 787.9872 T.
13. 1 mi., 100 mi.
14. 13,392 lb.

Page 493

1. 25. **6.** 48. **11.** 6.4. **16.** 2.1 in. **21.** 0.35 ft. **26.** 48.4 in.
2. 18. **7.** 36. **12.** 121. **17.** 66 in. **22.** 324 ft. **27.** 576 in.
3. 22. **8.** 33. **13.** 2.2 in. **18.** 125 in. **23.** 440 in. **28.** 512 in.
4. 27. **9.** 8.1. **14.** 1.4 ft. **19.** 77 yd. **24.** 420 ft. **29.** 520 ft.
5. 24. **10.** 3.5. **15.** 1.5 ft. **20.** 3.3 yd. **25.** 560 ft. **30.** 1440 in.
31. 96 rd., 32 rd. **33.** 13 rd.
32. 52 rd., 104 rd., 2. **34.** 68 rd., 204 rd., 3.

Page 494

1. 11. **2.** 9. **3.** 8. **4.** 12. **5.** 25. **6.** 14. **7.** 16. **8.** 18. **9.** 22 in.

Page 498

1. 57. **6.** 47. **11.** 77. **16.** 87 ft. **21.** 99 yd. **26.** $\frac{55}{57}$.
2. 61. **7.** 53. **12.** 79. **17.** 89 rd. **22.** $\frac{12}{23}$. **27.** $\frac{19}{91}$.
3. 63. **8.** 59. **13.** 82 ft. **18.** 91 in. **23.** $\frac{31}{32}$. **28.** $\frac{7}{33}$.
4. 71. **9.** 67. **14.** 97 ft. **19.** 95 ft. **24.** $\frac{33}{34}$. **29.** $\frac{17}{97}$.
5. 41. **10.** 73. **15.** 85 in. **20.** 83 yd. **25.** $\frac{44}{71}$.

Page 500

1. 111. **4.** 89.7. **7.** 34.3. **10.** 4607. **13.** 1.41. **16.** 2.828.
2. 234. **5.** 4.41. **8.** 9.07. **11.** 7008. **14.** 2.236. **17.** 3.316.
3. 10.5. **6.** 0.53. **9.** 251. **12.** 9812. **15.** 2.645. **18.** 12.649+rd.

Pages 501, 502

1. 85 ft.
2. 2.82 ft.
3. 36.05 in.
4. 95 in.
5. 125 ft.
6. 535 in., 85 in., 153 ft.
7. 22 ft. 6 in.
8. 16 ft. 6 in.
9. 43.60 ft.
10. 88.24 ft.
11. 16.40 rd.
12. 76.20 ft.
13. 8.49 in.
14. 9.76 in.
15. 21.93 ft.
16. 3.46 in.
17. 50 ft.
18. 7.5 mi.
19. 8.94 mi.

Page 503

2. 27.1488 in.
3. 46.3792 in.
4. 965.762 ft.
5. 1060.500 rd.
6. 127.260 ft.
7. 6.708 ft.
8. 21.54 ft.
9. 12.726 ft.
10. 22.624 ft.
11. 3.535 ft.
12. 10 in.
13. 3 in.
14. 5 ft.

Page 506

1. 17.
2. 19.
3. 21.
4. 18.
5. 24.
6. 27.
7. 31.
8. 36.
9. 51.
10. 54.
11. 61.
12. 67.
13. 82.
14. 91.
15. 96.
16. 98.
17. 84.
18. 92.
19. 95.
20. 99.
22. 2744.
23. 9261.
24. 15,625.
25. 27,000.
26. 166,375.
27. 216,000.
28. 421,875.
29. 970,299.

Page 509

1. 13.
2. 15.
3. 14.
4. 41.
5. 42.
6. 45.
7. 46.
8. 48.
9. 52.
10. 59.
11. 63.
12. 163.
13. 109.
14. 121.
15. 138.
16. 160.
17. 169.
18. 370.
19. 378.
20. 460.
21. 46.3.
22. 45.1.
23. 0.486.
24. 326.

Page 511

1. 214.3$\frac{4}{5}$ in.
2. 151.8 ft.
3. 1329$\frac{3}{7}$ in.
4. 16.06 ft.
5. 169.4 ft.
6. 20.02 in.
7. 14.74 ft.
8. 182.6 in.
9. 18.92 ft.
10. 42.4$\frac{2}{3}$ ft.
11. 55$\frac{11}{21}$ ft.
12. 202.4 ft.
13. 42 ft.
14. 18$\frac{6}{11}$ ft.
15. 31 ft.
16. 56 in.
17. 245 ft.
18. 30$\frac{1}{5}$ ft.
19. 21.7 in.
20. 1.26 ft.
21. 0.154 ft.
22. 53.4072 in.
23. 40.8408 in.
24. 8.79648 in.
25. 13.728792 in.
26. 7.0686 ft.
27. 19.3732 ft.
28. 17 in.
29. 27 in.
30. 31 in.
31. 29.9999+ ft.
32. 58+ ft.
33. 44 ft.
34. 308 in.
35. 484 in.
36. 572 in.
37. 660 in.
38. 96.8 in.
39. 11 in.
40. 23.8 ft.
41. 6.6 in.
42. 1.98 in.
43. 14.5 in

Page 513

1. $3\frac{1}{7}$ sq. in.
2. 1386 sq. ft.
3. 1386 sq. ft.
4. 35 ft.
5. 39.82 in., 126.12985 sq. in.
6. 7.7 in., 3.85 in., 46.585 sq. in
7. 78.54 sq. in., 226.9806 sq. in.
8. 22 ft., 38.5 sq. ft.
9. $7\frac{1}{4}$ in.

Page 515

1. 10,875 cu. in.
2. $0.341\frac{1}{3}$ cu. ft.
3. 325.92 cu. in.
4. 310.98 cu. in.
5. 178.383 cu. ft.
6. 199.076 cu. in.
7. $8\frac{1}{2}\frac{1}{9}$ in.
8. 9 in.
9. 8 in.
10. $9\frac{2}{7}\frac{0}{7}$.
11. 999.6 cu. in.
12. 70.528 cu. in.
13. 113.53076 cu. in.
14. 43.225 cu. ft.
15. 100.34375 cu. ft.
16. 2301.252 cu. in.
17. 226,981 cu. in.
18. 2885.794 cu. in.
19. 21 in., 11 in.
20. 7.5 in.

Page 516

1. $50,285\frac{5}{7}$ cu. ft.
2. $46\frac{2}{1}\frac{3}{2}\frac{3}{6}$ cu. ft.
3. $169,273\frac{2}{1}\frac{3}{5}$ gal.
4. $144\frac{3}{8}$ cu. ft.
5. 2079 cu. in.
6. 2618 cu. in., $11,455\frac{5}{7}$ cu. in.

Page 518

1. 105 sq. in.
2. 231 sq. in.
3. $62\frac{6}{7}$ sq. in.
4. 5092 sq. ft.
6. $188\frac{4}{7}$ sq. ft.
7. $244\frac{3}{4}$ sq. ft.
8. 110 sq. ft.
9. 27,456 sq. ft.
10. $1178\frac{4}{7}$ sq. ft.

Page 519

1. 160.332 cu. in.
2. 3933.384 cu. in.
3. 600.9404 cu. ft.
4. 667.038 cu. ft.
5. 21 sq. in.
6. 21 sq. in.
7. 27 sq. in.
8. 18 sq. in.
9. 93,537,284 cu. ft., 15,714,263,712 lb.

Page 520

1. 509.329$\frac{1}{8}$ cu. in., 1527.988 cu. in.
2. 134.904 cu. in., 404.712 cu. in.
3. 407.484 cu. in., 1222.452 cu. in.
4. 845.614 cu. in., 2536.842 cu. in.
5. $301\frac{5}{7}$ cu. in.
6. 55,440 cu. ft., 415,800 gal.
7. $718\frac{2}{3}$ cu. in.
8. $18,102\frac{6}{7}$ cu. in. $= 10\frac{1}{2}\frac{0}{1}$ cu. ft.

Page 522

1. 32 sq. in.
2. 272 sq. in.
3. 360 sq. in.
4. 140 sq. in.
5. 176 sq. in.
6. $592\frac{1}{2}$ sq. in.
7. $46.2\frac{1}{4}$ sq. in.
8. $40.21\frac{7}{8}$ sq. in.
9. 20.86 sq. in.
10. 10 sq. ft.
11. 340 sq. in.
12. $294\frac{1}{2}$ sq. in.
13. 10 in.
14. 50 in.
15. 2040 sq. ft.
16. $115\frac{1}{2}$ sq. ft.
17. 764,000 sq. ft.
18. 960 sq. ft.
19. $183\frac{1}{3}$ sq. ft.

Page 523

1. $63\frac{9}{14}$ sq. in.
2. 2,359,727,071,428$\frac{4}{7}$ sq. mi. ; or, say, 2,359,727,000,000 sq. mi.
3. $17\frac{6}{22}\frac{3}{4}$ sq. in.
4. $2828\frac{4}{7}$ sq. in.

Pages 524-527

1. $56\frac{185}{1344}$ cu. in.
2. 4851 cu. in.
3. 4,190,476,$190\frac{10}{21}$ cu. mi.
4. $1767\frac{9}{7}$ cu. ft.
5. $3688\frac{857}{456}$ cu. ft.
6. $22\frac{1}{24}$ cu. ft.
7. $53\frac{19}{81}$ cu. ft.
8. 268,190,476,-$190\frac{10}{21}$ cu. mi.
9. $463.433\frac{1}{7}$ cu. in.
10. $776.046\frac{6}{7}$ cu. in.
11. $\frac{5209}{2304}$ sq. in.
12. $\frac{165}{256}$ sq. in.
13. $\frac{289}{1296}$ sq. in.
14. $\frac{225}{4096}$ sq. in.
15. 8.2369 sq. ft.
16. 15.3664 sq. ft.
17. 306.25 sq. rd.
18. 0.1369 sq. in.
19. 31.28 sq. ft.
20. 46.72 sq. in.
21. $8\frac{19}{32}$ sq. ft.
22. $3.86\frac{1}{12}$ sq. ft.
23. 4.7724 sq. in.
24. 38 sq. ft.
25. 8.93 sq. ft.
26. $3037\frac{1}{2}$ sq. in.
27. 0.0777 sq. in.
28. 15.68 sq. in.
29. 3645 sq. in.
30. $3.37\frac{1}{6}$ sq. ft.
31. 18 in.
32. 2.1 in.
33. 3.4 in.
34. 1.9 in.
35. 11 in.
36. 17 in.
37. 64 in.
38. 6.4 ft.
39. 7.8 in.
40. 8.4 in.
41. 0.44 in.
42. 11 ft. 10 in.
43. 17.88 rd.
44. 13.076 ft.
45. 19.62 ft.
46. 12.369 in.
47. 18.76 rd.
48. 15.84 yd.
49. 2.74 in.
50. 2.457 yd.
51. 2.917 in.
52. 616 sq. in.
53. 745.36 sq. in.
54. 5003.46 sq. in.
55. 13,034.56 sq. ft.
56. 16,022.16 sq. ft.
57. 26,832.96 sq. yd.
58. 48,246.66 sq. rd.
59. 85,046.50 sq. rd.
60. 174,896.26 sq. yd.
61. 9.899 in.
62. 15.65 in.
63. 21 in.
64. 23.216 ft.
65. 24.249 ft.
66. 28.86 ft.
67. 3.569 yd.
68. 3.77 yd.
69. 6.71 yd.

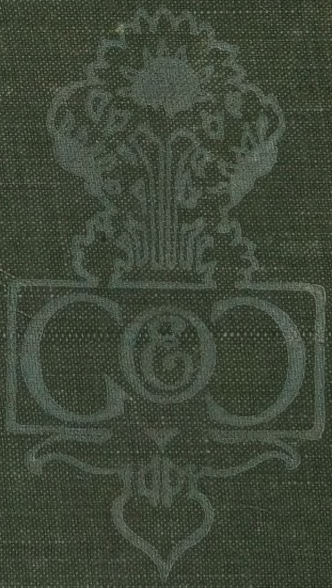
70. 24,848,438$\frac{10}{21}$ cu. in., 411,852$\frac{4}{7}$ sq. in.
71. 21,323,020$\frac{4}{21}$ cu. in., 371,913$\frac{1}{7}$ sq. in.
72. 1,023,065,476$\frac{4}{21}$ cu. in., 4,910,714$\frac{2}{7}$ sq. in.
73. 134,754.95 cu. in., 12,712.73$\frac{1}{7}$ sq. in.
74. 88,102.985 cu. in., 9576.41 sq. in.
75. 167,626.12 cu. in., 14,704.04$\frac{4}{7}$ sq. in.

76. 21.6 cu. ft.
77. 93.48 cu. in.
78. 71.34 cu. in.
79. 44.3$\frac{17}{18}$ cu. ft.
80. 25.432 cu. ft.
81. 439.899 cu. in.
82. 225.254$\frac{6}{7}$ cu. in.
83. 2928.080$\frac{4}{7}$ cu. ft.
84. 109.184$\frac{20}{21}$ cu. ft.
85. 23.533$\frac{5}{7}$ cu. ft.
86. 62.83 cu. in.
87. 427.038$\frac{6}{7}$ cu. in.
88. 28.189$\frac{1}{3}$ cu. in.
89. 498.968 cu. ft.
90. 20$\frac{152}{181}\frac{45}{44}$ cu. ft.
91. 70$\frac{20}{567}$ cu. ft.
92. 86$\frac{3}{7}$ lb.
93. 28 T.
94. 19.404 cu. in.
95. 2416$\frac{2}{63}$ lb.
96. 1,457,777$\frac{7}{9}$.
97. $\frac{2}{27}$ in.
98. 810$\frac{21}{21}\frac{5}{6}$ oz.
99. 1230$\frac{15}{242}$ oz.
100. 15.59 sq. ft.
101. 2.
102. 268,190,476,000 cu. mi., 201,143,000 sq. mi.

Pages 528-542

1. 35, 56.
2. 5, 35.
3. 140.
4. 487, 296.
5. 740$\frac{5}{18}$ lb.
6. $140.40.
7. 129.66$\frac{1}{3}$ lb.
8. $5.04.
9. 66$\frac{2}{3}$%.
10. 23$\frac{1}{13}$%.
11. 1′ 11$\frac{1}{5}$″.
12. 1,369,028$\frac{4}{7}$ T.
13. 36$\frac{24}{25}$ sq. in.
14. 12 da.
15. 24,942,881,600 lb.
16. 45 mi.
17. 12.
18. 375 T.
19. $10,890.
20. $5.25.
21. $80.
22. $550.
23. 66 ft.
24. $12.50 loss.
25. 20%, 16$\frac{2}{3}$%.
26. 20,558,000 bbl.
27. 5$\frac{1}{16}$′.
28. 769.692 sq. ft.
29. 2.
30. 6$\frac{1}{5}$ hr.
31. 1 yr. 2 mo.
32. 5$\frac{1}{5}$ sec.
33. 2$\frac{2}{3}$ da.
34. 80 da., 10$\frac{10}{11}$ da.
35. 5 da.
36. 76.
37. 8 hr.
38. 123 and 123.
39. 756.86967.
40. 424.26+ ft.
41. 3.71.
42. 45 ft.
43. 41 ft.
44. 25 ft.
45. 127.27+ ft.
46. 100.62 ft.
47. 160.
48. 3 hr. 30 min.
49. 128.
50. 70$\frac{5}{16}$ hr.
51. 10 hr.
52. $1250, $1750, $1500.
53. 8.8 lb.
55. $2.17.
56. 6 T.
57. 100 gal.
58. 8$\frac{1}{2}$ lb.
59. 75.
60. 486 sq. in., 729 cu. in.
61. 0.0220+ A.
62. 2$\frac{1}{4}$.

63. 98 sq. in.
64. 1,178,571$\frac{3}{7}$ ft.
65. 8 ft. 3 in.
66. 98 da.
67. 750.
68. $13\frac{888}{1760}$ mi.
69. \$24, \$40, \$48.
70. 45 A.
71. 9.
72. 6 da.
73. \$106.64.
74. 46.58+ ft.
75. 2.3535+.
76. $1\frac{1}{5}$ da.
77. 3 hr. 30 min.
78. 4%.
79. 1,456,700 bbl.
80. 2 hr. 24$\frac{4}{9}$ min.
81. 13,600 lb.
82. 45 mi.
83. 11.175 − mi.
84. \$85.50.
85. \$10.15.
86. $1\frac{1}{2}$.
87. 336 bu.
88. +174, −23, +7.
89. 2.
90. 283,104.
91. \$1700.
92. 28,032,232.8 A.
93. 1,860,012.
94. 5 wk.
95. 8426.88 gal.
96. $18\frac{9}{37}$%.
97. \$1.49.
98. 268,800.
99. 4.
100. 10 min.

101. 8.085 bbl., 26.565 bbl.
102. 34 cu. ft., 17 cu. ft., 102 cu. ft.
103. $10\frac{1}{8}$ bbl., $6\frac{21}{44}$ cu. yd.
104. 191,666$\frac{2}{3}$.
105. 750 yd., 16$\frac{2}{3}$ min.
106. \$1.25, \$1.60, \$2.20.
107. 10,800 ft.
108. $\frac{1}{2}$ hr., $1\frac{1}{2}$ mi. from first place.
109. \$17.90.
110. $\frac{4}{15}$ qt., $\frac{7}{30}$ qt.; therefore $\frac{56}{30}$ qt., or $1\frac{13}{15}$ qt.
111. 66 steps.
112. 5077$\frac{4}{5}$ T.
113. 729, $162\frac{162}{343}$ qt.
114. 1,608,921.6 T.
115. \$161, \$184.
116. \$42, \$48, \$60.
117. \$6.
118. 51 ft., 11, 7, 9.
119. 1 hr. 5 min. $27\frac{3}{11}$ sec.
120. 4 hr. 21 min. $49\frac{1}{11}$ sec., 7 hr. 38 min. $10\frac{10}{11}$ sec.

121. 2 hr. 27 min. $16\frac{4}{11}$ sec., 3 hr.
122. 5 hr. 10 min. $54\frac{6}{11}$ sec., 5 hr. 43 min. $38\frac{2}{11}$ sec., 5 hr. 27 min. $16\frac{4}{11}$ sec.
123. 9 hr. 5 min. $27\frac{3}{11}$ sec., 9 hr. 27 min. $16\frac{4}{11}$ sec.
124. 17 min. 30 sec. P.M.
125. The year 2878.
126. 1,418,000 bales.
127. $53\frac{13}{23}$%, $14\frac{146}{161}$%, $18\frac{150}{161}$%.
128. \$187.50, \$42.75.
129. \$148.
130. \$150, \$11.50.
131. \$1.228+, \$1.20, \$1.04, \$1.10, \$1.01, \$1.20, \$1.12, \$1.05.
132. 120 sec.
133. \$2650.
134. 28 mi.
135. $1\frac{1}{2}$ hr., 66 min.
136. $163\frac{7}{11}$ slates.
137. $2\frac{17}{54}$ lb., \$1.23+, \$42.120.